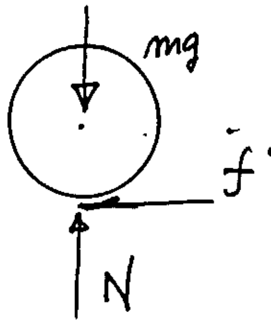
Oppgave 1

a) Arbeidsetning: $K_1 + U_1 = K_2 + U_2 - W_{12}^{nc}$



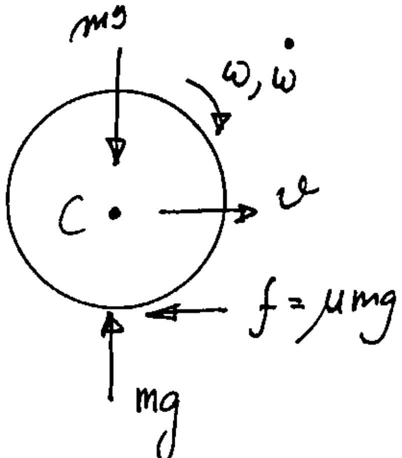
$N = mg$

$f = \mu N$

$$0 + \frac{1}{2}kb^2 = \frac{1}{2}mv_1^2 + 0 - (-\mu mgb)$$

$$\Rightarrow v_1 = \underline{\underline{\sqrt{\frac{k}{m}b^2 - 2\mu gb}}}$$

b)



Newtons lov: $-\mu mg = m\dot{v}$

$$\rightarrow \dot{v} = -\mu g$$

$$v = -\mu gt + C_1$$

$v = v_1$ for $t = 0$ gir $C_1 = v_1$ ⊃:

$$v = v_1 - \mu gt \quad (1)$$

Spinnsats om C: $\mu mg \cdot r = I_C \dot{\omega}$

$$\mu mgr = \frac{2}{5}mr^2\dot{\omega} \Rightarrow \dot{\omega} = \frac{5}{2}\frac{\mu g}{r}$$

$\omega = \frac{5}{2}\frac{\mu g}{r}t + C_2$. $\omega = 0$ for $t = 0$ gir $C_2 = 0$

⊃: $\omega = \frac{5}{2}\frac{\mu g}{r}t \quad (2)$

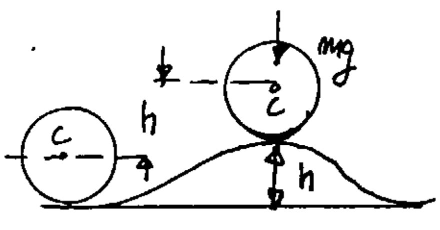
Ved ren rulling: $v_2 = \omega_2 r$.

(1) og (2) gir da: $v_2 = v_1 - \mu g t_2$ og $\omega_2 = \frac{5}{2}\frac{\mu g}{r}t_2$

Som gir $v_2 = \frac{5}{2}\mu g t_2$ og $\mu g t_2 = \frac{2}{5}v_2$

Derav: $v_2 = v_1 - \frac{2}{5}v_2 = \underline{\underline{\frac{5}{7}v_1}}$, $\omega_2 = \underline{\underline{\frac{5v_1}{7r}}}$ og $t_2 = \underline{\underline{\frac{2v_1}{7\mu g}}}$

c)

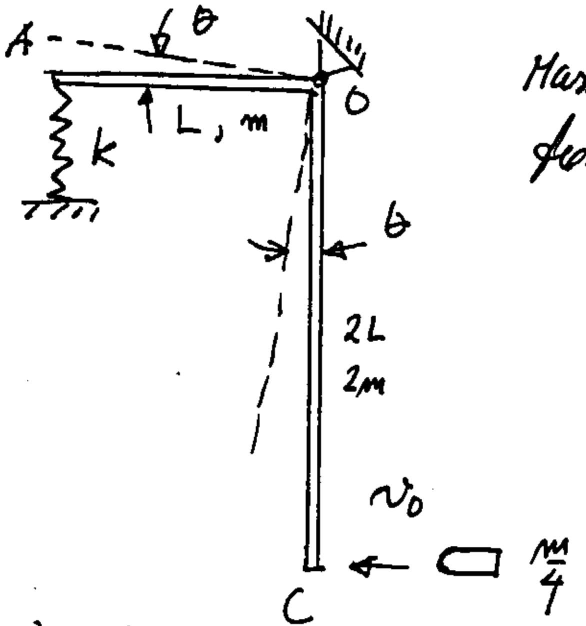


Bevaring av mekanisk energi:

$$mgh = \frac{1}{2}mv_2^2 + \frac{1}{2}I_C\omega_2^2$$

Som gir $h = \underline{\underline{\frac{5v_1^2}{14g}}}$

Oppgave 2



Bevegelse i horisontalplanet

Massetreghetsmoment om O for støtet: $I_0^f = \frac{1}{3}(mL^2 + 2m\,4L^2)$
$= 3mL^2$

a) Vinkelhastighet umiddelbart etter støtet. $= \omega_0$

Bevarelse av spinn om O:

$$\frac{m}{4} v_0 \cdot 2L = 3mL^2\omega_0 + \frac{m}{4}(2L)^2\omega_0$$
$$= 4mL^2\omega_0 = I_{tot}\cdot\omega_0$$

o: $\omega_0 = \frac{v_0}{8L}$

b) Svingeligning for små utslag.

Spinnsats om O: $\sum \tau_0 = I_{tot}\cdot\ddot{\theta}$

$$-L\theta\cdot k\cdot L = 4mL^2\ddot{\theta}$$

$\ddot{\theta} + \frac{k}{4m}\theta = 0$, $\omega_e = \frac{1}{2}\sqrt{\frac{k}{m}}$

c) Løsning av ligning: $\theta = A\sin\omega_e t + B\cos\omega_e t$

$\theta = 0$ for $t = 0$ og $\dot{\theta} = \omega_0$ for $\theta = 0$.

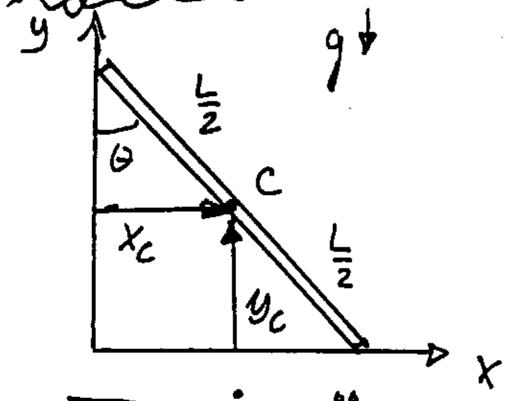
Gir $B = 0$ og $\dot{\theta} = A\omega_e\cos\omega_e t \Rightarrow \omega_0 = A\omega_e$

o: $A = \frac{\omega_0}{\omega_e}$ o: $\theta = \frac{\omega_0}{\omega_e}\sin\omega_e t$

$\theta_{maks} = \theta_0 = \frac{\omega_0}{\omega_e}$. $F_{maks} = Lk\theta_{maks} = Lk\frac{\omega_0}{\omega_e}$

$$= \frac{v_0}{4}\sqrt{km}$$

Oppgave 3



$$x_c = \frac{L}{2}\sin\theta, \quad \dot{x}_c = \frac{L}{2}\cos\theta\cdot\dot\theta$$

$$y_c = \frac{L}{2}\cos\theta, \quad \dot{y}_c = -\frac{L}{2}\sin\theta\cdot\dot\theta$$

$$\ddot{y}_c = -\frac{L}{2}\left(\sin\theta\cdot\ddot\theta + \cos\theta\cdot\dot\theta^2\right)$$

$$v_c^2 = \dot{x}_c^2 + \dot{y}_c^2 = \frac{\dot\theta^2 L^2}{4}$$

a) Finn $\dot\theta$ og $\ddot\theta$

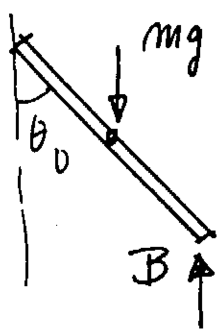
$$E_p + E_k = \text{konst} ; \quad E_k = \frac{1}{2}mv_c^2 + \frac{1}{2}I_C\dot\theta^2$$

$$= \frac{1}{2}m\dot\theta^2\cdot\frac{L^2}{4} + \frac{1}{2}\cdot\frac{1}{12}mL^2\dot\theta^2 = \underline{\frac{1}{6}m\dot\theta^2L^2}$$

$$0 + mg\frac{L}{2} = \frac{1}{6}m\dot\theta^2L^2 + \frac{mgL}{2}\cos\theta$$

$$\underline{\underline{\dot\theta^2 = \frac{3g}{L}(1-\cos\theta)}}, \quad \underline{\underline{\ddot\theta = \frac{3g}{2L}\sin\theta}}$$

b)



Newtons lov: $B - mg = m\ddot{y}_c$

$B = 0 \rightarrow \ddot{y}_c = -g$ eller

$$-\frac{L}{2}\left(\sin\theta_0\,\ddot\theta_0 + \cos\theta_0\,\dot\theta_0^2\right) = -g$$

Innsatt for $\ddot\theta_0$ og $\dot\theta_0^2$:

$$-\frac{3g}{2}\left[\frac{\sin^2\theta_0}{2} + \cos\theta_0 - \cos^2\theta_0\right] = -g \quad \text{eller}$$

$$9\cos^2\theta_0 - 6\cos\theta_0 + 1 = 0 \quad \text{med løsning:}$$

$$\cos\theta_0 = \frac{6\pm\sqrt{36-36}}{18} = \frac{1}{3} \quad \text{d:} \quad \underline{\underline{\cos\theta_0 = \frac{1}{3}}}, \quad \theta_0 \simeq \underline{\underline{70.53^\circ}}$$

c) $\theta = 90^\circ$.

$$B - mg = m\ddot{y}_c \Rightarrow B = m(g + \ddot{y}_c)$$

$$\ddot{y}_c\big|_{\theta=90} = -\frac{3g}{4} \Rightarrow B = \underline{\underline{\frac{mg}{4}}}$$