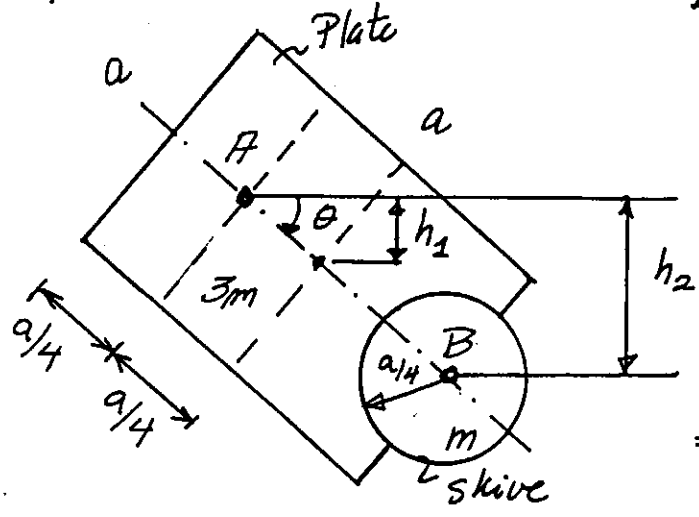
Oppgave 1



$$I_A^{Plate} = \frac{1}{12}(a^2+a^2)\cdot 3m + 3m\left(\frac{a}{4}\right)^2 = \frac{11}{16}ma^2$$

Skiva har bare translasjonsenergi.

$$U_\theta = -mgh_2 - 3mgh_1 = -mg(h_2+3h_2) = -\frac{3}{2}mga\sin\theta.$$

a) $K_0 + U_0 = K_\theta + U_\theta \Rightarrow$

$$0+0 = \frac{1}{2}mv_B^2 + \frac{1}{2}I_A^{Plate}\cdot\dot\theta^2 + U_\theta$$

$$= \frac{1}{2}m\left(\frac{3a}{4}\dot\theta\right)^2 + \frac{1}{2}\cdot\frac{11}{16}ma^2\dot\theta^2 - \frac{3}{2}mga\cdot\sin\theta$$

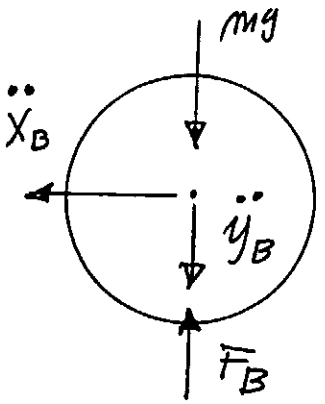
$$\Rightarrow \frac{5}{8}ma^2\cdot\dot\theta^2 = \frac{3}{2}mga\sin\theta \text{ som gir: } \dot\theta^2 = \underline{\underline{\frac{12}{5}\frac{g}{a}\sin\theta}}$$

Ved derivasjon: $\ddot\theta = \underline{\underline{\frac{6}{5}\frac{g}{a}\cos\theta}}$

b) $\theta = 0$.

$\ddot x_B = 0$ da $\dot\theta(0) = 0$

$\ddot y_B = \frac{6}{5}\frac{g}{a}\cdot\frac{3}{4}a = \frac{9}{10}g$



Ingen krefter i x-retn.

Newtons lov i y-retning:

$$-F_B + mg = m\ddot y_B = \frac{9}{10}mg$$

$$F_B = \underline{\underline{\frac{mg}{10}}}$$

Oppgave 1b forts

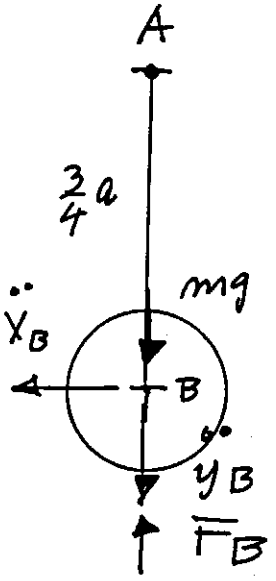
$\theta = 90^\circ$

$\ddot{y}_B = -\frac{3}{4}a \cdot \frac{12}{5}\frac{g}{a}$

$= -\frac{9}{5}g$

$F_B - mg = -m\ddot{y}_B = \frac{9}{5}mg$

$F_B = \underline{\underline{\frac{14}{5}mg}}$

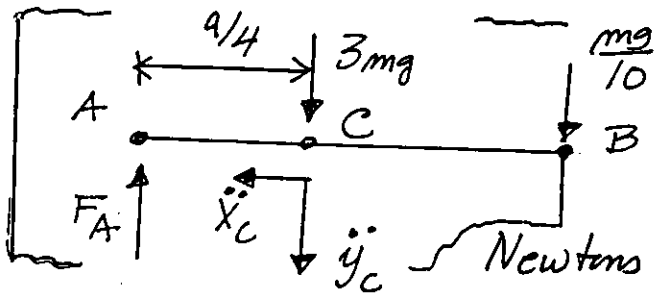


$\dot{\theta}^2 = \frac{12}{5}\frac{g}{a}$

$\ddot{\theta} = 0 \Rightarrow \ddot{x}_B = 0$

∑: ingen krefter i x-retning.

c) Plate. $\theta = 0$



$\dot{\theta} = 0 \rightarrow \ddot{x}_C = 0$

∑: Ingen krefter i x-retning.

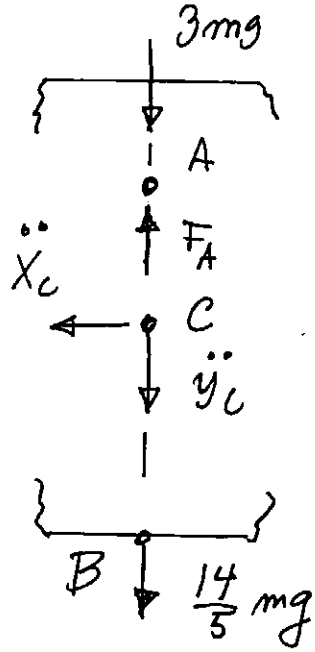
$\ddot{y}_C = \frac{6}{5}\frac{g}{a} \cdot \frac{a}{4} = \frac{3}{10}g$

Newtons lov:

$-F_A + 3mg + \frac{mg}{10} = 3m\ddot{y}_C = \frac{9}{10}mg$

$\rightarrow \underline{\underline{F_A = \frac{11}{5}mg}}$

$\theta = 90^\circ$



$\ddot{\theta} = 0 \rightarrow \ddot{x}_C = 0$ ∑: Ingen krefter i x-retning.

$\dot{\theta}^2 = \frac{12}{5}\frac{g}{a}$

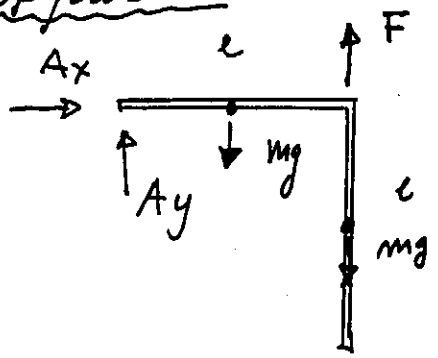
$\ddot{y}_C = -\frac{12}{5}\frac{g}{a} \cdot \frac{a}{4} = -\frac{3}{5}g$

$F_A - 3mg - \frac{14}{5}mg = -3m\ddot{y}_C = \frac{9}{5}mg$

$\underline{\underline{F_A = \frac{38}{5}mg}}$

Oppgave 2

a)



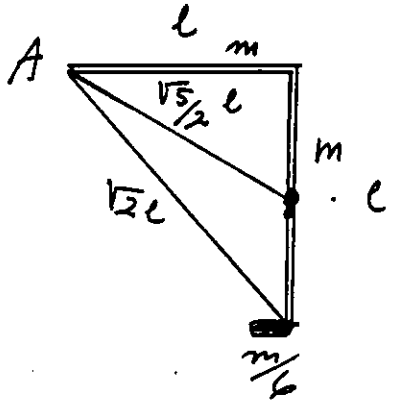
$\Sigma \tau_A = 0$:

$$F \cdot l - mg\frac{l}{2} - mg \cdot l = 0$$

$$F = \frac{3mg}{2}$$

$$F = k \cdot \delta_1 \rightarrow \delta_1 = \underline{\underline{\frac{3mg}{2k}}}$$

b)



Ingen ytre impulsmoment om A:

$$L_A^{for} = L_A^{etter} \rightarrow$$

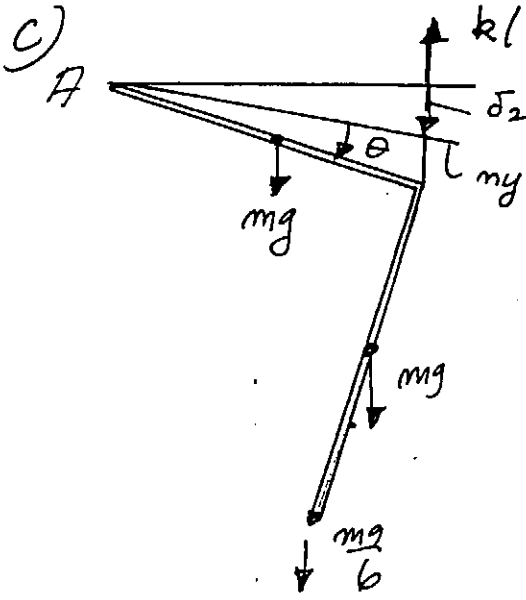
$$\frac{m}{6} v_0 \cdot l = I_A \cdot \omega_0$$

$$I_A = \frac{1}{3}ml^2 + \frac{1}{12}ml^2 + \frac{5}{4}ml^2 + \frac{m}{6} \cdot 2l^2$$

$$= 2ml^2$$

$$\frac{m}{6} v_0 l = 2ml^2 \omega_0 \rightarrow \underline{\underline{\omega_0 = \frac{1}{12}\frac{v_0}{l}}}$$

c)



Antar små utslag.

$$\Sigma \tau_A = I_A \cdot \ddot{\theta}$$

$$mg\frac{l}{2} + \left(mg + \frac{mg}{6}\right)l - k(\delta_2 + l\theta) \cdot l = I_A \ddot{\theta}$$

$$-\frac{5}{3}mgl - k(\delta_2 + l\theta) \cdot l = 2ml^2\ddot{\theta}$$

Likevekt: $\theta = 0$ og $\ddot{\theta} = 0 \Rightarrow$

$$\frac{5mg}{3} - k\delta_2 = 0 \rightarrow$$

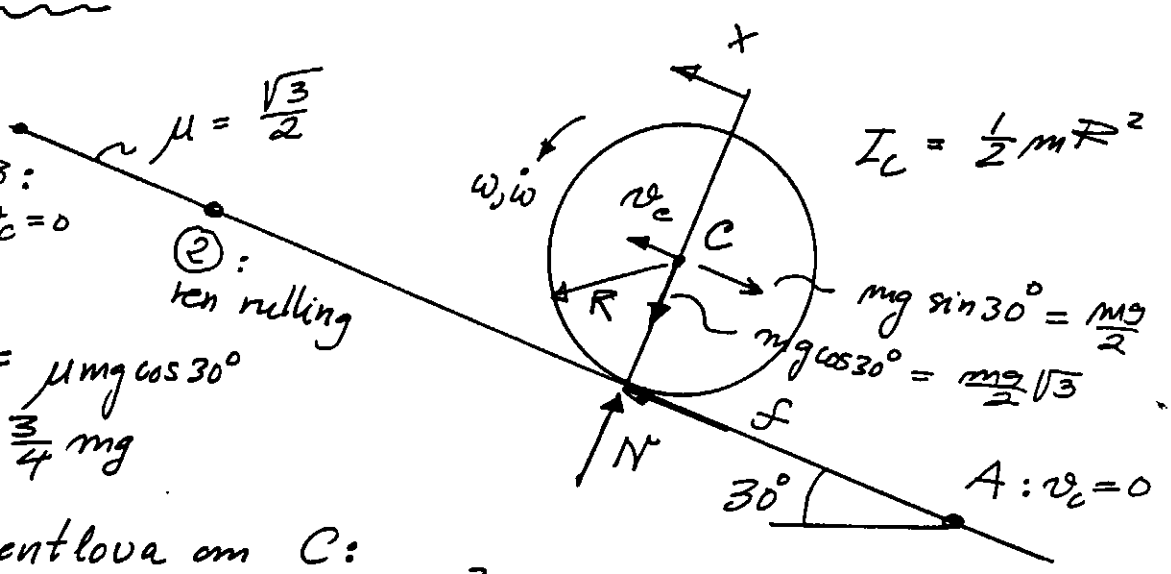
$$-kl^2\theta = 2ml^2\ddot{\theta} \rightarrow$$

$$\ddot{\theta} + \frac{k}{2m}\theta = 0$$

Egenfrekvensen $\underline{\underline{\omega_e = \sqrt{\frac{k}{2m}}}}$

Når vi regner fra likevektstillingen, holder tyngdekraften og forspenningskraften hverandre i likevekt. og får ingen innflytelse på egenfrekvensen.

Oppgave 3



$f = \mu N = \mu mg\cos 30^\circ = \frac{3}{4}mg$

a) Momentlova om C:

$$-fR = I_C\dot{\omega},\quad \dot{\omega} = -\frac{\frac{3}{4}mgR}{\frac{1}{2}mR^2} = -\frac{3g}{2R} \rightarrow$$

$\omega = \omega_0 - \frac{3g}{2R}t$ da $\omega = \omega_0$ for $t = 0$.

Newtons lov: $f - mg\sin 30^\circ = m\ddot{x}_C$

$$\frac{3}{4}mg - \frac{mg}{2} = m\ddot{x}_C \Rightarrow \ddot{x}_C = \frac{g}{4} \rightarrow$$

$v_C = \frac{g}{4}t$ da $v_C = 0$ for $t = 0$

Ren rulling i pkt. 2: $v_{C2} = \omega_2 R$ ved tiden $t = t_2$

$$(\omega_0 - \frac{3g}{2R}t_2)R = \frac{g}{4}t_2 \rightarrow t_2 = \frac{4\omega_0 R}{7g}$$

$$\Rightarrow v_{C2} = \frac{g}{4}t_2 = \underline{\underline{\frac{\omega_0 R}{7}}}$$

b) Mellom ② og B har vi ren rulling.

Newtons lov: $f - \frac{mg}{2} = m\dot{v}_C$

Momentlova om C: $-fR = I_C\dot{\omega} = I_C\frac{\dot{v}_C}{R}$ ved ren rulling.

$$\Rightarrow f = -\frac{1}{2}m\dot{v}_C$$

$$-\frac{1}{2}m\dot{v}_C - \frac{mg}{2} = m\dot{v}_C \rightarrow \dot{v}_C = -\frac{g}{3}$$

$v_C = v_{C2} - \frac{g}{3}t$, t regnes fra 0 i pkt. 2

$v_C = 0$ i B: $\frac{g}{3}t_B = v_{C2} = \frac{\omega_0 R}{7} \rightarrow t_B = \frac{3\omega_0 R}{7g}$

Totalt fra A–B: $t_{AB} = \frac{4\omega_0 R}{7g} + \frac{3\omega_0 R}{7g} = \frac{\omega_0 R}{g}$