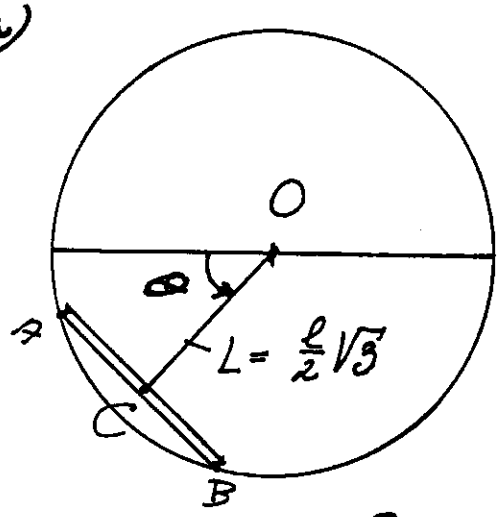
Løsning eksamen i fag 61105 MEKANIKK for fak. 7

Fredag 8. aug. – 1997

Oppgave 1

a)



Starter fra rc med $\theta = 0$

Ren rotasjon om O.

Energibevarelse:

$$K_\theta + U_\theta = K_0 + U_0 = 0$$

$$U_\theta = -mgL\sin\theta = -mg\frac{l}{2}\sqrt{3}\sin\theta$$

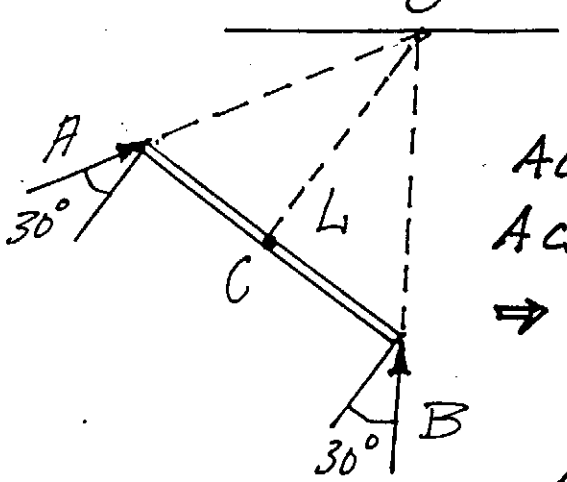
$$K_\theta = \frac{1}{2}I_0\dot{\theta}^2 = \frac{1}{2}\left(\frac{1}{12}ml^2 + mL^2\right)\dot{\theta}^2 = \frac{5}{12}ml^2\dot{\theta}^2$$

Får da: $mg\frac{l}{2}\sqrt{3}\sin\theta = \frac{5}{12}ml^2\dot{\theta}^2$ som gir

$$\dot{\theta}^2 = \frac{6}{5}\sqrt{3}\frac{g}{l}\sin\theta$$

Ved derivasjon: $\ddot{\theta} = \frac{3\sqrt{3}}{5}\frac{g}{l}\cos\theta$

b) $\theta = 60^\circ \rightarrow \dot{\theta}^2 = \frac{9}{5}\frac{g}{l}$, $\ddot{\theta} = \frac{3\sqrt{3}}{10}\frac{g}{l}$



Newtons lov loddrett og parallelt med stanga:

$$A\cos 30^\circ - mg\sin 60^\circ + B\cos 30^\circ = mL\dot{\theta}^2$$

$$A\cos 60^\circ + mg\cos 60^\circ - B\cos 60^\circ = mL\ddot{\theta}$$

$$\Rightarrow (A+B)\cdot\frac{1}{2}\sqrt{3} = \frac{7}{5}\sqrt{3}mg$$

$$(A-B)\cdot\frac{1}{2} = -\frac{mg}{20} \rightarrow$$

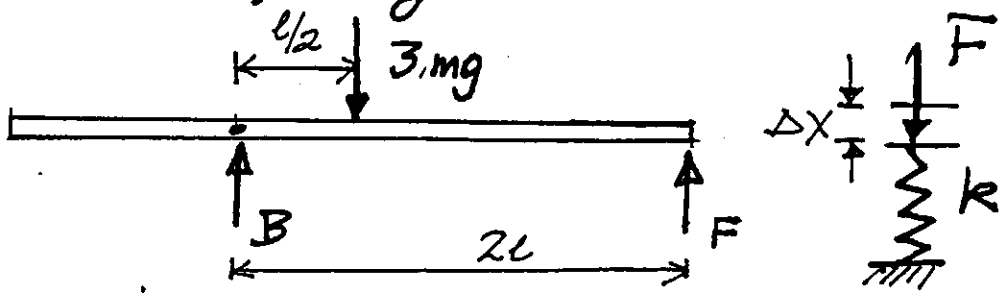
$$A + B = \frac{14}{5}mg$$

$$A - B = -\frac{mg}{10} \Rightarrow A = \frac{27}{20}mg$$

$$B = \frac{29}{20}mg$$

Oppgave 2

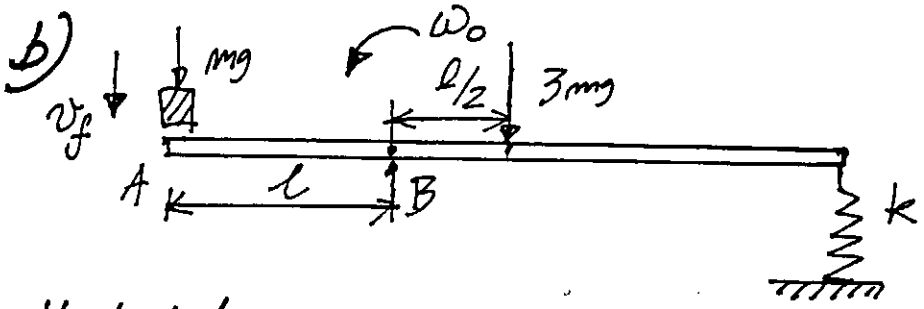
a) Sammentrykning Δx



$$\Sigma \tau_B = 0: \quad 3mg\frac{l}{2} - F \cdot 2l = 0 \rightarrow F = \frac{3mg}{4}$$

$$F = k \cdot \Delta x \rightarrow \Delta x = \frac{3mg}{4k}$$

b)



Hastighet av masse m rett før støtet = $v_f = \sqrt{2gh}$

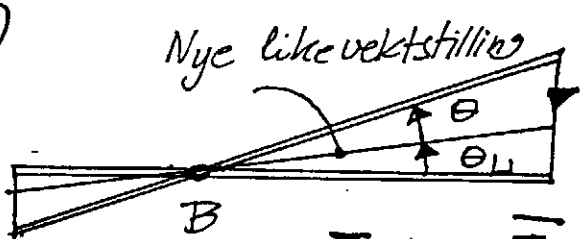
Bevarelse av spinn om B:

$$l m v_f = l m (l\omega_0) + I_B \omega_0 \rightarrow \omega_0 = \frac{l m v_f}{I_B + l^2 m}$$

$$I_B = \frac{1}{12} 3m (3l)^2 + 3m\left(\frac{l}{2}\right)^2 = 3ml^2$$

$$\rightarrow \omega_0 = \frac{\sqrt{2gh}}{4l}$$

c)



Regner utslag fra den nye likevektstillingen. Tyngden og forspenningen holder hverandre i likevekt her.

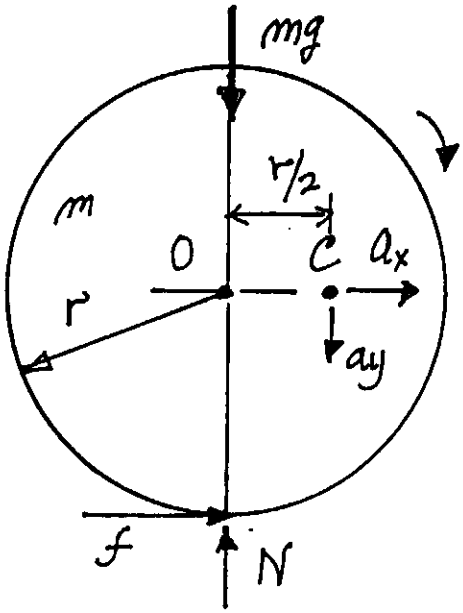
$$\Sigma \tau_B = I_B \ddot{\theta} = 4ml^2\ddot{\theta}$$

$$F = -k\theta \cdot 2l \Rightarrow \Sigma \tau_B = -4kl^2\theta$$

$$\Rightarrow -4kl^2\theta = 4ml^2\ddot{\theta} \rightarrow \ddot{\theta} + \frac{k}{m}\theta = 0$$

Derav: $\omega_e = \sqrt{\frac{k}{m}}$

Oppgave 3



$I_C = \frac{1}{2}mr^2$

a) I det bevegelsen starter er hastighetene $\equiv 0$
Pkt. O har bare horisontal akselerasjon.
Derav: $a_x = \underline{\dot{\omega} r}$, $a_y = \underline{\frac{\dot{\omega} r}{2}}$

b) Spinnsats om C: $-f \cdot r + N \cdot \frac{r}{2} = I_C \dot{\omega}$ (1)

Newtons lov horisontalt: $f = ma_x = m\dot{\omega} r$
$\rightarrow f \cdot r = \dot{\omega} r^2 m$ (2)

Newtons lov vertikalt: $mg - N = ma_y = m\dot{\omega}\frac{r}{2}$
$\rightarrow \frac{N \cdot r}{2} = mg\frac{r}{2} - \frac{\dot{\omega} r^2}{4} m$ (3)

(2) og (3) innsatt i (1):
$-\dot{\omega} r^2 m + mg\frac{r}{2} - \frac{\dot{\omega} r^2}{4} m = I_C \dot{\omega} = \frac{1}{2} m r^2 \dot{\omega}$
$\rightarrow \dot{\omega} = \underline{\frac{2g}{7r}}$

c) $f = \dot{\omega} r m = \underline{\frac{2mg}{7}}$
$N = mg - \frac{\dot{\omega} r}{2} m = \frac{6}{7} mg.$
$f \leq f_{maks} = \mu N \rightarrow \frac{2mg}{7} \leq \mu \frac{6}{7} mg \rightarrow \underline{\mu \geq \frac{1}{3}}$