L1

a) $\frac{dM}{dt} = -\alpha \Rightarrow M(t) = M_0 - \alpha t \quad (>0).$

$M(t)\frac{dV}{dt} = F$ (Ingen reaksjonskraft da sanden forlater vognen uten hast. i forh. til vognen).

$$dV = \frac{F}{M}\,dt = \frac{F\,dt}{M_0 - \alpha t}$$

$$V = \int \frac{F\,dt}{M_0 - \alpha t} = -\frac{F}{\alpha}\ln(M_0 - \alpha t) + C$$

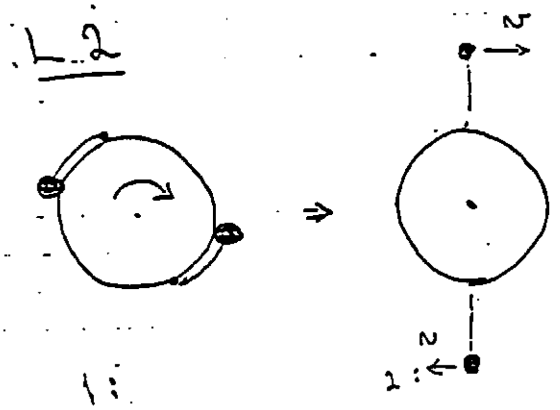
$$V(0) = 0 \Rightarrow C = \frac{F}{\alpha}\ln M_0 ,$$

$$\underline{\underline{V(t) = \frac{F}{\alpha}\ln\left(\frac{M_0}{M_0 - \alpha t}\right)}}$$

b) $\ln\frac{M_0}{M_0 - \alpha t} = -\ln\Big(1 - \underbrace{\frac{\alpha}{M_0}t}_{\ll 1}\Big) \approx \frac{\alpha}{M_0}t$

$V(t) \approx \frac{F}{\alpha}\frac{\alpha}{M_0}t = \underline{\frac{F}{M_0}t}$ som ventet.

L2



1: $L_1 = (2mR^2 + I)\,\omega = (2m + M/2)R^2\omega$.

2: $L_2 = 2m(R+l)\,v$

Ingen ytre momenter $\Rightarrow$ $L_1 = L_2$

$$(2m + M/2)R^2\omega = 2m(R+l)v \qquad (1).$$

Energibevarelse

$$\frac{1}{2}\left(2mR^2 + \frac{1}{2}MR^2\right)\omega^2 = 2\cdot\frac{1}{2}mv^2$$

$$(2m + M/2)R^2\omega^2 = 2mv^2 \qquad (2).$$

Løser (1) m.h.p. v og setter inn i (2) $\Rightarrow$.

$$(2m + M/2)R^2\omega^2 = 2m\,\frac{(2m+M/2)^2R^4\omega^2}{4m^2(R+l)^2}$$

$$\cancel{(2m+M/2)}\,\cancel{R^2}\,\cancel{\omega^2}\cdot 4m^{\cancel{2}}(R+l)^2 = 2\cancel{m}(2m+M/2)^{\cancel{2}}R^{\cancel{4}\,2}\cancel{\omega^2}$$

$$2m(R+l)^2 = (2m + M/2)R^2$$

$$2m\left[(R+l)^2 - R^2\right] = \frac{1}{2}MR^2$$

$$m = \frac{M}{4}\,\frac{R^2}{l\,[2R+l]} = \underline{\underline{\frac{M}{4}\,\frac{R^2/l^2}{1+2R/l}}}$$

$$R = l \Rightarrow \quad m = \frac{M}{4}\cdot\frac{1}{3} = \underline{\frac{M}{12}}$$