Exam 2023

1 Potential Gradient (Chapter 23)

In a three-dimensional space, the potential function is described as:

$V = 2 + 3x + 5y^3 + 7xy$.

Select the correct statements about the electric field $\vec{E}$ and its components in this region.

$E_x = -\left(\frac{\partial V}{\partial x}\right) = -(3 + 7y)$ $\quad E_y = -\left(\frac{\partial V}{\partial y}\right) = -(15y^2 + 7x)$ $\quad E_z = 0$

Correct statements:

1. The z component of the electric field is zero.

2. $E_y = -(15y^2 + 7x)$

2 Electric Forces Composition (Chapter 21).

A positive test charge, q, is released from rest at a distance r away from a charge +Q and at a distance 2r from a charge +2Q. For our purposes, the charge Q is bigger than the charge q and we can refer to the system as two-dimensional. The picture below describes the system of these three charges.

After being released, in which direction will the charge move?

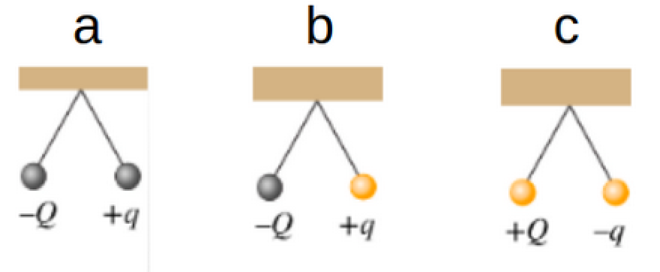
$|F_{Q;q}| = \frac{qQ}{r^2}$ $|F_{2Q;q}| = \frac{2qQ}{(2r)^2} = \frac{qQ}{2r^2}$ → $|F_{Q;q}| > |F_{2Q;q}|$ → The repulsive force of the charge Q is greater than the charge +2Q. The charge q will move to the right.

Let's refer to $\boldsymbol{F_{q,+2Q}}$ and $\boldsymbol{F_{+2Q,q}}$ as the magnitude of the force acting by the charge q on the charge +2Q and the magnitude of the force acting by the charge +2Q on the charge q, respectively. What is the relationship between the magnitude of these forces?

$$|F_{q,2Q}| = |F_{2Q,q}| \neq 0$$

3. Conductors and insulators (Chapter 21)

Consider the three scenarios in the picture below.

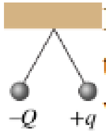


The black color indicates a conductive sphere, while the yellow color indicates an insulating material.

These spheres are charged with charges q and Q, where the magnitude of the charge Q is greater than the magnitude of the charge q. In the given scenarios, the spheres are allowed to swing freely and eventually come at rest in an equilibrium position. Determine whether the spheres in these scenarios are in contact with each other at rest or if they are separated from each other when at rest.

Correct statements:

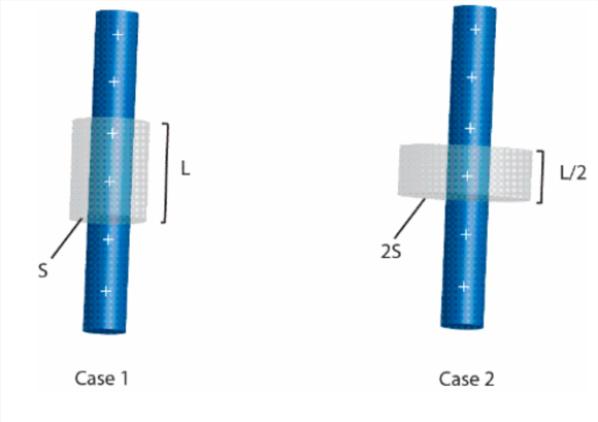
The spheres in the scenarios b and c re in contact at rest, while the spheres are separated at rest in the scenario a.



In this scenario, the two conducting spheres initially attract each other. When the spheres come into contact, the total charge is distributed equally between them, leaving each with a charge of $\frac{(q-Q)}{2}$. This again leaves the spheres with like charges, so they end up repelling each other and come to rest separated by some distance.

4. Electric flux (Chapter 22)

Consider the picture below. The two cases feature an infinitely long charged rod with a uniform linear charge density, λ, and a cylinder. In case 2, the cylinder is characterized by a radius r_2= 2S and a side length l_1 =L/2, while in case 1, the cylinder has radius r_1= S and side length l_1 =L. Compare the flux magnitudes, ϕ_1, out of the cylinder of case 1, and ϕ_2, out of the cylinder in case 2.



$\phi_1 = 2\phi_2$ The exercise can be solved with Gauss or calculating the flux.

$\phi_1 = \oint_{C_1} E(S)\,dA = E(S)\int dA = E(S)\,2\pi SL$

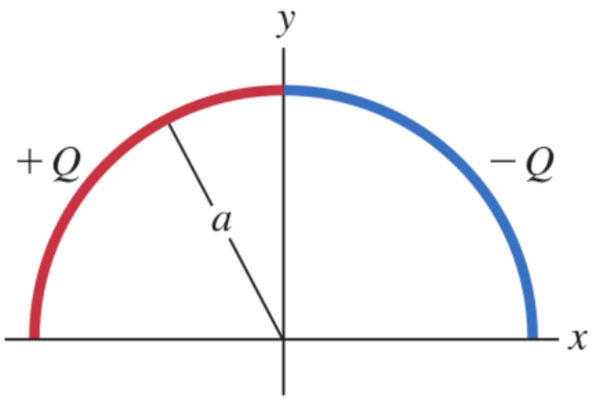
$\phi_2 = \oint_{C_2} E(2S)\,dA = E(2S)\int dA = E(2S)\,2\pi SL$

$E(2S) = \frac{\lambda}{2\pi\epsilon_0 2S}$

$E(S) = \frac{\lambda}{2\pi\epsilon_0 S}$

$\phi_1 = 2\phi_2$

5. Electric field calculation (Chapter 21)



A semicircle of radius a lies in the first and second quadrants with the center of curvature at the origin. Positive charge, +Q, is uniformly distributed on the left half of the semicircle, while a negative charge, -Q, is distributed on the right half of the semicircle.

(a)What is the direction of the total electric field vector produced by this charge distribution at the origin of the coordinate axes?

The electric field direction is +x. This can be observed the sign of the distributions and the relative direction of the Electric field.

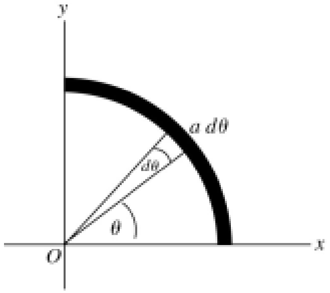
(b)What is the magnitude of the total electric field produced by this charge distribution at the origin of the coordinate axes?

$$E = \frac{Q}{\pi^2 \epsilon_0 a^2}$$

(b) Let's start considering that the right half semicircle is negatively charged, than the electric field vector will be pointing to the half semicircle.

$$dE=\frac{1}{4\pi\epsilon_0}\frac{|dQ|}{a^2}$$

$$\lambda=\frac{Q}{L}=\frac{2Q}{\pi a}=\frac{dQ}{dl}$$



$$dQ=\frac{2Q}{\pi a}dl=\frac{2Q}{\pi a}a\,d\theta$$

$$dE=\frac{1}{4\pi\epsilon_0}\cdot\frac{2Q\,d\theta}{a^2\pi}=\frac{Q\,d\theta}{2\pi^2\epsilon_0 a^2}$$

From construction

$$\vec{dE}=dE_x\hat{i}+dE_y\hat{j}$$

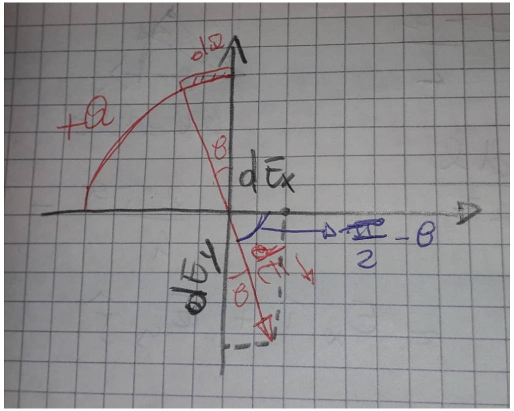
$$dE_x=dE\cos\theta \qquad dE_y=dE\sin\theta$$

$$E_x=\int_0^{\pi/2} dE\cos\theta=\frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\cos\theta d\theta=\frac{Q}{2\pi^2\epsilon_0 a^2}[\sin\theta]_0^{\pi/2}=\frac{Q}{2\pi^2\epsilon_0 a^2}$$

$$E_y=\int_0^{\pi/2} dE\sin\theta=\frac{Q}{2\pi^2\epsilon_0 a^2}\int_0^{\pi/2}\sin\theta d\theta=\frac{Q}{2\pi^2\epsilon_0 a^2}[-\cos\theta]_0^{\pi/2}=\frac{Q}{2\pi^2\epsilon_0 a^2}$$

$$\vec{E}=\frac{Q}{2\pi^2\epsilon_0 a^2}\hat{i}+\frac{Q}{2\pi^2\epsilon_0 a^2}\hat{j}$$

Let's consider the left half semicircle that is positively charged, meaning the total electric field vector will be pointing away from the half semicircle.



$$dE=\frac{1}{4\pi\epsilon_0}\frac{|dQ|}{a^2}$$

$$\lambda=\frac{Q}{L}=\frac{2Q}{\pi a}=\frac{dQ}{dl} \rightarrow dQ=\frac{2Q}{\pi a}dl=\frac{2Q}{\pi a}a\,d\theta$$

$$dE=\frac{1}{4\pi\epsilon_0}\cdot\frac{2Q\,d\theta}{a^2\pi}=\frac{Q\,d\theta}{2\pi^2\epsilon_0 a^2}$$

From construction now

$$\vec{dE}=dE_x\hat{i}+dE_y\hat{j} \rightarrow dE_x=dE\cos\left(\frac{\pi}{2}-\theta\right)=dE\sin\theta \quad dE_y=dE\sin\left(\frac{\pi}{2}-\theta\right)=dE\cos\theta$$

$$E_x=\int_{\pi/2}^{\pi} dE\sin\theta=\frac{Q}{2\pi^2\epsilon_0 a^2}\int_{\pi/2}^{\pi}\sin\theta d\theta=\frac{Q}{2\pi^2\epsilon_0 a^2}[-\cos\theta]_{\pi/2}^{\pi}=\frac{Q}{2\pi^2\epsilon_0 a^2}$$

$$E_y=\int_{\pi/2}^{\pi} dE\cos\theta=\frac{Q}{2\pi^2\epsilon_0 a^2}\int_{\pi/2}^{\pi}\cos\theta d\theta=\frac{Q}{2\pi^2\epsilon_0 a^2}[\sin\theta]_{\pi/2}^{\pi}=\frac{-Q}{2\pi^2\epsilon_0 a^2}$$

$$\rightarrow \vec{E}=\frac{Q}{2\pi^2\epsilon_0 a^2}\hat{i}-\frac{Q}{2\pi^2\epsilon_0 a^2}\hat{j}$$

The total electric field of the whole semicircle is:

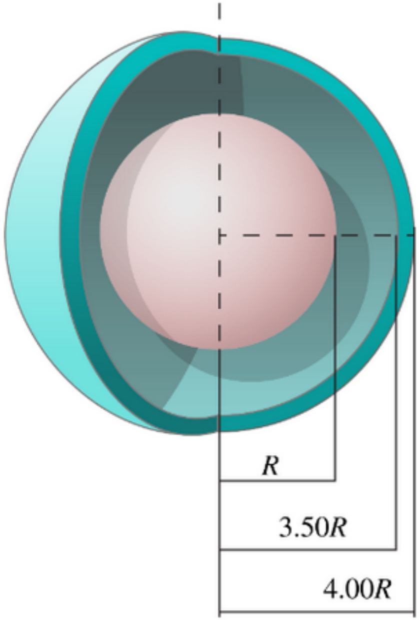
$$\vec{E}_{Total}=\frac{Q}{\pi^2\epsilon_0 a^2}\hat{i}$$

While the module is:

$$E=\frac{Q}{\pi^2\epsilon_0 a^2}$$

6. Gauss’s Law (Chapter 22)

The insulating sphere with a radius of R = 0.340 m in the figure below carries a uniform charge density of 151 nC/m³. The sphere is located at the center of a spherical conducting shell with inner radius R_1 = 3.50R and outer radius R_2 = 4.00R. The conducting shell carries a total charge of Q = -11.9 nC. Calculate the magnitude of the electric field at different radial distances from the center of the charge distribution.



What is the magnitude of the electric field at the point P, which is at a distance d= 0.560 R from the center of the sphere?

P is inside the insulating sphere. Considering Gauss's law:

$$\oint \vec{E}\, d\vec{A} = Q_{inc}/\epsilon_0$$

$$\oint \vec{E}\, d\vec{A} = E\,A = E(4\,\pi(0.560\,R)^2) = \frac{Q_{incl}}{\epsilon_0}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3} = \frac{Q_{incl}}{\frac{4}{3}\pi(0.560\,R)^3} \rightarrow Q_{incl} = \rho \cdot \frac{4}{3}\pi(0.560\,R)^3$$

$$E = \frac{Q_{incl}}{\epsilon_0\, 4\,\pi(0.560\,R)^2} = \frac{(\rho\frac{4}{3}\pi(0.560\,R)^3)}{\epsilon_0\, 4\,\pi(0.560\,R)^2} = 1080\, N/C = 1.08\, kN/C$$

What is the magnitude of the electric field at the point P, which is at a distance d= 3.90 R from the center of the Sphere?

P is outside the insulating sphere and inside the conducting shell. The E inside a conductor is zero.

What is the magnitude of the electric field at the point P, which is at a distance d= 3.40 R from the center of the sphere?

P is between the insulating sphere and the conducting shell. Considering Gauss's law:

$$\oint \vec{E}\, d\vec{A} = E\,A = E(4\,\pi(3.40\,R)^2) = \frac{Q_{incl}}{\epsilon_0}$$

$$Q_{incl} = \rho\frac{4}{3}\pi R^3 = 24.8\, nC$$

$$E = \frac{Q_{incl}}{\epsilon_0\, 4\,\pi(0.340\,R)^2} = \frac{\rho\frac{4}{3}\pi R^3}{\epsilon_0\, 4\,\pi(0.340\,R)^2} = 167\, N/C$$

What is the magnitude of the electric field at the point P, which is at a distance d= 4.30 R from the center of the sphere?

P is outside the conducting shell. We need to sum the charges from the conducting shell and the insulating core. Considering Gauss's law:

$$\oint \vec{E}\, d\vec{A} = E\,A = E\left(4\,\pi(4.40\,R)^2\right) = \frac{Q_{incl}}{\epsilon_0}$$

$$Q_{incl} = \rho\frac{4}{3}\pi R^3 - 11.9\,nC = 24.8\,nC - 11.9\,nC = 12.9\,nC$$

$$E = \frac{Q_{incl}}{\epsilon_0\, 4\,\pi(4.30\,R)^2} = 54.5\,N/C$$

7. Capacitor (Chapter 24)

A parallel plate capacitor consists of two parallel metal plates separated by a distance d. We consider the plates to be infinitely large. Let's assume there is air between the plates. The top plate has a positive charge, while the bottom plate has a negative charge. No voltage source is connected to the plates.

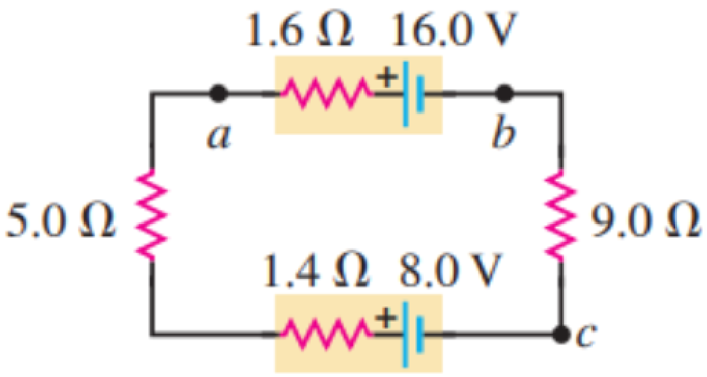
Which physical quantity remains unchanged when we increase the distance d between the plates?

Correct answer: The Electric field between the plates.

The electric field of a parallel capacitor with plates infinitely large does not depend on the distance between the plates!

8. Kirchhoffs' rules (Chapter 26)

In the circuit below there are two batteries and two resistors. Calculate the current passing through the circuit, as well as the potentials V_{ab} and V_{ac}.



What is the current magnitude and direction of the circuit above?

We need to apply the Kirchhoff's loop rule.

$$16V - 1.6\Omega I - 5.0\Omega I - 1.4\Omega I - 8.0V - 9.0\Omega I = 0$$

$$I = \frac{8.0V}{1.6\Omega + 5.0\Omega + 1.4\Omega + 9.0\Omega} = 0.47A$$ The direction is counterclockwise

What is the potential V_{ab} and V_{ac}?

We need to apply the Bookkeping loop rule.

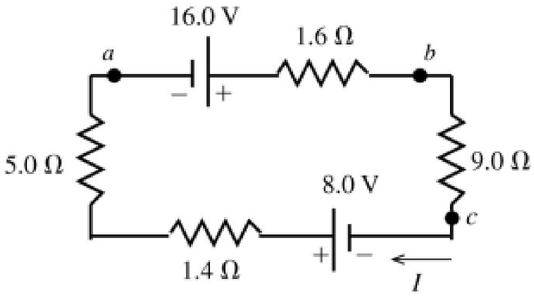
$$V_{ab} = 16V - 1.6\Omega * 0.47A = 15.2V$$

$$V_{ac} = -9.0\Omega \times 0.47A + 16V - 1.6\Omega \times 0.47A = 11.0V$$

Remove the 16 V battery and reinsert it with opposite polarity. In other words, place the negative terminal of the battery closer to the point a.

What is now the current magnitude and potential V_{ab}?

Kirchhoff's loop rule, assume counterclockwise direction.



$$-16\,V - 1.6\,\Omega\, I - 5.0\,\Omega\, I - 1.4\,\Omega\, I - 8.0\,V - 9.0\,\Omega\, I = 0$$

$$I = \frac{-24.0\,V}{1.6\,\Omega + 5.0\,\Omega + 1.4\,\Omega + 9.0\,\Omega} = -1.41\,A$$

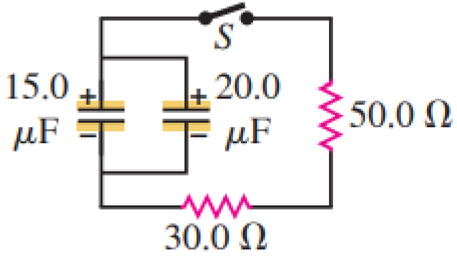
The current is clockwise! I = 1.41 A.

Bookkeping loop rule with I in clockwise direction.

$$V_{ab} = -16\,V + 1.6\,\Omega * 1.41\,A = -13.7\,V$$

9. RC circuit (Chapter 26)

In the following circuit, both capacitors are initially charged at 45.0 V.



How long after closing the switch S will the potential across each capacitor V be reduced to 10.0 V?
Hint: Consider that the potential (V) is proportional to the charge (Q) and follows the same equation as Q.

$$C_{eq} = 35\,\mu F \qquad R_{eq} = 80\,\Omega \qquad C = \frac{Q}{V} \qquad C = \frac{Q_0}{V_0} \rightarrow \frac{Q}{Q_0} = \frac{V}{V_0} \rightarrow \ln\left(\frac{Q}{Q_0}\right) = \ln\left(\frac{V}{V_0}\right) = -1.50$$

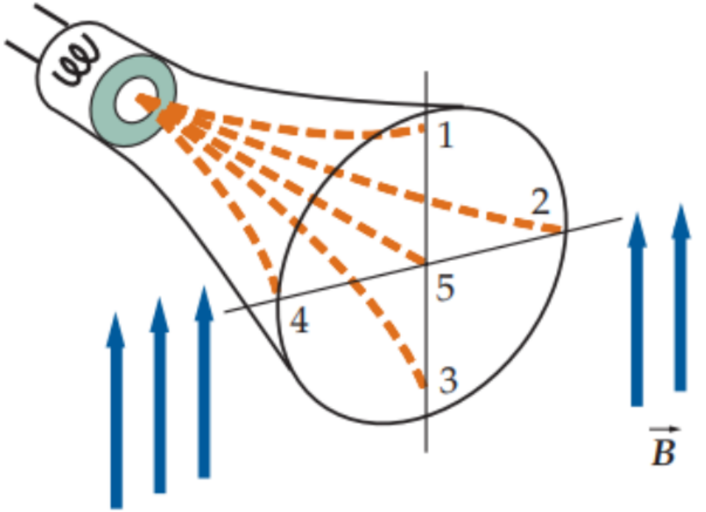
$$Q = Q_0 e^{-t/RC} \rightarrow \ln\left(\frac{Q}{Q_0}\right) = -t/RC \rightarrow t = -R_{eq} C_{eq} \cdot \ln\left(\frac{Q}{Q_0}\right) = 4211\,\mu s = 4.21\,ms$$

What is the current after when the potential is reduced to 10 V?

$$\ln\left(\frac{I}{I_0}\right) = \frac{-t}{R_{eq} C_{eq}} = -1.50 \qquad I_0 = \frac{V_0}{R_{eq}} = \frac{45\,V}{80\,\Omega} \qquad I = I_0 e^{(-1.50)} = 0.126\,A$$

10. Electrons in a magnetic field (Chapter 27)

Consider the cathode-ray tube below. Electrons are emitted from the cathode, and a magnetic field is directed vertically upward with respect to the horizontal axis of the cathode ray tube.



Which one of the dashed path to the face of the tube is followed by the emitted electrons?

Emitted electrons will be deflected and follow the path 2.

11. Magnetic field (Chapter 28)

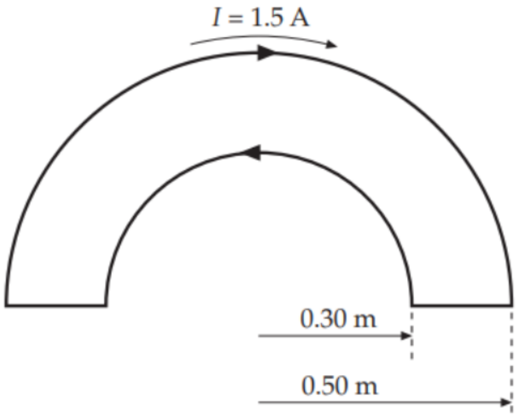
Correct statements:

Inside the material of a bar magnet, the magnetic field due to the bar magnet points from the magnet’s south pole toward its north pole.

If a current loop simultaneously has its current doubled and its area cut in half, then the magnitude of its magnetic moment remains the same.

12. Magnetic moment (Chapter 27)

A wire loop consists of two semicircles connected by straight segments, as in the figure below. The radii of the two semicircles are specified in the picture. A current of 1.5 A flows through the wire in the clockwise direction.



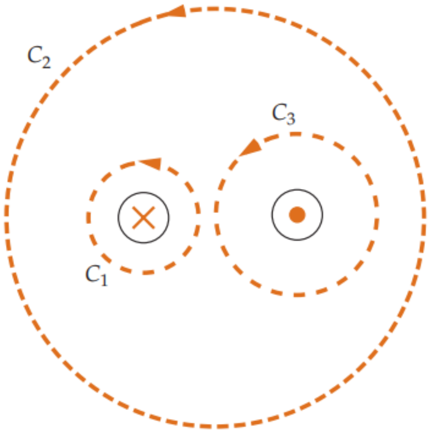
What is the magnetic moment of the current loop?

$\mu = I\,A = 1.5\,A \times Area = 0.38\,m^2$

$Area = Area_{big} - Area_{small} = \frac{\pi}{2}\left[(0.50\,m)^2 - (0.30\,m)^2\right] = 0.25\,m^2$ According to the right hand rule, the direction of the magnetic field is into the page.

13 Ampere's Law (Chapter 28)

Consider the picture below. The current into the page is 8.0 A and the current out of the page is 8.0 A. Furthermore, C_1 C_2 and C_3 are all circular paths. For each path, evaluate the line integral $\oint \vec{B} \cdot \vec{dl}$ in the counterclockwise direction and rank their absolute values from the highest to the lowest.



$$\oint \vec{B}\vec{dl} = \mu_0 I$$

$$\oint_{C_1} \vec{B}\vec{dl} = \mu_0 I = -8.0\,A\,\mu_0 \quad ; \quad \vec{B} \text{ and } \vec{dl} \text{ are antiparallel.}$$

$$\oint_{C_2} \vec{B}\vec{dl} = \mu_0 I = m_0(8.0\,A - 8.0\,A) = 0 \quad \text{; The total current is zero.}$$

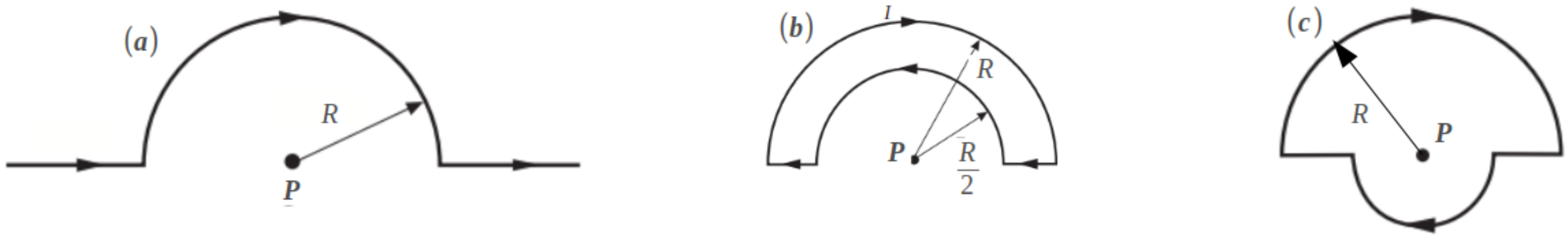
$$\oint_{C_3} \vec{B}\vec{dl} = \mu_0 I = 8.0\,A\,\mu_0$$

The correct answer is:

$$\left|\oint_{C_3} \vec{B}\vec{dl}\right| = \left|\oint_{C_1} \vec{B}\vec{dl}\right| > \left|\oint_{C_2} \vec{B}\vec{dl}\right|$$

14. Biot and Savart (Chapter 28)

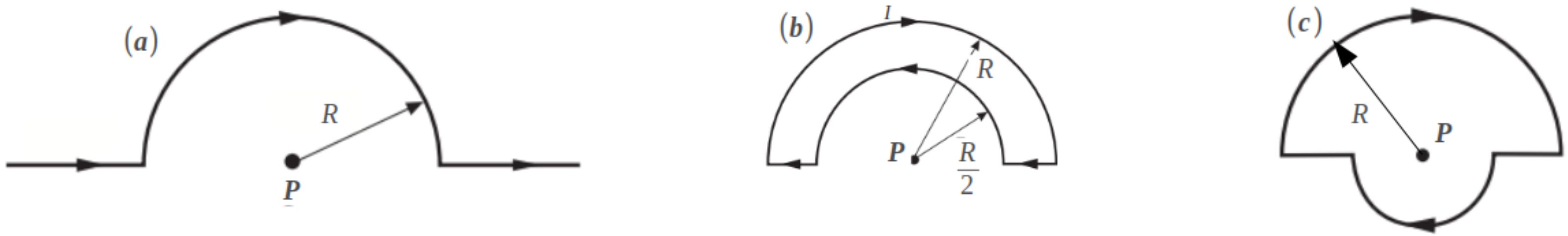
Consider the following wires, each with a different shape but crossed by the same current I. Let +x be the axes out of the page. Calculate the magnetic field using the Biot and Savart law and rank the wires in (a), (b) and (c) m from the highest to the lowest magnetic field that they generate.



(a) The straight segments of the wire do not contribute because the vector segment dl is parallel to vector distance r. The magnetic field of a circular loop in the center of the loop is:

$$B=\frac{\mu_0 I}{2R}$$

This is a semicirle, and positive x axis is out of the page. Then: $\vec{B}_a=-\frac{\mu_0 I}{4R}\hat{i}$ $B_a=\frac{\mu_0 I}{4R}$



(b) The straight segments of the wire do not contribute.

$$\vec{B}_{Tot}=\vec{B}_R+\vec{B}_{R/2} \qquad \vec{B}_R=-\frac{\mu_0 I}{4R}\hat{i} \qquad \vec{B}_{R/2}=+\frac{\mu_0 I}{4(R/2)}\hat{i} \qquad \vec{B}_b=\vec{B}_R+\vec{B}_{R/2}=\frac{\mu_0 I}{4R}(2-1)\hat{i}=\frac{\mu_0 I}{4R}\hat{i} \rightarrow B_b=\frac{\mu_0 I}{4R}$$

(c) The straight segments of the wire do not contribute.

$$\vec{B}_{Tot}=\vec{B}_R+\vec{B}_{R/2} \qquad \vec{B}_R=-\frac{\mu_0 I}{4R}\hat{i} \qquad \vec{B}_{R/2}=-\frac{\mu_0 I}{4(R/2)}\hat{i}$$

It is negative because the current is clockwise

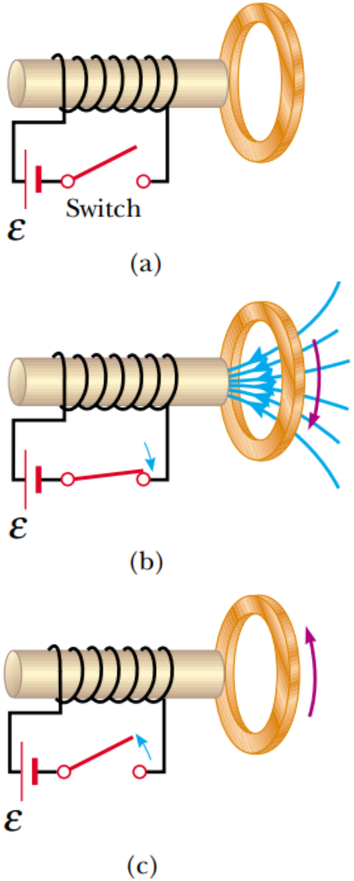
$$\vec{B}_c=\vec{B}_R+\vec{B}_{R/2}=\frac{\mu_0 I}{4R}(-2-1)\hat{i}=-3\frac{\mu_0 I}{4R}\hat{i}$$

$$\rightarrow B_c=3\frac{\mu_0 I}{4R}$$

Considering that only the magnitudes should be considered, the correct answer is: $B_c > B_b = B_a$

15. Lenz’s Law (Chapter 29)

A metal ring is placed near a solenoid as shown in the figure below:



What happens as soon as the switch of the circuit containing the solenoid is closed?

A magnetic field is created from the right to the left. An electromotive force (emf) in the ring is induced, counteracting the change in the magnetic flux and generating a magnetic field from left to right.

What happens after the switch has been closed for several minutes?

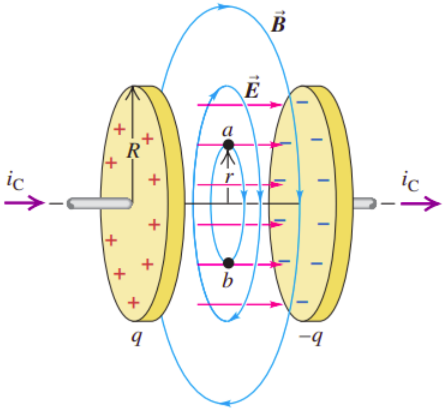
The magnetic field is constant. The induced emf is zero because there is not change in the magnetic flux.

What happens at the instant the switch is opened again after the current has circulated in the solenoid for several minutes?

The magnetic field from the left to the right suddenly becomes zero. An emf is induced in the ring, counteracting the change in the magnetic flux and generating a magnetic field from right to left.

16. Displacement current (Chapter 29)

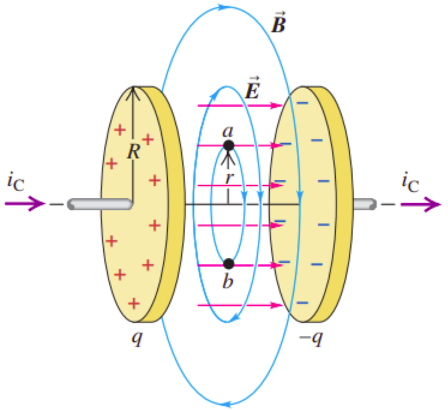
The parallel-plate capacitor in the figure below is being charged. The circular plates have a radius of 4.00 cm, and at a particular instant, the conduction current in the wires is 0.520 A.



What is the displacement current density in the air-space between the plates?

According to the generalization of the Ampere's law by Maxwell, the conduction current is equal to the displacement current.

$$J_D = \frac{i_D}{\pi r^2} = 0.520 \frac{A}{\pi(4.00\,cm)^2} = 103\,A/m^2$$



Consider the induced magnetic field between the plates.
Rank the magnetic fields at various points located at a distance r from the axis of the capacitor plates, starting with the highest value and descending to the lowest value.

Consider the following points for evaluation.
P_1, r_1 =2.00 cm; P_2, r_2= 1.00cm; P_3, r_3 =3.00 cm; P_4, r_4 =4.00cm and P_5, r_5 = 5.00 cm.

The points P_1, r_1 =2.00 cm; P_2, r_2= 1.00cm; P_3, r_3 =3.00 cm and P_4, r_4 =4.00cm are included in the area beetween the plates. We can use the formula to see that the B increase linearly with r:

$$B=\frac{\mu_0}{2\pi}\frac{r}{R^2}I=Constant\times\frac{r}{R^2}$$

$$B_1=Constant\times\frac{2.00\,cm}{16.00\,cm^2}\qquad B_2=Constant\times\frac{1.00\,cm}{16.00\,cm^2}\qquad B_3=Constant\times\frac{3.00\,cm}{16.00\,cm^2}\qquad B_4=Constant\times\frac{4.00\,cm}{16.00\,cm^2}$$

$$B_4>B_3>B_1>B_2$$

P_5 at r_5 = 5.00 cm is outside the region, and it feels a B generated by a long straight conductor carrying a current i distant r_5 = 5.00 cm from the conductor.

$$B_5=\frac{\mu_0}{2\pi}\frac{I}{r}=Const\times\frac{1}{5.00\,cm}=const\times 0.2$$

$$B_4>B_5>B_3>B_1>B_2$$

17. Microwaves (Chapter 32)

Microwave ovens are commonly used in kitchens to cook food. These microwaves ovens typically operates at a frequency of 10^{10} Hz.

What is the wavelength of these microwaves?

$$\lambda = \frac{c}{\nu} = \frac{3 \cdot 10^8\, m/s}{10^{10}} = 3 \cdot 10^{-2}\, m/s = 3\, cm$$

Grading scale when grades are assigned using percentage points

A: 89–100 points

B: 77–88 points

C: 65–76 points

D: 53–64 points

E: 41–52 points

F: 0–40 points