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Suggested solution for continuation 2019 Exam in Electricity and Magnetism

NOTE: The solutions below are meant as guidelines for how the problems may be solved and do not necessarily contain all the detailed steps of the calculations.

(1) Use $E = -dV/dx$ to identify that graph D is the correct one.

(2) There is no polarization in vacuum (the ε_0 region), so the field lines have to show the electric polarization. The **E** and **D** fields would both exist in the vacuum region.

(3) The circular motion is caused by the Lorentz-force, so we obtain $evB_0 = mv^2/r$. Since $v = \sqrt{v_0^2 + v_0^2}$, we get $r = \sqrt{2}m_e v_0/eB_0$.

(4) There must be zero current through the wires connected to the capacitors under stationary conditions, giving $I_2 = I_4 = 0$ immediately. Therefore, $I_1 = I_3$ and inspection reveals that they are both equal to $V_0/4R$.

(5) The charges on the same line as P will cancel each others electric fields. The two other charges provide a net electric field horizontally toward the right direction, thus vector 3 is correct.

(6) The current amplitude through L drops linearly with frequency while the current amplitude through C increases linearly with frequency. Therefore, the total current $I = I_L + I_C$ must have a minimum at a particular frequency.

(7) Magnetic hysteresis means that the magnetization of a material at a given external magnetic **B**-field depends on the history of the material, for instance in which magnetization direction the material was saturated before the **B**-field assumed its present value. Therefore, the magnetization of a material is not uniquely determined by the present value of the **B**-field alone.

(8) Using Gauss law gives $E(r) = \lambda/(2\pi r\varepsilon_0) = 2k\lambda/r$.

(9) The velocity is parallell with the magnetic field at the location of the particle, and thus no force acts on the particle.

(10) The student should be able to derive the resonance frequency of this circuit and show that the impedance at this frequency equals $|Z| = R$. This is a minimum and it is only zero if $R = 0$, thus generally finite.

(11) Faraday's law of induction states that the electromotive force is $\mathcal{E} = -d\Phi/dt$. Here, $A = l_b l$ where l_b is the height of the part of the loop which is inside the magnetic field at a given time. The speed in negative z-direction is $v = -dl_b/dt$. Therefore, we obtain

$$\mathcal{E} = -d(BA)/dt = Bl(-dl_b/dt) = Blv. \tag{1}$$

The induced current tries to counteract the change in magnetic flux and thus has to flow counterclockwise.

(12) We use Gauss' law with Gauss-surfaces just outside the two cavities. These Gauss-surfaces are thus entirely inside the metallic sphere. The electric field at these surfaces must therefore be zero, since they are inside the metallic sphere, so the net charge inside the surfaces also has to be zero. We conclude that there is zero charge on the surface of cavity 1 and charge $-Q$ on the surface on the surface of cavity 2. The metallic sphere is supposed to be neutral in total, so the remaining charge $+Q$ has to reside on its outer surface.

(13) According to one of Maxwell's laws, $\iint \mathbf{B} \cdot d\mathbf{A} = 0$, the magnetic flux can never be positive out of a surface.

(14) The student should be able to derive that the magnetic field inside the solenoid center is approximately constant and equal to $B = \mu_0 IN/l$ where l is the length of the solenoid. Putting in the numbers gives $B = 1.6$ mT.

(15) The electric field has to be normal to the propagation direction and can thus take any direction in the yz-plane.

(16) The average value of the magnitude of the Poynting-vector is equal to the intensity of the wave. The effect transported by the wave is thus $P = IA = 170$ J/min.

(17) An electrically neutral object can still be polarized and thus act similarly to a dipole. In this way, an electric force may be exerted on it by an electrically charged object.

(18) In semiconductors, more charge carriers are excited and become available as temperature increases since they acquire more thermal energy. This allows them to move freely and carry current. Therefore, the resistivity drops. This is in contrast to a metal where the collisions between electrons and lattice-vibrations (phonons) become more frequent as temperature rises and causes an increased resistivity.

(19) Using the total electric field is always correct. In some cases, only the conservative or non-conservative part of it contributes, but it is always correct to use the total field.

(20) In paramagnets, the magnetic moments in the material align themselves with the external field whereas they revert to a non-ordered state when the external field is turned off. This is opposite to diamagnetic materials. Ferromagnetic and antiferromagnetic materials exhibit magnetic order even without any external field.

Problem 21:

Kondensator og forskyvningsstrøm.

a) Forskyvningsstrømmen er I_d og er under oppladning av kondensatoren lik ledningsstrømmen som tilføres platene, dvs.

$$I_d = \frac{dQ}{dt} = I_0 = \underline{10,0\,\mu A}$$

med samme retning som I, dvs. i positiv retning (mot høyre).

b) Finner først total resistans i dielektrikumet mellom platene:

$$R = \rho\frac{d}{A} = 2,0 \cdot 10^{12}\,\Omega\,m\,\frac{2,00 \cdot 10^{-3}\,m}{50,00 \cdot 10^{-2}\,m^2} = 8,00 \cdot 10^9\,\Omega.$$

Strømmen I går fra positiv til negativ plate, som vi har definert som positiv retning. Derfor minustegn (Q avtar) i oppgitt uttrykk: $I = -dQ/dt$. Vi bruker videre for kondensatoren $Q = CV$ og Ohms lov $V = RI$, som gir

$$I(t) = -\frac{dQ}{dt} = -C\frac{dV}{dt} = -CR\frac{dI}{dt} \quad \Rightarrow \quad \frac{dI}{I} = -\frac{dt}{RC}.$$

Løsning av denne diff.likn. med $I = I_0'$ ved $t = 0$ gir

$$I(t) = I_0' \exp\left(-\frac{t}{RC}\right),$$

der

$$RC = 8,00 \cdot 10^9\,\Omega \cdot 17,7\,nF = 141,6\,s \qquad \text{og} \quad I_0' = \frac{V_0}{R} = \frac{Q_0}{RC} = \frac{10\,\mu C}{141,6\,s} = 70,62\,nA.$$

Merk at I_0' er ulik I_0 i oppgave a). Etter $t = 1,00\,min = 60\,s$ er

$$I(60\,s) = 70,62\,nA \cdot \exp\left(-\frac{60}{141,6}\right) = 70,62\,nA \cdot 0,655 = 46,23\,nA = \underline{46\,nA}.$$

[Kunne også beregnet RC fra $R \cdot C = \rho\frac{d}{A} \cdot \epsilon_r\epsilon_0\frac{A}{d} = \rho \cdot \epsilon_r\epsilon_0 = 2,0 \cdot 10^{12}\,\Omega\,m \cdot 8,00 \cdot 8,85 \cdot 10^{-12}\,F/m = 141,6\,s.$]

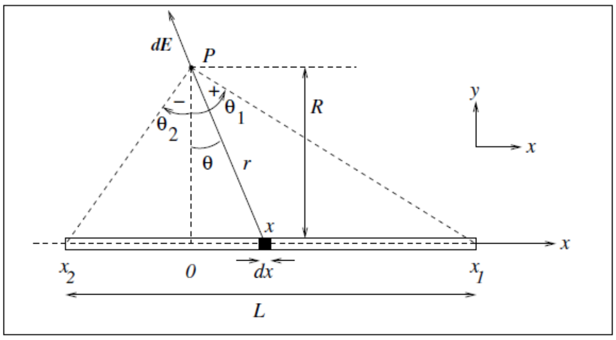
c) Med samme begrunnelse som i **a)** er $I_d = \frac{dQ}{dt}$. I **a)** økte $Q(t)$ og I_d hadde positiv retning (mot høyre). Nå avtar $Q(t)$ slik at I_d er negativ, dvs. negativ retning (mot venstre). Ved $t = 60$ s er

$$I_d(60\,s) = -I(60\,s) = \underline{-46\,nA}.$$

Problem 22:

a) Med "linjeladning" (dvs. ladning per lengdeenhet) λ må ladningene dq og Q på henholdsvis en liten lengde dx og på hele staven bli

$$\mathrm{d}q = \lambda\, dx \qquad Q = \lambda L$$



b) Elektrisk felt i pkt. P i avstand r fra lengdeelement $\mathrm{d}x$ i posisjon x (som vist i figuren):

$$\mathrm{d}\vec{E} = k\,\lambda\frac{\mathrm{d}x}{r^2}\hat{r}$$

der som vanlig $k = 1/4\pi\varepsilon_0$. Fra figuren ser vi at denne vektoren har komponentene

$$\mathrm{d}E_x = -\mathrm{d}E\sin\theta = -\frac{k\,\lambda\,\mathrm{d}x}{r^2}\sin\theta \qquad \mathrm{d}E_y = \mathrm{d}E\cos\theta = \frac{k\,\lambda\,\mathrm{d}x}{r^2}\cos\theta$$

Her har vi valgt $x = 0$ når $\theta = 0$, og fortegnet stemmer med oppgaveteksten, dvs $\theta > 0$ når $x > 0$. Vi bruker tipset i oppgaven og uttrykker $\mathrm{d}x$ og $1/r^2$ ved vinkelen θ:

$$\left.\begin{array}{lll} x = R\tan\theta & \Rightarrow & \mathrm{d}x = \dfrac{R\mathrm{d}\theta}{\cos^2\theta} \\ r = \dfrac{R}{\cos\theta} & \Rightarrow & \dfrac{1}{r^2} = \dfrac{\cos^2\theta}{R^2} \end{array}\right\} \Rightarrow \frac{\mathrm{d}x}{r^2} = \frac{\mathrm{d}\theta}{R}$$

De søkte komponentene E_x og E_y av feltet $\vec{E}$ i punktet P fra hele staven får vi ved å integrere $\mathrm{d}E_x$ og $\mathrm{d}E_y$:

$$E_x = \int_{\text{staven}} \mathrm{d}E_x = -\frac{k\,\lambda}{R}\int_{\theta_2}^{\theta_1}\sin\theta\mathrm{d}\theta = \frac{k\,\lambda}{R}\Big|_{\theta_2}^{\theta_1}\cos\theta = \underline{\frac{\lambda}{4\pi\varepsilon_0 R}(\cos\theta_1 - \cos\theta_2)}$$

$$E_y = \int_{\text{staven}} \mathrm{d}E_y = \frac{k\,\lambda}{R}\int_{\theta_2}^{\theta_1}\cos\theta\mathrm{d}\theta = \frac{k\,\lambda}{R}\Big|_{\theta_2}^{\theta_1}\sin\theta = \underline{\frac{\lambda}{4\pi\varepsilon_0 R}(\sin\theta_1 - \sin\theta_2)}\,.$$

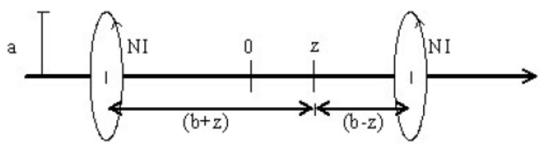
KOMMENTAR: Her kunne en ha vært "uheldig" og startet med sammenhengen $x = r\sin\theta$, som gir $\mathrm{d}x = r\cos\theta\,\mathrm{d}\theta + \sin\theta\,\mathrm{d}r$, ettersom både θ og r varierer med x. Da må vi uttrykke r og $\mathrm{d}r$ med θ og $\mathrm{d}\theta$. Vi har $\cos\theta = R/r$, dvs $r = R/\cos\theta$, og dermed $\mathrm{d}r = -R\dfrac{1}{\cos^2\theta}(-\sin\theta)\,\mathrm{d}\theta$. Da er

$$\frac{\mathrm{d}x}{r^2} = \frac{r\cos\theta\,\mathrm{d}\theta + \sin\theta\,\mathrm{d}r}{r^2} = \cos\theta\,\mathrm{d}\theta\cdot\frac{1}{r} + \sin\theta\,\mathrm{d}r\frac{1}{r^2} = \cos\theta\,\mathrm{d}\theta\cdot\frac{\cos\theta}{R} + \sin\theta\,\frac{R\sin\theta}{\cos^2\theta}\,\mathrm{d}\theta\frac{\cos^2\theta}{R^2} = \frac{\mathrm{d}\theta}{R}\left(\cos^2\theta + \sin^2\theta\right) = \frac{\mathrm{d}\theta}{R}\,.$$

c) Far away from the rod, the rod itself will look like a point-particle. Therefore, we should have $E_y = Q/4\pi\varepsilon_0 R^2$.

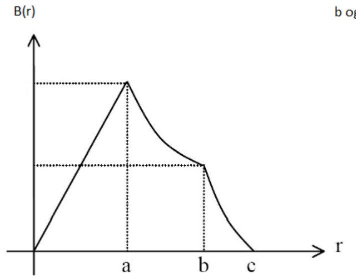
Problem 23:

Helmholtzspoler.



a) Velger origo midt mellom spolene og skal finne B-feltet i et punkt z på aksen (se figur). Avstandene fra dette punktet og til henholdsvis venstre og høyre spole vil være henholdsvis $(b+z)$ og $(b-z)$. Vi utleder uttrykket for B-feltet fra en sirkulær strømsløyfe, bruker superposisjonsprinsippet, og får følgende B-felt som ifølge høyrehåndsregelen er retta langs aksen mot venstre:

$$B(z) = N\cdot B_{\text{sloyfe}}(b+z) + N\cdot B_{\text{sloyfe}}(b-z) = \frac{\mu_0 N I a^2}{2}\left[\left(\frac{1}{a^2+(z+b)^2}\right)^{3/2} + \left(\frac{1}{a^2+(z-b)^2}\right)^{3/2}\right].$$



b og c) Med strømmen jamt fordelt over tverrsnittet på lederne blir $B(r)$ kvalitativt som $H(r)$ i tidligere øving ($B = \mu_0 H$). Vist i figuren til venstre.

Med bare overflatestrømmer blir skissa forenklet ved at $B \neq 0$ bare mellom lederne ($r \in [a, b]$), vist i figuren til høyre.

$$B = \mu_0 \frac{I}{2\pi r} \quad \text{for } r \in [a, b]\,, \qquad B = 0 \quad \text{ellers.}$$