Exam 2024

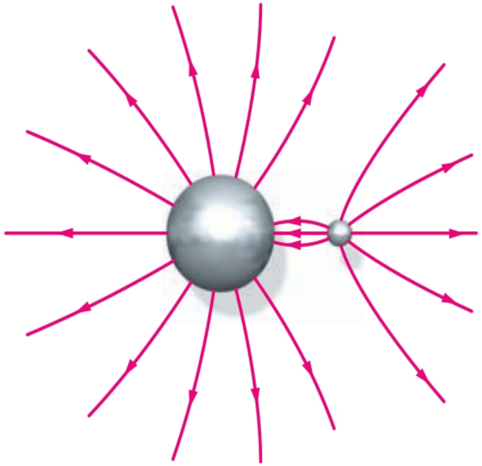
1. Electric field lines (Chapter 21) (4 Points max.)

Consider the electric field lines of two conducting spheres are shown in the figure.

What is the sign of the charge on the bigger sphere A and on the smaller sphere B?

Both sphere are positively charged. (2 points)

Both spheres have more outward than inward electric field lines.



Can you establish the relative magnitudes of the charges on the spheres?

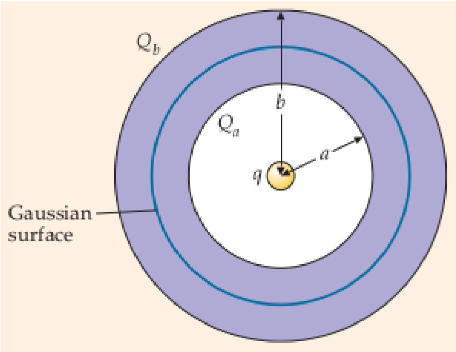
The spheres are carrying the same amount of charge. (2 points)

Sphere A: 11 field lines out and 3 field lines in. Net 8 field lines are out of the sphere.

Sphere B: 8 field lines out.

The electric field intensity is proportional to the charge. The Electric field is also is proportional to the net number of field lines. Hence, the spheres have the same charge.

2. Potential of a conducting shell (Chapter 23) (7 Points max.)



An uncharged spherical conducting shell has an inner radius a and an outer radius b. A positive point charge +q i located at the center of the shell. Assume that the potential V at the distance r = ∞ from the center is zero.

Find the charge on the inner surface, i.e. Q_{inner} , and on the outer surface, i.e. Q_{outer}, of the conductor.

$Q_{inner}=-q, Q_{outer}=+q$ *Because of the property of the conductor with a cavity.* (1 Point)

Potential generated by a charge +Q on a point distant r from it: $V=k\frac{q}{r}$

Potential generated by a spherical conductor of radius R on a point on the surface of the conductor: $V=k\frac{q}{R}$

Potential generated by a spherical conductor of radius R on a point inside the conductor: $V=k\frac{q}{R}$

We have three charges in the conductor. Following the same phylosophy:

$V=k\frac{q}{r}$ for r > b. This case reduce to the situation where all charge is concentrate in the center of the sphere $Q_{total}=q+q-q=q$. (1 Point)

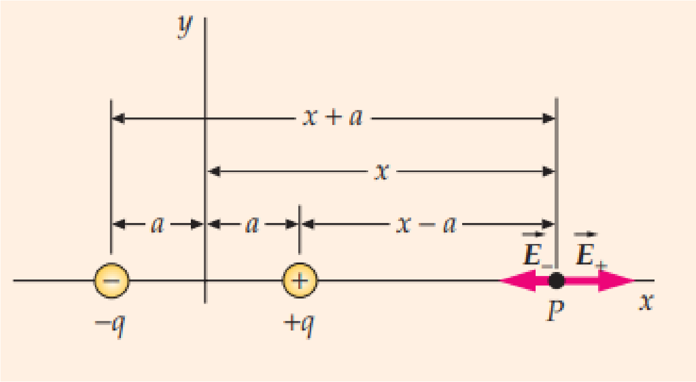
$V=k\frac{q}{b}$ for a < r < b. The charge $+q$ and the Q_{inner} charge are concentrated on the center of the sphere. Q_{outer} is on the outer surface of conductor, meaning that the potential generated by it, is constant. Hence: $V_{total}=k\left(\frac{q}{r}-\frac{q}{r}+\frac{q}{b}\right)=k\frac{q}{b}$. (3 Points)

$V=k\left(\frac{q}{r}-\frac{q}{a}+\frac{q}{b}\right)$ for 0 < r < a. Both, Q_{inner} charge and Q_{outer} are on the surface the center of the spheres, meaning that they generate constant potentials.

(2 Points)

3. Electric field calculation(Chapter 21) (Max. Points 6)

Consider the figure below:



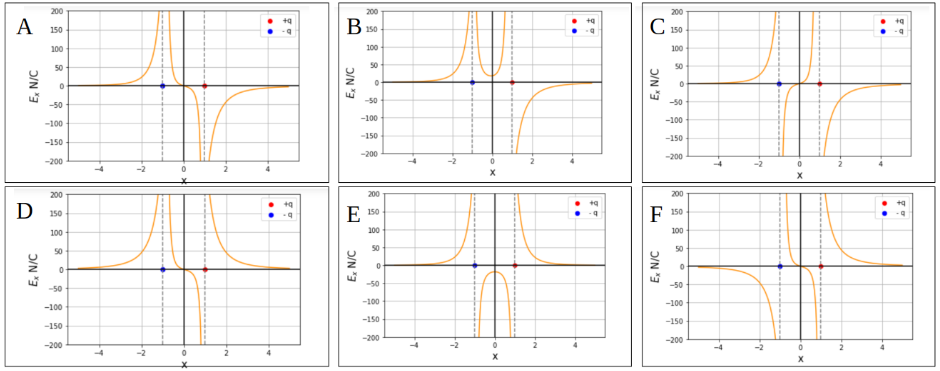
1.Electric field on the x axis at an arbitraty point x > a ?

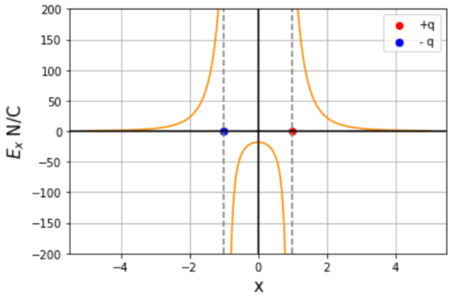
$$\vec{E}_{total}(r)=\vec{E}_+(r)+\vec{E}_-(r)=\frac{kq}{(x-a)^2}\hat{i}-\frac{kq}{(x+a)^2}\hat{i}=kq\left[\frac{(x+a)^2}{(x-a)^2}-\frac{(x-a)^2}{(x+a)^2}\right]\hat{i}=kq\left[\frac{4ax}{(x^2-a^2)^2}\right]\hat{i}$$ (2 points)

2.Electric field on the x axis at an arbitrary point x >> a ?

$$\vec{E}_{total}(r)\approx kq\left[\frac{4a}{x^3}\right]\hat{i}$$ (2 points)

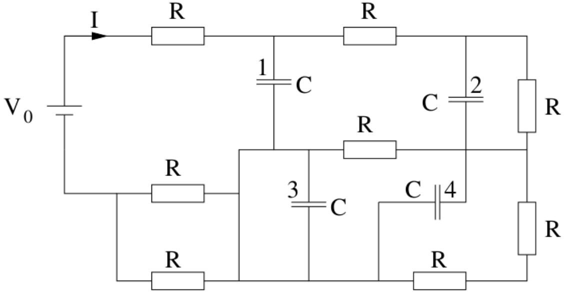
3. Consider the case of q = 1.0 nC and a = 1.0 m. Which graph among the plots below represent the electric field versus x for all distances x?





(2 points)

4. DC currents (Chapter 26) (Max. Points 8)



R = 1.0 M Ω
C = 1.0 nF
V_0= 1.25 kV

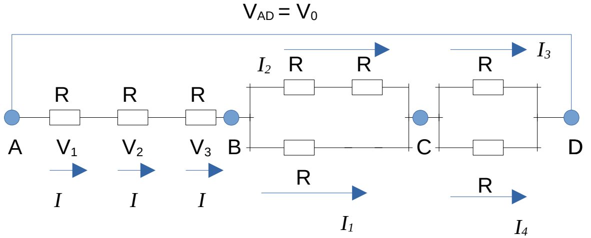
In the circuit below, the DC voltage source V_0 has been connected for a sufficient amount of time such that the currents in the circuit and the charges on the capacitors no longer change.

Determine the current I, as well as the charges Q_1, Q_2, Q_3, and Q_4 on the capacitors labeled 1, 2, 3, and 4 respectively.

From the equivalent resistance we find the current I.

$$R_t = R + R + R + \left(\frac{1}{R} + \frac{1}{R+R}\right)^{-1} + \left(\frac{1}{R} + \frac{1}{R}\right)^{-1} = \frac{25R}{6} \longrightarrow I = \frac{V_0}{R_t} = \frac{6V_0}{25R} = \frac{6 \cdot 1.25 \cdot 10^3}{25 \cdot 10^6} = 0.3\,\mathrm{mA}$$

(Points 3)

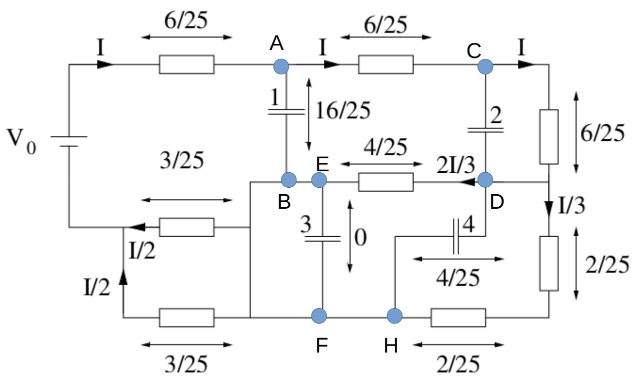


$$V_{AB} = I\,R_{eq} = I \cdot 3R = \frac{6}{25}\frac{V_0}{R} \cdot 3R = \frac{18}{25}V_0 \longrightarrow V_1 = \frac{V_{AB}}{3} = V_2 = V_3 = \frac{6}{25}V_0$$

$$V_{BC} = I\,R_{eq} = I \cdot \frac{2R}{3} = \frac{6}{25}\frac{V_0}{R} \cdot \frac{2R}{3} = \frac{4}{25}V_0 \longrightarrow I_1 = \frac{V_{BC}}{R} = \frac{4}{25}\frac{V_0}{R} = \frac{2}{3}I \quad \text{and} \quad I_2 = I - I_1 = \frac{I}{3}$$

$$V_{CD} = I\,R_{eq} = I \cdot \frac{R}{2} = \frac{6}{25}\frac{V_0}{R} \cdot \frac{R}{2} = \frac{3}{25}V_0 \longrightarrow I_3 = \frac{V_{CD}}{R} = \frac{3}{25}\frac{V_0}{R} = \frac{I}{2} = I_4$$

This is the resulting circuit with the potential on each resistance.



To find the potential V_{ab} of a point a respect to b, for example, you can use the **bookkeeping rule**. Start at b and add the potential changes you encounter going from b to a. Use the same sign convention.

$$V_{AB}=\frac{4}{25}V_0+\frac{6}{25}V_0+\frac{6}{25}V_0=\frac{16}{25}V_0=V_1$$ → *Potential on capacitor 1.*

$$V_{CD}=\frac{6}{25}V_0=V_2$$ → *Potential on capacitor 2.*

$$V_{EF}=\frac{2}{25}V_0+\frac{2}{25}V_0-\frac{4}{25}V_0=0=V_3$$ → *Potential on capacitor 3.*

$$V_{DH}=\frac{2}{25}V_0+\frac{2}{25}V_0=\frac{16}{25}V_0=V_4$$ → *Potential on capacitor 4.*

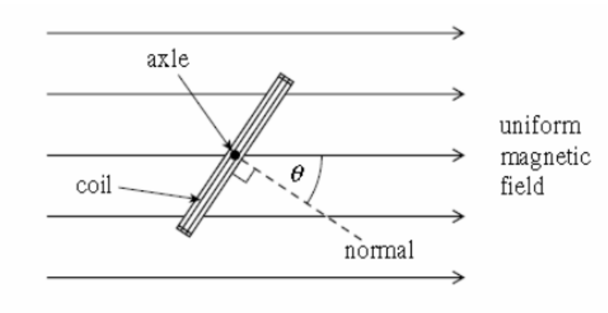
Replacing all the values, we finally obtain the charges on the capacitors. →

$$Q_1=V_1C=\frac{16}{25}V_0C=0.8\mu\text{C} \qquad Q_2=V_2C=\frac{6}{25}V_0C=0.3\mu\text{C}$$

$$Q_3=V_3C=0 \qquad Q_4=V_4C=\frac{4}{25}V_0C=0.2\mu\text{C}$$

(Points 5)

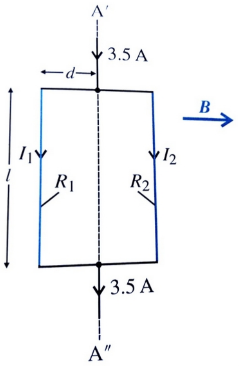
5. Coil in a magnetic field (Chapter 27)



A rectangular coil comprises 200 turns of wire of dimensions 10 cm × 15 cm. The coil is placed in a uniform magnetic field of magnitude 0.16 T directed such that the field lines make an angle with the normal to the plane of the coil. Calculate the magnitude of the torque on the coil if a current of 120 mA flows in it.

$\tau = NIABsin(\theta) = 200 \times 120 \times 10^{-3}\,A \times 10 \times 10^{-2} \times 15 \times 10^{-2} \times 0.16\,T \times 0.42 \approx 0.024\,Nm$ (Points 5)

6. Torque of the force (Chapter 27) (Max Points 9)



The problem requires to calculate the forces on the right inside and left hand side and the currents. I_{total} = 3.5 A, rectangle dimension: 3cm x 4cm, both of the long sides have L = 4 cm. The current on the left is I_1 and the wire has R = 2.0.
The current on the right is I_2 and the wire has R = 5.0. The resistance in the side 2d is negligeable. The plane is parallel to the uniform magnetic field. The angle between the magnetic field B and the currents I_1 and I_2 is $\theta=\frac{\pi}{2}$

What is the torque?

$R_1 I_1 = R_2 I_2 \rightarrow 2 I_1 = 5 I_2$ also $I = I_1 + I_2 \rightarrow I_1 = 2.5\,A;\ I_2 = 1.0\,A.$ (Points 2)

module of magnetic force on the left $F_1 = I_1 lB \cdot \sin(90^\circ) = 2.5\,A \times 4.0 \cdot 10^{-2}\,m \times 2.4\,T = 24 \cdot 10^{-2}\,N$

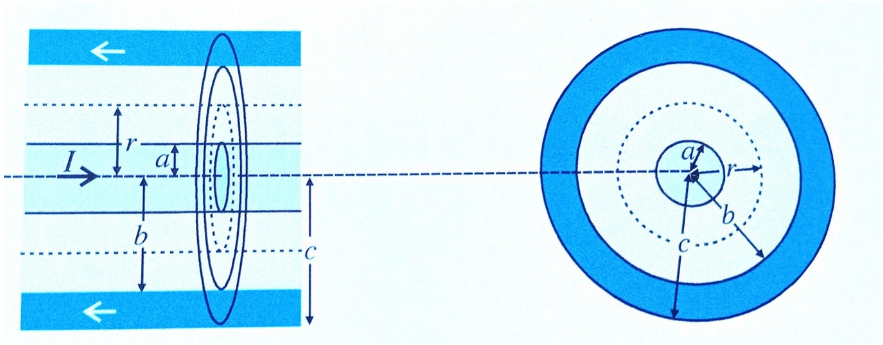
(Points 2)

module of magnetic force on the left $F_2 = I_2 lB \cdot \sin(90^\circ) = 1.0\,A \times 4.0 \cdot 10^{-2}\,m \times 2.4\,T = 9.6 \cdot 10^{-2}\,N$

$\tau = (F_1 - F_2)\, d/2 = (24 - 9.6) \cdot 10^{-2}\,N \cdot 1.5 \cdot 10^{-2}\,m = 0.0022\,N \cdot m$ (Points 3)

Viewing the loop from the direction of the current I, the direction of the torque is counterclockwise. (Points 2)

7. Ampere’s law (Chapter 28) (Max Points 7)



Consider two coaxial conducting wires in the illustration below. The inner wire has a radius a, and the outer conducting shell has an inner radius b and a thickness equal to c-b. Calculate the magnetic field strength as a function of the distance r from the axis of the wires using Ampère's law. The current flows in the central conductor and returns in the opposite direction in the outer conductor, as shown by the arrows in the illustration.

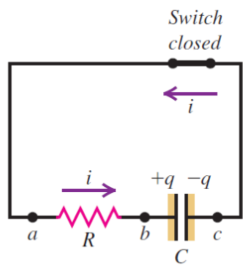
Magnitude of B at r < a ? $\oint_L \vec{B}\vec{dl} = \mu_0 I_{inc} \rightarrow$ The included current I_inc is $I_{incl} = J \cdot A = \frac{I}{\pi a^2} \cdot \pi r^2 \rightarrow B \cdot 2\pi r = \mu_0 I \frac{r^2}{a^2} \rightarrow B = \frac{\mu_0 I r}{2\pi a^2}$ (Points 2)

Magnitude of B at b > r > a ? $B = \mu_0 \frac{I}{2\pi r}$ (Points 1)

Magnitude of B at c > r > b ? $\oint_L \vec{B}\vec{dl} = \mu_0 I_{inc} \rightarrow > B \cdot 2\pi r = \mu_0 I - \mu_0 I \frac{(r^2 - b^2)}{(c^2 - b^2)} \rightarrow B = \frac{\mu_0 I (c^2 - r^2)}{2\pi r (c^2 - b^2)}$ (Points 2)

Magnitude of B at r > c ? $\oint_L \vec{B}\vec{dl} = \mu_0 I_{inc} \rightarrow I_{incl} = I - I = 0 \rightarrow B = 0$ (Points 2)

8. Discharcing a capacitor (Chapter 26) (Max Points 6)



In the circuit below, after charging the capacitor, the battery has been removed. When the switch is closed (t=0s), the potential difference across the capacitor is 100 V. At t =10 s, the potential difference across the capacitor is measured to be 1.0V.

Determine the time constant of the circuit

$$Q=Q_0 e^{-t/RC}=CV=CV_0 e^{-t/RC} \rightarrow RC=-t/\ln(V/V_0) \rightarrow RC=-10\,s/\ln(1/100)=\frac{-10\,s}{-4.6}=2.17\,s$$

(Points 2)

Determine the V after t = 17s

$$V(t=17\,s)=100\,V\,e^{-17\,s/2.17\,s}=0.04\,V\,s$$

(Points 2)

Determine the change on R to have a potential tre times the V at t = 17s

$$(RC)_{NEW}=-17\,s/\ln(0.04*3/100)=\frac{-17\,s}{-6.21}=2.53\,s$$

$$\frac{(RC)_{NEW}}{RC}=\frac{2.53\,s}{2.17\,s}=1.17 \quad \rightarrow (R)_{NEW}=1.17\,R$$

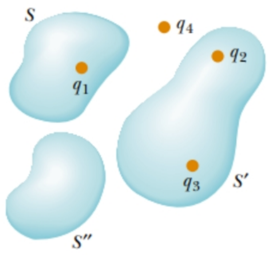
(Points 2)

9. Gauss law Conceptual (Max Points 6)

Your friend likes to rub his feet on the carpet and then touch you to give you a shock. While trying to escape the shocks, you find a hollow metal cylinder large enough to enter inside. In which of the following scenarios will you not be shocked?

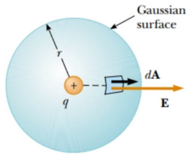
○ You step inside the cylinder, making contact with the inner surface, while your charged brother touches the outer metal surface. (Points 2)

Consider the charge distribution shown in the Figure. Which charges contribute to the total electric field at a chosen point on the surface S' ?



All the charges in the picture are contributing to the electric field on the surface S'. (Points 2)

Which statement is true if the charge in the figure below were inside but not at the center of the spherical Gaussian surface?



○ The Gauss Law would still be valid, however, the Electric field would not be everywhere perpendicular to the surface and the situation would not possess enough symmetry to evaluate the electric field. (Points 2)

10. EMwaves (Chapter 32) (Max Points 6)

FM transmits at frequency of 10^{10} Hz.

What is the wavelength of these radiowaves?

$$\lambda=\frac{c}{v}=\frac{3\cdot 10^8 m/s}{100\cdot 10^6 s^{-1}}=3\cdot 10^{-2} m=3 m$$

(Points 2)

Assume that this FM signal is a sinusoidal wave propagating in the z direction with the electric field pointing always in the x direction with magnitude = 2.0 V/m. What is the magnitude of the magnetic field?

$$B=\frac{E}{c}=\frac{2.0 V/m}{3\cdot 10^8 m/s}=6.7\cdot 10^{-9} T$$

(Points 2)

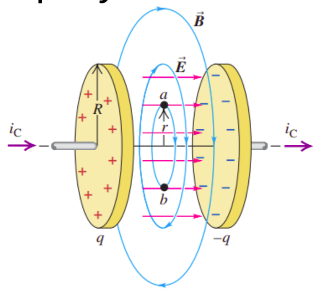
What is the direction of the magnetic field?

The direction of the magnetic field is parallel to the y direction.

(Points 2)

11. Displacement current (Chapter 29) (Max Points 6)

The parallel-plate capacitor in the figure below is being charged. The circular plates have area of 100 m² and radius r = r_0. The capacitor as a capacitance of C = 30 pF and it is connected in series to a battery of 70V and to a 2.0 Ω resistor. At the istant t = t_0 the circuit is closed and the electric field between the plates is changing most rapidly.



What is the current?

$$Q=Q_0(1-e^{-t/RC})=CV(1-e^{-t/RC}) \longrightarrow \left(\frac{dQ}{dt}\right)_{t=0}=\frac{CV}{RC}(1-e^{-t/RC})_{t=0}=\frac{V_0}{R}=35\,A \qquad I=\frac{V}{R}=\frac{70\,V}{2.0\,\Omega}=35\,A$$

(Points 2)

What is the change of E?

$$E=\frac{\sigma}{\epsilon_0}=Q/A=I \quad \rightarrow \frac{dE}{dt}=\frac{dQ/dt}{\epsilon_0 A}=4.0\times 10^{14}\,V/(m\cdot s)$$

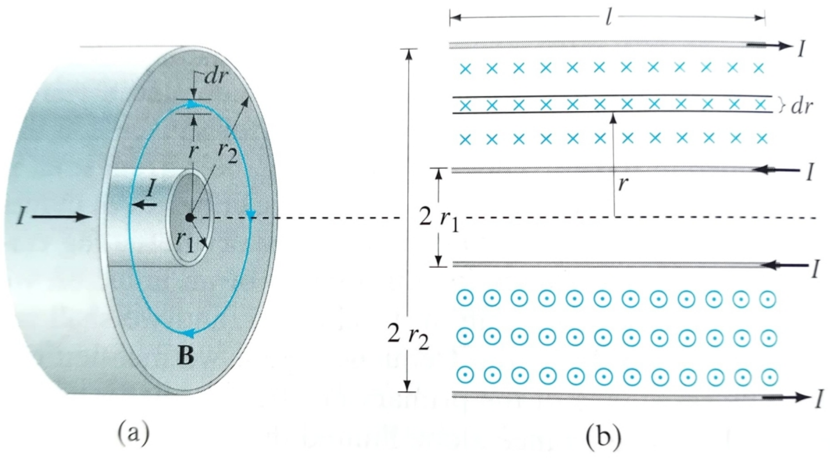
(Points 2)

What is the magnetic field induced at r = r_0?

$$\oint_L \vec{B}\cdot d\vec{l}=\mu_0\epsilon_0\frac{d\Phi_E}{dt}\rightarrow(B2\pi r)=\mu_0\epsilon_0\frac{d}{dt}(\pi r^2 E)=\mu_0\epsilon_0\pi r^2\frac{dE}{dt}=1.2\,T\times 10^{-4}$$

(Points 2)

12. Inductance (Chapter 30) (Max Points 7)



$B = \frac{\mu_0 I}{2\pi r}$ We can calculate the flux of B through the small square dr and l:

$$d\Phi_B = B(l dr) \rightarrow \Phi_B = \int_{r_1}^{r_2} \frac{\mu_0 I}{2\pi r} l\, dr = \frac{\mu_0 I l}{2\pi} \int_{r_1}^{r_2} \frac{dr}{r} = \frac{\mu_0 I l}{2\pi} \ln \frac{r_2}{r_1}$$

$$L = \frac{\Phi_B}{I} = \frac{\mu_0 l}{2\pi} \ln \frac{r_2}{r_1}$$

(Points 7)

13. Power station (Chapter 31) (Max Points 6)

An average of 120 kW of electric power is sent to a small town from a power plant far away. The transmission lines have a total resistance of 0.40 Ω. This resistance does not include the load of the town.

$$I=\frac{P}{V}=\frac{1.2\times10^5 W}{2.4\times10^2 V}=500\,A$$

$$P_{loss}=I^2R=(500\,A)^2(0.40\,A)=100\,kW$$

(Points 2)

$$I=\frac{P}{V}=\frac{1.2\times10^5 W}{2.4\times10^4 V}=5.0\,A$$

$$P_{loss}=I^2R=(5.0\,A)^2(0.40\,A)=10\,W$$

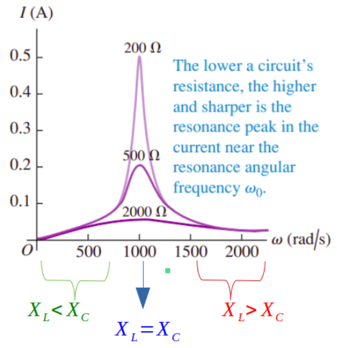
(Points 2)

This exercise shows that it is convenient to step up the voltage prior transmission to reduce the power loss. In the first case the power loss is 83% of the original value, in the second case the power loss during the transmission is 0.01 %, i.e. very little.

(Points 2)

14. AC RLC series circuit (Max Points 4)

$V = 100V, L = 2.0H, C = 0.50\mu F$



- *<u>Increasing the resistance of the resistor in an AC LRC circuit results in a higher and sharper peak in the current near the resonance frequency.</u>*

  WRONG -> Look at the graph.

- *<u>At resonance, the impedance equals the resistance R.</u>* **TRUE** -> $Z = \sqrt{(R^2 - (X_L - X_C)^2)} \rightarrow X_L = X_C \rightarrow Z = R$ (Points 2)

- *<u>At resonance, the current is in phase with the voltage applied.</u>* **TRUE** The instantanpus voltage accross a resistor is always in phase with the current. (Points 2)

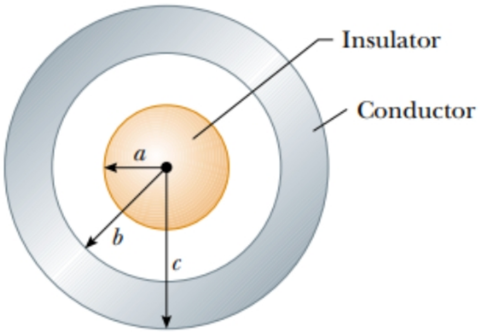
- Inductors tend to pass high-frequency current. - > **WRONG** $X_L = \omega L \rightarrow I = V_L/(\omega L) \rightarrow$ high ω means filtered or small current.

- *<u>Capacitors tend to pass low-frequency current.</u>* -> **WRONG** $X_c = 1/(\omega C) \rightarrow I = V_C / X_C = (\omega C) * V_C \rightarrow$ Low ω means small current.

- *<u>For pure resistors, the average power is zero.</u>* -> **WRONG**. The average power is always positive because is the product of the instantanous current and voltage that are always in phase.

- The phase angle does not depend on the resistance. -> **WRONG** $\tan\phi = \frac{(X_L - X_C)}{R} = \frac{\omega L - (1/\omega C)}{R}$

15. Gauss law (Chapter 22) (Max Points 13)



Magnitude of E at r < a ? Apply Gauss Law $E(r)\cdot 4\pi r^2 = Q_{inc}/\epsilon_0 \rightarrow Q_{inc} = \rho\cdot\frac{4}{3}\pi r^3 = \frac{Q}{\frac{4}{3}\pi a^3}\cdot\frac{4}{3}\pi r^3 = Q\frac{r^3}{a^3} \rightarrow \quad E = kQ\frac{r}{a^3}$ (Points 2)

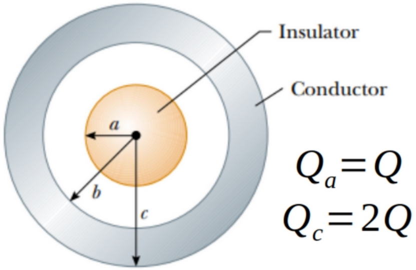
Magnitude of E at b > r > a ? Apply Gauss Law $E(r)\cdot 4\pi r^2 = Q_{inc}/\epsilon_0 \rightarrow \quad E = k\frac{Q}{r^2}$ (Points 1)

Magnitude of E at c > r > b ? Apply Gauss Law $E(r)=0$ Because inside the conductor the electric field is zero (Points 2)

Magnitude of E at r > c ? Apply Gauss Law $E(r)\cdot 4\pi r^2 = Q_{inc}/\epsilon_0 \rightarrow \quad E = k\frac{Q}{r^2}$ (Points 1)

Q_{inner} and Q_{outer} $Q_{inner} = -Q, \quad Q_{outer} = +Q$ Because of the induction phenomena in a cavity of a conductor (Points 1)

10. B Gauss law (Chapter 22)



Magnitude of E at c > r > b ? Apply Gauss Law $E(r) = 0$ Because inside the conductor the electric field is zero (Points 2)

Magnitude of E at r > c ? Apply Gauss Law $E(r) \cdot 4\pi r^2 = Q_{inc} / \epsilon_0 \rightarrow \quad E = k\,3\frac{Q}{r^2}$ (Points 2)

Q_{inner} and Q_{outer} $Q_{inner} = -Q, Q_{outer} = +3Q$ Because of the induction phenomena in a cavity of a conductor (Points 2)

Grading scale is different becasue the maximum is now 98.

Grading scale when grades are assigned using percentage points

A: 89–100 points

B: 77–88 points

C: 65–76 points

D: 53–64 points

E: 41–52 points

F: 0–40 points

A: 87–98 points

B: 75–86 points

C: 63–74 points

D: 51–62 points

E: 39–50 points

F: 0–38 points