NTNU – Trondheim
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Department of Physics

Examination paper for

TFY4250/FY2045 Quantum Mechanics 1

(Kontinuasjonseksamen)

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Examination date: August , 2015

Examination time (from-to): 0900-1300

Permitted examination support material:

Calculators, Rottmann (formula collection)

Other information:

N/A

Language: English

Number of pages: 3 (including this one)

Number of pages enclosed: 2

Checked by:

Date Signature

1) A particle of mass *m* is in the state

$$\psi(x,t) = A\, e^{-a\left[\left(\frac{mx^2}{\hbar}\right)+it\right]},$$

where A and a are positive real constants.

a) Find A.
b) For what potential energy function $V(x)$ does ψ satisfy the Schroedinger equation?
c) Calculate the expectation values of x and p.

2) The Hamiltonian for a certain three-level system is represented by the matrix

$$\boldsymbol{H} = \begin{pmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{pmatrix},$$

where a, b and c are real numbers (assume that $a - c \neq \pm b$). If the system starts out in the state

$$| s(0) > = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

what is $| s(t) >$?

3) An electron is in the spin state

$$\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}.$$

a) Determine the normalization constant A.
b) Find the expectation values of S_x, S_y and S_z.
c) Find the standard deviations σ_{S_x}, σ_{S_y} and σ_{S_z}.

4) Find the best bound E_{gs} for the one-dimensional harmonic oscillator using a trial wave function of the form

$$\psi(x) = \frac{A}{x^2 + b^2}$$

where A is determined by normalization and b is the adjustable parameter.

5) Consider a quantum system with just three linearly independent states. Suppose the Hamiltonian, in matrix form, is

$$\boldsymbol{H} = V_0\begin{pmatrix} (1-\varepsilon) & 0 & 0 \\ 0 & 1 & \varepsilon \\ 0 & \varepsilon & 2 \end{pmatrix},$$

where V_0 is a constant, and ε is some small number $(\varepsilon \ll 1)$.

a) Write down the eigenvectors and eigenvalues of the unperturbed Hamiltonian H^0 $(\varepsilon = 0)$.
b) Solve for the exact eigenvalues of $\boldsymbol{H}$. Expand each of them as a power series in ε, up to second order.
c) Use first- and second-order non-degenerate perturbation theory to find the approximate eigenvalues for the state that grows out of the non-degenerate eigenvector of H^0. Compare with the exact result.

Formulas:

$$i\hbar\frac{\partial\psi}{\partial t} = \hat{H}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$E_n^{(2)} = \sum_{m}^{m\neq n} \frac{\left|\langle\psi_m^{(0)}|H_1|\psi_n^{(0)}\rangle\right|^2}{E_n^{(0)} - E_m^{(0)}}$$