NTNU Trondheim, Institutt for fysikk

Examination for FY2450, Astrophysics

This examination paper consists of 7 pages (including this title page)

Units, masses and other values: Constants and useful formulae are listed on pages 6 and 7 of this paper.

Permitted Examination Aids: Simple calculator

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Please write your candidate number on every page and number the pages.

Please write only on ONE side of each sheet you hand in.

Answer ALL questions

Examination for FY2450, Astrophysics

Part A: Multiple Choice Questions

Choose a **single** *answer for each question in Part A unless otherwise stated. Write the answer down in the official examination paper that you hand in.* **[Part Marks]**

Question 1

A pure continuous spectrum of light is formed by:

(a) the expanding shell of gas in a Planetary Nebula

(b) the photosphere of the Sun

(c) a black-body radiator

(d) none of the above

[1]

Correct answer: c

Question 2

Why can't we see what is near the center of our Galaxy with regular optical telescopes?

(a) Because at night we do not face the right direction to point our telescopes there

(b) Because there is too much gas and dust there for visible light to go through

(c) Because it is so far away that light has not had time to reach us yet

(d) Because there is a black hole there that does not let any light escape

[1]

Correct answer: b

Question 3

Are all galaxies moving away from us?

(a) Yes

(b) No, all distant galaxies do, but some nearby ones do not

(c) No, all nearby galaxies do, but the distant ones do not

(d) No, elliptical galaxies do, but spiral galaxies do not

[1]

Correct answer: b

Question 4

How can we find the distance to a nearby galaxy, like Andromeda?

(a) We use the parallax method

(b) We find its temperature and use the HR diagram

(c) We look for variable stars such as Cepheids in it

(d) We find its velocity and use Hubble's law

[1]

Correct answer: c

Question 5 [Part Marks]

In a hypothetical galaxy it is observed that the rotational velocity of stars in a galaxy, v, about a point A is proportional to r, where r is the distance from A, i.e. $v \propto r$. This implies that the mass in the galaxy varies as [Hint: Use some of the formulae in the appendix to answer this question]

(a) r

(b) r^2

(c) r^3

(d) r^4

[4]

Answer: Equating centripetal and gravitational force one finds: $v^2 \propto GM/r$. *Since* $M = 4/3\pi r^3 \rho$, *if* $M(r) \propto r^3$ *we obtain the observed behaviour*

Question 6

Star A has a surface temperature of 4,000 K while star B is 40,000 K on its surface. Assuming that both have the same radius, indicate **all** the statements that are true:

(a) Star B is more luminous than A (integrated over all wavelengths) *True, from Stefan's law*

(b) Star A emits more at infrared wavelengths than star B *True, from Wien's displacement law*

(c) The wavelength at which the emission of star B peaks is "redder" than the corresponding wavelength for star A *False, from Wien's displacement law*

(d) The radiation spectrum of star B peaks in the infrared range (Hint: The surface of the Sun is $\sim$ 6000 K) *False, from Wien's displacement law; the Sun is predominantly yellow, and a body hotter than the Sun would radiate at a peak wavelength that is shorter than yellow.*

[4]

Part B: Full Answers

Question 7

(a) Sketch a Hertzsprung-Russell diagram and label the axes. Indicate the locus of main-sequence stars, giant stars, and white dwarfs. **[5]**
See any textbook for solution here. One mark for each correctly labeled axis, one mark for each correctly sketched type of object

(b) Indicate in which direction along the main-sequence the radius and the lifetime of the stars increases. *One mark for correct lifetime "direction" and one mark for radius.* **[2]**

Question 8

The AGN MCG-6-30-15 observed with the Chandra X-ray Observatory exhibits variability in the timescale of 10^4 seconds.

(a) Calculate the upper limit placed on its size by this observation. **[2]**

Solution: $R = c \times \Delta T = 3 \times 10^{10} \times 10^4 = 3 \times 10^{14}$ cm. Minus half point for incorrect numerical result. Minus one point if no units.

(b) The radius of the event horizon of a black hole is called the Schwarzschild radius. Write down the Schwarzschild radius of a black-hole of mass M in the classical approximation. To do so, derive the escape velocity of a non-propelled body at a distance r from a star (or stellar remnant) of mass M.

Solution: Esc velocity

$$v_{\rm esc} = \sqrt{\frac{2GM}{r}}$$

[2 points for correct derivation starting from kinetic and potential energy.]

Setting $v = c$ *gives* $R_s = 2GM/c^2$. *[2 points for correct expression]* **[4]**

(c) Hence, and using the result you obtained in (a) calculate the maximum mass of a supermassive black hole that can fit inside the X-ray emitting region of MCG-6-30-15. Express your answer in terms of solar masses. **[3]**
Solution:

$$M = \frac{R_s \times c^2}{2G} = \frac{3 \times 10^{14}\ \mathrm{cm} \times (3 \times 10^{10} \mathrm{cm/s})^2}{2 \times 6.7 \times 10^{-8} \mathrm{erg/g^2/cm}} = 2. \times 10^{42}\ \mathrm{g} = 10^9\ \mathrm{M}_\odot$$

[2.5 points for correct calculation even if incorrect r *was used from (a)]*

(d) The Eddington luminosity of an object with mass M and radius r is given by the expression

$$L_{\rm E} = \frac{4\pi c G M m_p}{\sigma_{\rm T}}$$

Briefly describe the physical origin of this fundamental upper limit qualitatively.

Solution: 2 points for explanation which explicitly states fundamental limit due to balance of pressure/gravity

Using your results in part (c) calculate the Eddington luminosity of the engine powering the emission of MCG-6-30-1. Express your answer in terms of the solar luminosity. **[4]**

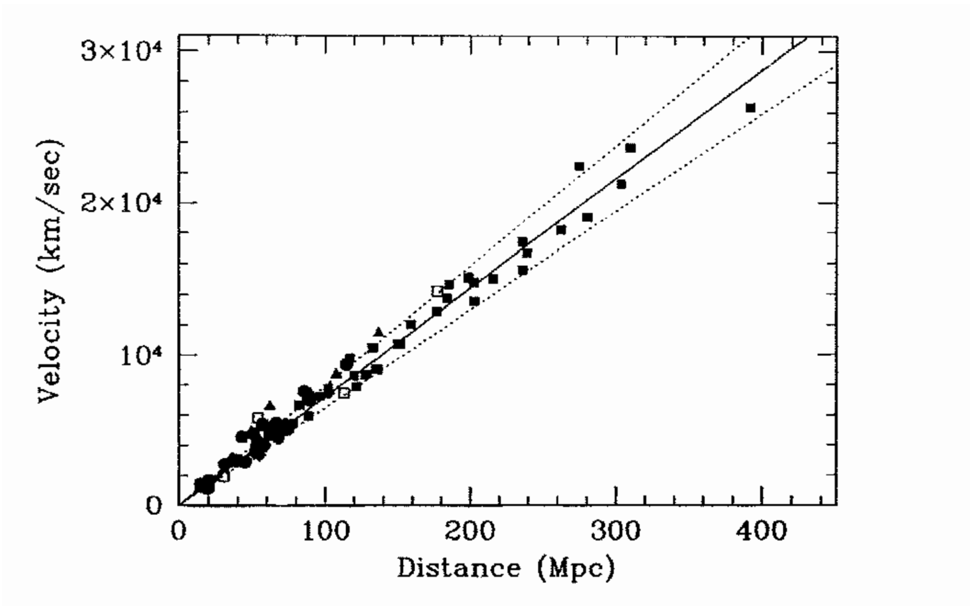
$L = 4 \times 3.1415 \times c \times G \times M \times m_p/\sigma_T = 1.3 \times 10^{47}$ *erg/s* $= 3.4 \times 10^{13} L_\odot$
1 point for numerical answer which is correct given the assumed mass

(e) Assume that MCG-6-30-1 radiates with the Eddington luminosity that you calculated in (d) and that the emission that we observe is due to accretion onto the supermassive black hole. Assuming further that 10% of rest-mass is converted to radiation during the accretion process, calculate the mass accretion rate in MCG-6-30-1. Express your answer in solar masses per year.

Answer: $L = 0.1 \times M \times c^2$ *and therefore* $M = L/0.1/c^2 = 1.43 \times 10^{27}$ g/s $= 4.5 \times 10^{34}$ g/yr $\sim 12 {\rm M}_\odot/{\rm yr}$
2 points for numerical answer which is correct given the starting assumed value of the Eddington luminosity **[3]**

Question 9

The plot below shows the the recessional velocity of galaxies as a function of their distance



(a) Use the diagram to estimate the Hubble constant H_0.

Answer:

$$H_0 = \frac{\Delta v}{\Delta d} \approx \frac{3 \times 10^4 \text{ km/s}}{400 \ Mpc} \sim 75 \text{ km s}^{-1}\text{Mpc}^{-1}$$

[Answers around $50 - 100$ km $\text{s}^{-1}\text{Mpc}^{-1}$ *get full marks. -0.5 point for units that do not reveal the physical meaning (e.g. inverse time).]* **[2]**

(b) Use the answer to part (a) to estimate the age of the Universe.

Answer:

$$t = \frac{1}{H_0} = 3 \times 10^{24} \text{ cm}75 \times 10^5 \text{ cm s} \sim 4 \times 10^{17} \text{ s} \sim 12.7 \text{ Gyr}$$

[-0.5 points for answers in units that don't give an intuitive sense of the timescale (e.g. seconds).] **[2]**

(c) The Friedman equation can be written as

$$H^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}.$$

Briefly describe the meaning of k, Λ, R.

1 point for each correct definition: k *: curvature parameter.* Λ *: cosmological constant.* R *: scale factor* **[3]**

(d) Using your answer in (a) and the Friedmann equation, calculate the present day value of the critical density for a Universe. Express your answer in solar masses per cubic megaparsec.

Answer:

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} \approx 3.1 \times 10^{44} \text{ g/Mpc} \approx 8.1 \times 10^{10} \text{ M}_\odot \text{ Mpc}^{-3}$$

-1 point for setting the curvature term to zero
-1 point for setting the cosmological constant term to zero
-Full points for answer in the correct ballpark with requested units.
-Minus 0.5 point for wrong numerical answer with correct units and correct procedure

[3]

[Total Marks = 45]

END OF PAPER

Constants

Gravitational constant	$G = 6.7 \times 10^{-8}$ erg cm g^{-2}
Speed of light	$c = 3 \times 10^{10}$ cm s^{-1}
Planck's constant	$h = 6.6 \times 10^{-27}$ erg s
	$\hbar = h/2\pi = 1.05 \times 10^{-27}$ erg s
Boltzmann's constant	$k = 1.4 \times 10^{-16}$ erg K^{-1}
Stefan-Boltzmann constant	$\sigma = 5.7 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
Radiation constant	$\alpha = 4\sigma/c = 7.6 \times 10^{-15}$ erg cm^{-3} K^{-4}
Proton mass	$m_p = 1.7 \times 10^{-24}$ g
Electron mass	$m_e = 9.1 \times 10^{-28}$ g
Electron charge	$e = 4.8 \times 10^{-10}$ esu
Electron volt	1 eV$= 1.6 \times 10^{-12}$ erg
Erg	1 erg $= 1$ cm$^2 \cdot$ g $\cdot$ s^{-2}
Thomson cross section	$\sigma_{\mathrm{T}} = 6.7 \times 10^{-25}$ cm^2
Angstrom	1 Å$= 10^{-8}$ cm
Solar Mass	$M_\odot = 2 \times 10^{33}$ g
Solar Luminosity	$L_\odot = 3.8 \times 10^{33}$ erg s^{-1}
Solar Radius	$r_\odot = 7.0 \times 10^{10}$ cm
Astronomical Unit	1 AU $= 1.496 \times 10^{13}$ cm
Parsec	1 pc $= 3.086 \times 10^{18}$ cm
Megaparsec	1 Mpc $= 3.086 \times 10^{24}$ cm

Useful relations and formulae

Rest-mass energy of body with mass m

$$E = mc^2.$$

Non-relativistic kinetic energy of a body with mass m and velocity v

$$E = \frac{1}{2}mv^2.$$

Gravitational potential energy for two bodies with masses M, and m a distance r apart

$$E = -\frac{GMm}{r}.$$

Centripetal force

$$F = -\frac{mv^2}{r}.$$

Gravitational force

$$F = -\frac{GMm}{r^2}.$$

Wien's displacement law: wavelength of maximum intensity

$$\lambda_{\max} = 2900 \text{ Å} \left(\frac{10^4 \text{ K}}{T}\right)$$

$$h\nu_{\max} = 2.4 \text{ eV} \left(\frac{10^4 \text{ K}}{T}\right).$$

Luminosity of black body with temperature T

$$L = 4\pi R^2 \sigma T^4.$$