NTNU Trondheim, Institutt for fysikk

Examination for FY2450, Astrophysics

This examination paper consists of 9 pages (including this title page)

Units, masses and other values: Constants and useful formulae are listed on pages 8 and 9 of this paper.

Permitted Examination Aids: Simple calculator

Contact: Foteini Oikonomou, 453 92 241

Please write your candidate number on every page and number the pages.

Please write only on ONE side of each sheet you hand in.

Answer ALL questions

Examination for FY2450, Astrophysics

Part A: Multiple Choice Questions

Choose a **single** *answer for each question in Part A unless otherwise stated. Write the answer down in the official examination paper that you hand in.* **[Part Marks]**

Question 1
There are a number of different "standard candles" that are very useful in astronomy for estimating distances to extragalactic objects. Which of the following are standard candles? [1 point for each correct answer up to a maximum of 6 if no incorrect answers. -1 point for each incorrect answer.]

(a) Type Ic SNe

(b) Type Ia SNe

(c) Gamma-ray bursts (Active Galactic Nuclei)

(d) Massive compact halo objects (MACHOs)

(e) Cepheid variables

(f) Lyα forest

[6]

Correct answers: b,e. There was an inconsistency between the pdf and the latex version of the exam for item c. Whatever you answered in c it will be graded as correct.

Question 2
Which of the following phenomena/epochs is associated with the formation of elements heavier than iron?

(a) Epoch of recombination

(b) Epoch of reionisation

(c) Big Bang nucleosynthesis

(d) Stellar nucleosynthesis

[2]

Correct answer: d

Question 3

A candle has a power in the visual band of 3 Watts (3×10^7erg/s). When this candle is placed at a distance of 3 km it has the same apparent brightness as a certain star. Assume that this star has the same luminosity as the Sun in the visual band ($\sim 10^{33}$ erg/s). How far away is the star?

(a) 5.60 pc

(b) 11 kpc

(c) 0.56 pc

(d) 0.11 pc

[4]

Answer: (c) Same flux:

$$\frac{3 \times 10^7 \text{ erg s}^{-1}}{(3 \text{ km})^2} = \frac{10^{33} \text{ erg s}^{-1}}{d^2}$$

$$d = \sqrt{\frac{10^{33} \text{ erg/s} * (3 \text{ km})^2}{3 \times 10^7 \text{ erg s}^{-1}}} = 0.56 \text{ pc}$$

Question 4

A radio interferometer, operating at a wavelength of 1 cm, consists of 100 small dishes, each 1 m in diameter, distributed within a 1 km diameter circle. What is the angular resolution of a single dish? What is the angular resolution of the interferometer array for a source directly overhead?

(a) $\theta_{\text{single dish}} \approx 0.01^\circ$, $\theta_{\text{array}} \approx 0.04''$

(b) $\theta_{\text{single dish}} \approx 0.7^\circ$, $\theta_{\text{array}} \approx 3''$

(c) $\theta_{\text{single dish}} \approx 0.01^\circ$, $\theta_{\text{array}} \approx 3''$

(d) $\theta_{\text{single dish}} \approx 0.7^\circ$, $\theta_{\text{array}} \approx 0.04''$

[4]

Answer: (b)

$$\theta_{\text{single dish}} \approx 1.22\frac{\lambda}{D_1} \sim 1.22\frac{1}{100} \text{ rad} \sim 0.7^\circ$$

$$\theta_{\text{array}} \approx \frac{\lambda}{D_{\text{array}}} \approx \frac{1}{1000 \cdot 100} \sim 10^{-5} \text{ rad} \sim 2''$$

NB: There was a typo in (b) - it should have been 2 arcsec not 3. Therefore, everyone is given full points.

Question 5

A certain star has a radius that is 500 times the radius of the Sun, and a temperature that is lower than that of the Sun by a factor of two. What is the star's luminosity in units of the luminosity of the Sun?

(a) $L_{\rm star} = 0.06 L_\odot$

(b) $L_{\rm star} = 16 L_\odot$

(c) $L_{\rm star} = 1.6 \times 10^4 L_\odot$

(d) $L_{\rm star} = 4.0 \times 10^6 L_\odot$

[2]

Answer:

$$L = \left(\frac{500 R_\odot}{R_\odot}\right)^2 \left(\frac{T_\odot}{2T_\odot}\right)^4 L_\odot \approx 1.6 \times 10^4 L_\odot$$

Question 6

A star is at a distance of 200 pc from the Earth. Find it's parallax angle θ''.

(a) $\theta'' = 0.005''$

(b) $\theta'' = 20''$

(c) $\theta'' = 0.4'$

(d) $\theta'' = 20'$

[2]

Answer:

$$D = 1 \text{ pc}/\theta''$$
$$\theta'' = 1\text{pc}/200\text{pc}$$
$$\theta'' = 0.005''$$

Question 7

The present day temperature of the CMB is $T_0 \approx 2.7$K. What was the temperature of the CMB at the surface of last scattering (redshift $z \approx 1100$)?

(a) $T \approx 2.7$ K

(b) $T \approx 3000$ K

(c) $T \approx 3 \times 10^6$ K

(d) $T \approx 3 \times 10^{12}$ K.

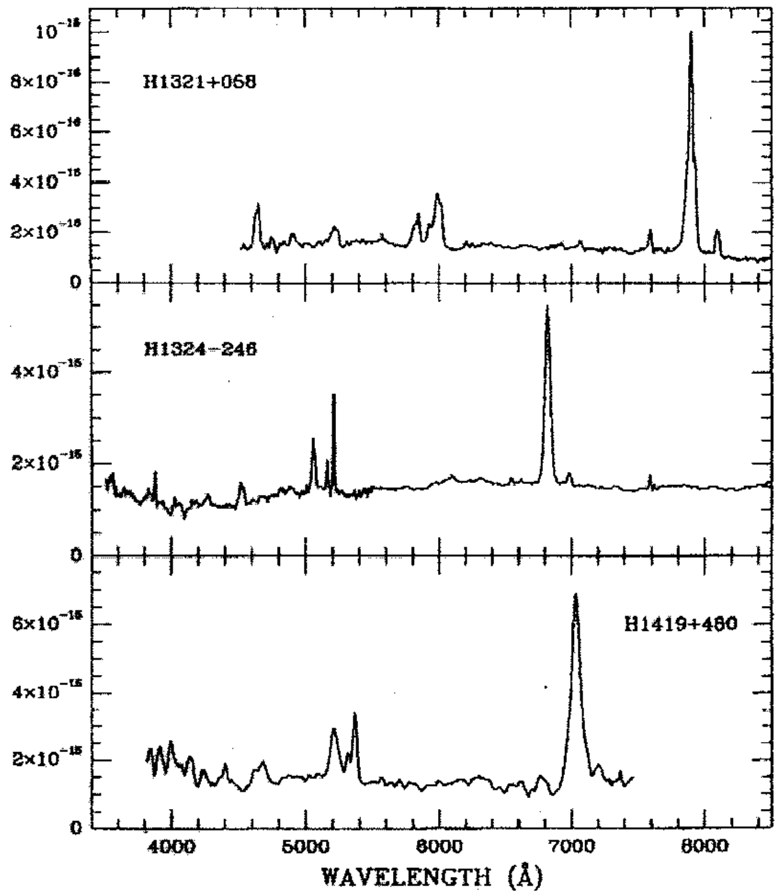
[2]

Answer: $T_{\rm rec} = T_0(1 + z_{\rm rec}) \approx 3000$ K.

Part B: Full Answers

Question 8

The plot below shows three quasar spectra. The most prominent line in each spectrum is due to Hα with a rest wavelength of 6563 Å. For the purpose of this problem, assume a value for the Hubble constant $H_0 = 70$ km s $^{-1}$ Mpc^{-1}.



(a) For the most distant of these quasars, find its approximate redshift $z = (\lambda - \lambda_0)/\lambda$. **[2]**

Note the typo in the definition of redshift! It should have been $z = (\lambda - \lambda_0)/\lambda_0$.
Correct answer: $\lambda \approx 7900$ Å. $z \approx (7900 - 6560)/6560 = 0.204$
Answer that also gets full points: $\lambda \approx 7900$ Å. $z \approx (7900 - 6560)/7900 = 0.17$.

[0 points if wrong quasar used with formula already provided). 1 point if the correct formula was used ($z = (\lambda - \lambda_0)/\lambda_0$) with the wrong quasar.]

(b) What is the approximate recession velocity of this quasar? **[2]**

Answer: $z \approx v/c$, $v \approx 0.2c \approx 60.000$ *km/s.*

[Formula: 1 point for using the correct formula.

Numerical answer: 1 point for correct numerical answer with correct units, 0.5 point for incorrect numerical answer with correct units. 0 points for numerical answer if no units.]

(c) Estimate the distance to the quasar. **[2]**
Answer: $v \approx H_0/d$. $d \sim 60000$ km/s $/H_0. = 60000/70 = 860$ *Mpc.*

[Formula: 1 point for using the correct formula.
Numerical answer: 1 point for correct numerical answer with correct units, 0.5 point for incorrect numerical answer with correct units. 0 points for numerical answer if no units.]

(d) The energy flux from this quasar is 3×10^{-13} erg cm^{-2} s^{-1}. Find the luminosity of the quasar. **[2]**
Answer: $L = 4\pi d^2 f \approx 12(860 \text{ Mpc})^2 \cdot 3 \times 10^{-13} \approx 3 \times 10^{43}$*erg/s.*

[Formula: 1 point for using the correct formula.
Numerical answer: 1 point for correct numerical answer with correct units, 0.5 point for incorrect numerical answer with correct units. 0 points for numerical answer if no units.]

(e) A model for the central object in this quasar involves a supermassive black hole of mass 10^7 $M_\odot$. Is this model consistent or not with the concept of an Eddington limit? Explain.
Answer: $L_{\text{Edd}} \approx 10^{38}(M/M_\odot) \approx 10^{45}$ *erg/s for a* 10^7 $M_\odot$ *black hole. [Max 2 points: 1 point for using the expression correctly, Numerical answer: 1 point for correct answer in erg/s. 0.5 point if numerical error.] Since* $L \ll L_{\text{Edd}}$ *this is consistent with the supermassive BH model. [Max 2 points: 1 point if the wrong conclusion is reached because of numerical error in part (d) or first part of part (e).]* No points for simply stating what the Eddington limit is. 0.5 points for writing that it is consistent but with the wrong reasoning / explanation for what Eddington limit is. 1 point for giving the correct reasoning but not writing whether it is consistent or not. **[4]**

Question 9

The orbits of several stars have been measured orbiting the massive black hole near the center of our Galaxy. One of them has an orbital period of 15 years, and the orbital radius is 0.12 second of arc (as seen from the Earth). The distance to the Galactic centre is approximately 8 kpc.

(a) What is the orbital radius of this star? **[2]**
Answer: $\theta'' = 0.12'' = 6 \times 10^{-7}$ rad, $d = 8$ kpc.

$$r \sim \theta \text{ (in radians)} \cdot 8 \text{ kpc} \approx 6 \times 10^{-7} \text{ rad} \cdot 8 \text{ kpc} \approx 1.4 \times 10^{16} \text{ cm}.$$

[-1 point for any answer using θ in degrees or arcmin, arcsec etc.]

(b) Starting from Newton's second law $F = ma$, write down Kepler's third law which connects the radius and orbital period of a star orbiting a massive central object with mass $M_{\rm bh}$. (Assume that the star is far enough away that Newton's law of gravity applies) **[4]**

Answer:

$$F = ma = m\frac{v^2}{r}\text{[1 point]}$$
$$\frac{GmM}{r^2} = m\frac{v^2}{r}\text{[1 point]}$$
$$\frac{GM_{\rm bh}}{r} = v^2$$
$$v = \frac{2\pi r}{P}\text{[1 point]}$$
$$\frac{GM_{\rm bh}}{r} = \left(\frac{2\pi r}{P}\right)^2\text{ [1 point]}$$

c) Hence compute the mass of the black hole in the centre of the Milky Way. Express your answer in terms of $M_\odot$. **[2]**

Answer: $P = 15$ yr and $r \approx 1.4 \times 10^{16}$ cm.

$$M = \frac{4\pi^2 r^3}{P^2 G} \approx 3.6 \times 10^6 M_\odot$$

1 point for correct expression.
1 point for correct numerical answer in terms of $M_\odot$. 0.5 point for correct numerical answer in other units. 0.5 point for those who realised that their answer here is wrong.

Question 10

The Robertson-Walker metric describes a homogeneous and isotropic, expanding Universe

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \alpha(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right]$$

Here, $\alpha(t)$ is the scale factor at time t and k the curvature parameter. By considering the path of a photon, show that in a flat universe the comoving distance between an object and the origin is

$$r = c\int_{t_e}^{t_0} \frac{\mathrm{d}t}{\alpha(t)}$$

where t_e and t_0 are the time of emission and observation, respectively. Explain your reasoning. **[2]**

Answer: Note that there was a typo in the question and the RW metric reads

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \alpha^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right]$$

Any answer that demonstrates some understanding of the issue will be given full points. Likewise any answer that ignores that there was a typo and gets to the correct result will be given full points.

A photon has $ds^2 = 0$, because it travels at the speed of light. [0.5 point. 0 points here if no explanation why.]. In a flat Universe k = 0 [0.5 point].

For a radial trajectory the RW metric gives $\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + \alpha(t)\mathrm{d}r^2$, from which result follows immediately. [1 point. - 0.5 point if no explanation why the angular terms were set to zero.].

Hence show that, in an expanding universe, the observed light will be redshifted such that

$$\frac{\lambda_e}{\alpha(t_e)} = \frac{\lambda_0}{\alpha(t_0)}$$

where λ_e is the emitted wavelength and λ_0 is the observed wavelength.

[6]

Model Answer: If a wave crest is emitted at time t_e and observed at time t_0, then the comoving proper distance is given by the previous sub-question. Considering the emission and reception of the next wave crest, which is emitted at $t_e + \lambda_e/c$ and observed at $t_0 + \lambda_0/c$, gives

$$r = c\int_{t_e+\lambda_e/c}^{t_0+\lambda_0/c} \frac{\mathrm{d}t}{\alpha(t)}$$

[1 point. Here one could also consider the interval from t_e to $t_e + \mathrm{d}t_e$ and from t_0 to $t_0 + \mathrm{d}t_0$.]

Now subtract from both integrals the common interval $c\int_{t_e+\lambda_e/c}^{t_0} \frac{\mathrm{d}t}{\alpha(t)}$, giving

$$c\int_{t_e}^{t_e+\lambda_e/c} \frac{\mathrm{d}t}{\alpha(t)} = c\int_{t_0}^{t_0+\lambda_0/c} \frac{\mathrm{d}t}{\alpha(t)}.$$

[3 points for arriving at this expression with an explanation in words or with a sketch as we did in class]

But in this small time interval (assuming, reasonably, that $\lambda \ll c/H_0$) we can neglect the change in α, so this equation becomes

$$\frac{c}{\alpha(t_e)}\int_{t_e}^{t_e+\lambda_e/c} \mathrm{d}t = \frac{c}{\alpha(t_0)}\int_{t_0}^{t_0+\lambda_0/c} \mathrm{d}t.$$

which integrates trivially to give the required answer.

[1 point for correctly stating that the change of α in the small time interval from t_e to $t_e + \mathrm{d}t_e$ and from t_0 to $t_0 + \mathrm{d}t_0$ can be neglected. 1 point for correctly arriving at the conclusion that

$$\frac{c}{\alpha(t_e)}\int_{t_e}^{t_e+\lambda_e/c} \mathrm{d}t = \frac{c}{\alpha(t_0)}\int_{t_0}^{t_0+\lambda_0/c} \mathrm{d}t.$$

based on previous working or arguments].

[-Alternate correct solutions, e.g. starting from Hubble's law $v = r \cdot H_0$ and performing the integration are also given full points.
- Solutions starting from the definition of redshift $\alpha(t_0)/\alpha(t_e) = 1 + z$ are given 4 points (since the answer is correct but the question asked to use the definition of comoving distance in a flat Universe).]

Question 11

A neutron star whose mass is $1.4M_\odot$ and radius is 10 km is in orbit about a normal star. Matter is flowing from the normal star onto the neutron star at a rate of 10^{17}g/s.

(a) Assume that all the gravitational potential energy of the infalling matter is being converted to radiation. Compute the bolometric luminosity of the star. (You may ignore any relativistic effects and assume the standard Newtonian formula for the gravitational potential energy.) **[4]**

(b) Suppose that the surface of the neutron star radiates as a blackbody of temperature T. Compute T. **[4]**

(c) Calculate the wavelength and hence name the spectral band of radiation that is being emitted. **[5]**

Answer:

(a)

$$L \approx \frac{GM\dot{M}}{r} \approx 2 \times 10^{37} \text{ erg/s}$$

[Formula: 2 points for using the correct equation. Numerical answer: Maximum of 2 points. 1.5 point for numerical error due to one wrong quantity. 1 point for numerical error due to two wrong quantities etc. -1 point if no units or completely wrong units.]

(b)

$$L = 4\pi R^2 \sigma T^4$$

$$T = \left(\frac{L}{4\pi\sigma R^2}\right)^{1/4} = 1.2 \times 10^7 \text{ K}$$

[Formula: 1 point for using the correct formula] [Numerical answer: 3 points for correct numerical answer. 2 points if error carried over from part (a). -1 point if no units or completely wrong units.]

(c) From Wien's displacement law $E \approx 2.9$ keV or $\lambda \sim 3\mathring{A}$.

[Up to a max of 4 points for correct estimate of wavelength or photon energy. Formula: 2 points for using the correct formula. Numerical answer: 1.5 points if error carried over from part (a) or (b). -1 point if no units or completely wrong units.] This is X-ray radiation. [1 point]

[Total Marks = 63]

END OF PAPER

Constants

Gravitational constant	$G = 6.7 \times 10^{-8}$ erg cm g^{-2}
Speed of light	$c = 3 \times 10^{10}$ cm s^{-1}
Planck's constant	$h = 6.6 \times 10^{-27}$ erg s
	$\hbar = h/2\pi = 1.05 \times 10^{-27}$ erg s
Boltzmann's constant	$k = 1.4 \times 10^{-16}$ erg K^{-1}
Stefan-Boltzmann constant	$\sigma = 5.7 \times 10^{-5}$ erg cm^{-2} s^{-1} K^{-4}
Radiation constant	$\alpha = 4\sigma/c = 7.6 \times 10^{-15}$ erg cm^{-3} K^{-4}
Proton mass	$m_p = 1.7 \times 10^{-24}$ g
Electron mass	$m_e = 9.1 \times 10^{-28}$ g
Electron charge	$e = 4.8 \times 10^{-10}$ esu
Electron volt	1 eV$= 1.6 \times 10^{-12}$ erg
Erg	1 erg $= 1$ cm$^2 \cdot$ g $\cdot$ s^{-2}
Thomson cross section	$\sigma_{\mathrm{T}} = 6.7 \times 10^{-25}$ cm^2
Angstrom	1 Å$= 10^{-8}$ cm
Solar Mass	$M_\odot = 2 \times 10^{33}$ g
Solar Luminosity	$L_\odot = 3.8 \times 10^{33}$ erg s^{-1}
Solar Radius	$r_\odot = 7.0 \times 10^{10}$ cm
Solar effective temperature	$T_\odot = 5780$K
Astronomical Unit	1 AU $= 1.496 \times 10^{13}$ cm
Parsec	1 pc $= 3.086 \times 10^{18}$ cm
Megaparsec	1 Mpc $= 3.086 \times 10^{24}$ cm

Useful relations and formulae

Rest-mass energy of body with mass m

$$E = mc^2.$$

Non-relativistic kinetic energy of a body with mass m and velocity v

$$E = \frac{1}{2}mv^2.$$

Gravitational potential energy for two bodies with masses M, and m a distance r apart

$$E = -\frac{GMm}{r}.$$

Centripetal force

$$F = -\frac{mv^2}{r}.$$

Gravitational force

$$F = -\frac{GMm}{r^2}.$$

Wien's displacement law: wavelength of maximum intensity

$$\lambda_{\rm max} = 2900\ \text{Å}\ \left(\frac{10^4\ \rm K}{T}\right)$$

$$h\nu_{\rm max} = 2.4\ {\rm eV}\ \left(\frac{10^4\ \rm K}{T}\right)^{-1}.$$

The $^{-1}$ was missing here. Luckily nobody seems to have used this formula.

Luminosity of black body with temperature T

$$L = 4\pi R^2 \sigma T^4.$$

Eddington luminosity of object with mass M and radius r

$$L_{\rm E} = \frac{4\pi c G M m_p}{\sigma_{\rm T}}$$