NTNU Trondheim, Institut for fysikk

Examination for FY2450 Astrophysics 1

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Possibles languages for your answers: Bókmal, English, German, Nynorsk.
Allowed tools: Pocket calculator, mathematical tables
Some formulas and numerical values of constants can be found at the end.
Grades: 18.6.2008

1. The Sun.

a) Calculate the apparent visual magnitude $m_{V,\odot}$ of the Sun. (2 pt)
b) Calculate the radius of the Sun. (2 pt)
c) Estimate the central temperature of the Sun. (Hint: Apply the virial theorem to one particle.) (4 pt)
d) Estimate the central pressure of the Sun. (4 pt)

a) Inserting the distance $d = AU = 1.496 \times 10^{13} cm = 4.85 \times 10^{-6} pc$ into $m - M = 5\log(d/10\text{pc})$ gives as result

$$m_{V,\odot} = M_{V,\odot} + 5\log\frac{d}{10\text{pc}} \approx 4.79 - 31.6 = -26.78\,.$$

b) Approximating the Sun as blackbody radiator, $L = 4\pi R^2 \sigma T^4$, it follows

$$R_\odot = \left(\frac{L_\odot}{4\pi\sigma}\right)^{1/2}\left(\frac{1}{T_\odot}\right)^2 = \left(\frac{3.84\times10^{33} erg/s}{4\pi\times5.670\times10^{-5} ergs^{-1}cm^{-2}K^{-4}}\right)^{1/2}\left(\frac{1}{5784K}\right)^2 = 6.9\times10^{10} cm\,.$$

c) The gravitational potential energy of a proton at the center of the Sun is approximately given by

$$\langle E_{\text{grav}}\rangle = -\frac{GM_\odot m_p}{R_\odot} \approx -3.2\text{keV}/c^2$$

For a thermal velocity distribution of a Maxwell-Boltzmann gas we obtain

$$\langle E_{\text{kin}}\rangle = \frac{3}{2}kT = -\frac{1}{2}\langle E_{\text{grav}}\rangle \approx 1.6\text{ keV}/c^2\,.$$

Hence our estimate for the central temperature of the Sun is $T_c \approx 1.1\,\text{keV}/c^2 \approx 1.2\times10^7$ K.
d) The simplest estimate for the central pressure $P_c = P(0)$ of the Sun is obtained by converting the hydrostatic equilibrium equation into a difference equation,

$$P_c \sim \frac{3GM_\odot^2}{4\pi R_\odot^4}\,.$$

A lower bound for the central pressure $P_c = P(0)$ may be derived with $P(R) \approx 0$ by

$$P_c = \int_0^R \frac{\mathrm{d}P}{\mathrm{d}r}\,\mathrm{d}r = G\int_0^M \mathrm{d}M\,\frac{M}{4\pi r^4}\,,$$

where we used the continuity equation to substitute $\mathrm{d}r = \mathrm{d}M/(4\pi r^2 \rho)$ *by* $\mathrm{d}M$. *Replacing r by the stellar radius* $R \geq r$, *we obtain a lower limit for the central pressure,*

$$P_c = G \int_0^M \mathrm{d}M \, \frac{M}{4\pi r^4} > G \int_0^M \mathrm{d}M \, \frac{M}{4\pi R^4} = \frac{M^2}{8\pi R^4}\,.$$

Inserting values for the Sun, it follows

$$P_c > \frac{M_\odot^2}{8\pi R_\odot^4} = 4 \times 10^8 \mathrm{bar}\,.$$

Alternatively one may use a constant average density $\rho_\odot \approx 1g/cm^3$ *or use the ideal gas law* $P = nkT = R\rho T/\mu$.

2. Energy.

a) Which of the following energy resources – 1. internal heat, 2. gravitational potential energy, 3. nuclear fusion reactions, 4. nuclear fission reactions, 5. others – is the main energy source of a

white dwarf star ...1... (1 pt)

main-sequence star ...3... (1 pt)

active galactic nucleus ...2... (1 pt)

b. Photons produced in the center of a supergiant star with radius $R = 100R_\odot$ escape from this star in roughly: (1 pt)

☐ 2.3 seconds

☐ 230 seconds

☐ a month

☐ 10 years

⊗ many million years

☐ never: they cannot escape, because they are trapped inside the event horizon.

b) Since the interaction length $l_{\rm int}$ *of photons is much smaller than* $R_\odot$ *and they are scattered isotropically, they perform a random walk and need much longer than* $R_\odot/c \sim 230$ *s. For an estimate see solutions to exercise sheet of week 3.*

3. Hertzspung-Russel diagram.

a) Sketch schematically a Hertzsprung-Russell diagram including the main-sequence and (super-) giant stars, white dwarfs and neutron stars. (2 pt)

b) Indicate in which direction along the main-sequence the i) radius and the ii) life-time of stars increases. (2 pt)

Compare with the script or any textbook. Neutron stars are not contained in a standard Hertzspung-Russel diagram, because i) they are not luminous enough or ii) emit mainly non-thermal radiation. A diagram without labels for axes has no sense.

4. Nucleosynthesis.

Explain how different elements are synthesized. (2 pt)

(Most part of) light elements up to Li-7 was formed when the expanding universe at $t \sim 1$ s cooled down to $T \sim 0.2$ MeV. Heavier elements up to Fe-56, the stablest element, are produced by fusion in stars. Even heavier elements are suppossed to be produced in supernova explosions.

5. Dark matter.

Give two observations that indicate the presence of dark matter. (2 pt)

i) Velocity curves of galaxies, ii) virial mass of cluster of galaxies, iii) structure formation requires non-baryonic matter, iv) CMB and SNIa require $\Omega_m \sim 0.3$ but $\Omega_b \sim 0.04$, v) the bullet cluster.

6. Cosmology.

The (first) Friedmann equation can be written as

$$H^2 = \frac{8\pi}{3} G\rho - \frac{kc^2}{R^2} + \frac{\Lambda c^2}{3}$$

a) Explain (briefly!) the meaning of H, Λ, k and R. (2 pt)
b) Which one of the four quantities H, Λ, k and R is determined by the loose phrase "The size of the observable universe is 4 Gpc"? (1 pt)
c) Estimate the distance to a galaxy with redshift $z = 0.1$. (1 pt)
d) How is the critical density $\rho_{\rm cr}$ defined? Estimate the present value of $\rho_{\rm cr}$. (3 pt)
e) The current number density of protons and of photons is $n_p = 2.5 \times 10^{-7}$ cm^3 and $n_\gamma = 411/cm^3$, respectively. What were the values of n_p and n_γ at redshift $z = 10$? (2 pt)

a) The Hubble parameter $H = \dot{a}/a$ gives the expansion rate of the universe. (For small distance, $v = H_0 d$ ['Hubbles law'], determines the recession velocity of galaxies.).
The cosmological constant Λ accounts for a possible intrinisc (constant) energy density and pressure of the vacuum, with E.o.S. $\rho = -p$.
The three cases $k = -1, 0, +1$ distinguishe the geometry of space, i.e. give the curvature of 3-dimensional hypersurfaces with $t = \text{const.}$; $k = 0$ flat. (N.B.: The simple correspondance $k = 1$, expansion stops for finite t holds only for $\Lambda = 0$).
R is the curvature radius of the Universe. (The quantity a in $H = \dot{a}/a$ is the scale factor, its time-dependence gives the relative distance of two observers at rest.)
b) The size of the horizon determines via $4Gpc \approx ct_0 \approx c/H_0$ the value of the Hubble constant.
c) Roughly 10% of 4 Gpc, i.e. 400 Mpc.
d) The density of a flat universe equals by definition the critical density $\rho_{\rm cr}$. Thus it is obtained by setting $k = 0$ in the Friedmann eq., $\rho_{\rm cr} = 3H_0^2/(8\pi G)$. (N.B. This holds also for $\Lambda \neq 0$ with $\rho_\lambda = \frac{\Lambda}{8\pi G}$). The value of H_0 can be estimated from the horizon distance (cf. b.) or or via $H_0 \approx 1/t_0$.
e) Without interactions, both the number of photons and of protons is conserved. Thus they scale as $(1+z)^3 = (a_0/a)^3 = 11^3$, or $n_i(z = 10) = 11^3 n_i(z = 0)$.