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Examination, course FY2450 Astrophysics
Wednesday 23rd May, 2012
Time: 09.00 – 13.00

Allowed to use: Calculator, translation dictionary, printed or hand written notes covering a maximum of one side of A5 paper.

On pages 5 and 6 you will find a table of the properties of main sequence stars, a list of physical and astronomical constants in CGS and SI units and the equations of stellar colour.

Answer all questions, each of the 15 sub-problems will be weighted equally in the grading

Please answer questions in English.

1. (a) The Kepler satellite observes variations in the light curve of a star with a spectral type of F5V. A planetary transit is observed with a repeat period of 6 months. Estimate the mean distance between the star and the planet. How does this orbital radius compare to the so-called "habitable zone" for an Earth-like planet in orbit around this F5V star?

(b) High resolution spectroscopic observations of the atomic lines in the photosphere of this star show its radial velocity changes by up to ±20 m/s over the six month orbital period. Estimate the mass of the planet.

(c) The fading time of the observed light curve as the planet both begins and ends its transit across this star's disk is measured to be 3000 seconds. Estimate the density of the planet. Compared to Solar System objects, what sort of planet is this?

(d) Photometric observations of this star reveal a B-band apparent magnitude (m_B) of +8.0 and a V-band apparent magnitude (m_V) of +7.0. Estimate the distance to the planetary system and the minimum diameter optical telescope required to spatially resolve the star and the planet at maximum separation.

(e) Estimate the planet's mean surface temperature. At what wavelength of light does the planet's continuum spectrum reach maximum intensity? What is the ratio of the spectral luminosity of the planet to that of the star at this particular wavelength of light?

2. (a) Starting from the definition of the gravitational potential energy between two point masses (relative to infinity):

$U = -Gm_1m_2/r$

where m_1 and m_2 are the point masses, r is their separation and G is the gravitational constant, show how the gravitational potential energy of a star can be approximated by:

$U = -(3/5)GM^2/R$

where M is the mass of a spherical star of uniform constant density and R is the star's radius.

(b) Use this result to calculate how long the Sun could use this stored gravitational energy to maintain its current luminosity (the gravitational lifetime or Kelvin time) without any further sources of energy.

(c) The equation of hydrostatic equilibrium can be used to relate the pressure gradient (dP/dr) and the density ($\rho(r)$) in a star:

$dP/dr = -[GM(r)/r^2]\ \rho(r)$

where M(r) is the total mass contained within a sphere of radius r. Use this, and an appropriate equation of state, to estimate the temperature in the core of the Sun assuming that the Sun has a constant uniform density and is composed entirely of ionised hydrogen.

(d) Give an example of a step-by-step reaction sequence inside the Sun's core that converts protons to alpha particles.

(e) As the Sun evolves during its main sequence lifetime, explain why the core slowly gets hotter. How does the photosphere of the Sun respond to this core heating – illustrate your answer with a sketch of the Sun's path as an evolutionary track relative to the zero-age main sequence on a Hertzsprung-Russell diagram during this period of evolution.

3. (a) The material in a spiral galaxy is in an approximate circular orbit a distance r from the galaxy's centre of mass. The mass contained within a sphere interior to r provides the gravitational acceleration to keep the material in a circular orbit. The orbital speed of the local standard of rest (LSR) in the Milky Way is assumed to be 220 km/s at a distance 8.5 kpc from the centre of mass. Calculate the mass contained within the orbit of the LSR. Express the answer in solar masses.

(b) The rotation curve of The Galaxy is observed to be "flat" out to the edge of the visible disk. Thus v(r) is constant (equal to v_0). Use this information to show that the mass density at a distance r from the galactic centre is given by:

$\rho(r) = v_0^2/(4\pi G r^2)$

where G is the gravitational constant.

(c) Sketch the orbital period of material in the Milky Way as a function of distance from the galactic centre and explain why it is unlikely that spiral structure in the Galaxy could be carried by the visible matter in the Galaxy if the age of the Galaxy is comparable to the age of the Universe.

(d) The spiral structure in a galaxy is believed to be due to slow-moving "density waves". Explain why you would expect to see enhanced star formation in the spiral arms of galaxies.

(e) If a heliocentric recession velocity of +35 km/s is measured from mm observations of the CO rotational lines in a molecular cloud along a line of sight with galactic latitude = 0° and galactic longitude = +30°, estimate the kinematic distance(s) to the molecular cloud. You should get two possible answers. Describe how you might resolve this distance ambiguity observationally.

Properties of main sequence stars

Spectral type	M_V[1]	*B-V*	$T_{eff}(K)$[2]	M/M_{Sun}	R/R_{Sun}	L/L_{Sun}
O5	-6	-0.45	35000	39.8	17.8	3.2×10^5
B0	-3.7	-0.31	21000	17.0	7.6	1.3×10^4
B5	-0.9	-0.17	13500	7.1	4.0	6.3×10^2
A0	+0.7	+0.0	9700	3.6	2.6	7.9×10^1
A5	+2.0	+0.16	8100	2.2	1.8	2.0×10^1
F0	+2.8	+0.30	7200	1.8	1.4	6.3
F5	+3.8	+0.45	6500	1.4	1.2	2.5
G0	+4.6	+0.57	6000	1.1	1.05	1.3
G5	+5.2	+0.70	5400	0.9	0.93	7.9×10^{-1}
K0	+6.0	+0.81	4700	0.8	0.85	4.0×10^{-1}
K5	+7.4	+1.11	4000	0.7	0.74	1.6×10^{-1}
M0	+8.9	+1.39	3300	0.5	0.63	6.3×10^{-2}
M5	+12.0	+1.61	2600	0.2	0.32	7.9×10^{-3}

(1) Absolute V-band magnitude

(2) Effective surface temperature

Physical constants

speed of light	c	2.998×10^{10} cm s^{-1}	2.998×10^8 m s^{-1}
gravitational constant	G	6.673×10^{-8} dyne cm^2 g^{-2}	6.673×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann constant	k	1.381×10^{-16} erg K^{-1}	1.381×10^{-23} J K^{-1}
Planck's constant	h	6.626×10^{-27} erg s	6.626×10^{-34} J s
Stefan–Boltzmann constant	σ	5.670×10^{-5} erg cm^{-2} K^{-4} s^{-1}	5.670×10^{-8} W m^{-2} K^{-4}
Wien displacement constant	$\lambda_{max}T$	2.898×10^{-1} cm K	2.898×10^{-3} m K
Rydberg constant	R	1.097×10^5 cm^{-1}	1.097×10^7 m^{-1}
mass of proton	m_p	1.6726×10^{-24} g	1.6726×10^{-27} kg
mass of neutron	m_n	1.6749×10^{-24} g	1.6749×10^{-27} kg
mass of electron	m_e	9.1096×10^{-28} g	9.1096×10^{-31} kg
mass of hydrogen atom	m_H	1.6735×10^{-24} g	1.6735×10^{-27} kg

Astronomical constants

astronomical unit	AU	1.496×10^{13} cm	1.496×10^{11} m
parsec	pc	3.086×10^{18} cm	3.086×10^{16} m
solar mass	M_{Sun}	1.989×10^{33} g	1.989×10^{30} kg
solar radius (mean)	R_{Sun}	6.960×10^{10} cm	6.960×10^8 m
solar luminosity	L_{Sun}	3.839×10^{33} erg s^{-1}	3.839×10^{26} J s^{-1}
Earth mass	M_E	5.977×10^{27} g	5.977×10^{24} kg
Earth radius (mean)	R_E	6.371×10^8 cm	6.371×10^6 m
Jupiter mass	M_J	1.899×10^{30} g	1.899×10^{27} kg
Jupiter radius (mean)	R_J	6.991×10^9 cm	6.991×10^7 m

The equations of stellar colour

Planck's empirical law: Energy per second per frequency interval per unit area

$I(\nu,T) = [2h\nu^3/c^2] / [\exp(h\nu/kT) - 1]$

Planck's empirical law: Energy per second per wavelength interval per unit area

$I(\lambda,T) = [2hc^2/\lambda^5] / [\exp(hc/\lambda kT) - 1]$

Wien's displacement law: wavelength of maximum intensity

$\lambda_{max}T = 2.898 \times 10^6$ nm K

Stefan-Boltzmann law: Integrated energy per second per unit surface area

$E = \sigma T^4$

Integrated energy per second from a sphere: e.g. the total (bolometric) luminosity of a star

$L = 4\pi R^2\sigma T^4$