NTNU – Trondheim
Norwegian University of
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NTNU, Department of Physics

Exam FY2450 Spring 2018

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Permitted examination support material:
Approved calculator
Rottmann: Matematisk Formelsamling
Rottmann: Matematische Formelsammlung
Barnett & Cronin: Mathematical Formulae
Angell og Lian: Fysiske størrelser og enheter: navn og symboler

The problem set consists of five pages. Read carefully.

Problem 1

a) State Kepler's three laws of planetary motion.

b) Consider a planet of mass m moving in an elliptical orbit, see Fig. 1. Using polar coordinates (r, θ), the motion can be described by $r = r(\theta)$,

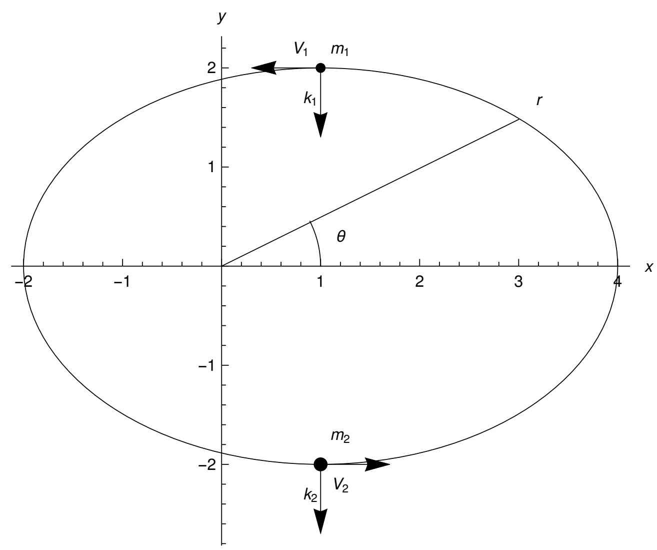

Figure 1: Planet moving in an elliptical orbit.

where

$$r(\theta) \;=\; \frac{r_0}{1 - e\cos\theta} \; . \tag{1}$$

Here $r_0 = \frac{L^2}{GMm^2}$, e is the eccentricity, L is the angular momentum of the planet, M is the solar mass, and G is Newton's constant of gravitation. The velocity of the planet in polar coordinates is given by

$$\mathbf{v} \;=\; \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta \; , \tag{2}$$

where $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are unit vectors in polar coordinates. Show that the speed v is given by

$$v \;=\; |\mathbf{v}| = A\sqrt{1 + e^2 - 2e\cos\theta} \; , \tag{3}$$

and determine the dimensionful constant A. Hint: $\frac{L}{m} = r^2\dot{\theta}$.

c) A light source on the planet is continually emitting light whose frequency in the rest frame of the source is ω'. The light is emitted in the negative y-direction, $\mathbf{k} = -|\mathbf{k}|\mathbf{e}_y$. Let α be angle the between the velocity vector $\mathbf{v}$ and the wavevector $\mathbf{k}$. Show that

$$\cos\alpha \;=\; \frac{e - B\cos\theta}{\sqrt{1+e^2-2e\cos\theta}}\,, \tag{4}$$

and determine the dimensionless constant B.

d) An observer at rest in an inertial frame whose coordinate system is given by (r,θ). The observer receives the photons, measuring the frequency ω. Calculate the Doppler shift, i.e. calculate the ratio $\frac{\omega}{\omega'}$.

Problem 2

Consider a curved space-time whose metric in spherical coordinates (r,ϕ,θ) is given by

$$ds^2 = -\left(1-\frac{M}{r}\right)^2 c^2dt^2 + \left(1-\frac{M}{r}\right)^{-2} dr^2 + r^2\left(d\theta^2+\sin^2\theta d\phi^2\right)\;, \tag{5}$$

where t is a time coordinate and M is a constant.

a) What is the dimension of M? Find the singular points of the metric and classify them.

b) A stationary source located at $r_A > r_s = M$ emits light of frequency ω_A in the radial direction. Calculate the frequency ω_B of the absorbed light by a stationary detector located at $r_B > r_A$. What happens as $r_A \to r_s$?

Problem 3

In this problem, we set $G = c = 1$. Friedmann's equations for a homogeneous and isotropic universe are

$$
\begin{aligned}
3\frac{\dot{a}^2 + k}{a^2} &= 8\pi\rho + \Lambda \ , \\
-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2 + k}{a^2} &= 8\pi p - \Lambda \ .
\end{aligned}
$$

a) Define the quantities a, k, ρ, p, and Λ.

b) What are the possible values of k? What is the spatial geometry of the universe for each value of k?

c) Define the concepts homogeneous and isotropic.

d) Consider the case $k = 1$ and $\rho_r = 0$. Show that there is a solution to the Friedmann equations, where a and ρ_m are independent of time. Express a and ρ_m in terms of Λ.

e) The solution in **d**) is called Einstein's static universe. Find the cosmological redshift in this universe.

Problem 4

a) A gravitationally bound molecular cloud is collapsing due to gravity. The temperature of the gas is increasing and at some point it is so high that fusion of protons can take place in the core of the star. Explain briefly the mechanism behind the heating of the gas. What is the approximate temperature required for fusion?

b) Define the concept of binding energy of a nucleus.

c) The triple-α process is a very important fusion process in the interior of stars. It consists of the two steps

$$
\begin{aligned}
{}^{4}\mathrm{He} + {}^{4}\mathrm{He} &\rightarrow {}^{8}\mathrm{Be} \,, && (6)\\
{}^{4}\mathrm{He} + {}^{8}\mathrm{Be} &\rightarrow {}^{12}\mathrm{C} + 2\gamma \,. && (7)
\end{aligned}
$$

Does this process take place in all stars independently of the mass?

Useful formulas

$$\frac{\omega}{c} = \frac{\omega'}{c} \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}\cos\alpha} \,. \tag{8}$$
