NTNU – Trondheim
Norwegian University of
Science and Technology

NTNU, Department of Physics

Exam FY2450 Spring 2019

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May 28 2019
09:00-13:00

Permitted examination support material:
Approved calculator
Rottmann: Matematisk Formelsamling
Rottmann: Matematische Formelsammlung
Barnett & Cronin: Mathematical Formulae
Angell og Lian: Fysiske størrelser og enheter: navn og symboler

The problem set consists of four pages. Read carefully. Good luck! Bonne chance! Viel Glück! Veel succes! Lykke til!

Problem 1

a) Define tidal force. Consider the system Earth-Moon. Explain how tidal forces give rise to the tides on Earth.

b) Consider the system Earth-Moon-Sun. How often do high tide and low tide occur? Draw a figure and explain when we get neap tide and spring tide. How often do neap tide and spring tide occur?

Hint:

(a) It takes Earth 24h to complete a rotation with respect to the Sun and it takes 24h 50min to complete a rotation with respect to the Moon.

(b) It takes 29.5 days from new moon to new moon.

Problem 2

We consider the Schwarzschild geometry outside a spherically symmetric mass distribution with a total mass M. Using the coordinates (ct, r, θ, ϕ), the line element is given by

$$ds^2 = -\left(1 - \frac{2GM}{c^2 r}\right) c^2 dt^2 + \left(1 - \frac{2GM}{c^2 r}\right)^{-1} dr^2 + r^2 d\Omega^2 \, . \tag{1}$$

a) What is the interpretation of the coordinate time t? What is a physical singularity? What is a coordinate singularity? What is a horizon? Give an example of each from the Schwarzschild geometry above.

b) One can introduce the so-called Kruskal–Szekeres coordinates X and T defined as

$$T = \left(\frac{c^2 r}{2GM} - 1\right)^{\frac{1}{2}} e^{\frac{c^2 r}{4GM}} \sinh\left(\frac{c^3 t}{4GM}\right) \, , \tag{2}$$

$$X = \left(\frac{c^2 r}{2GM} - 1\right)^{\frac{1}{2}} e^{\frac{c^2 r}{4GM}} \cosh\left(\frac{c^3 t}{4GM}\right) \, , \tag{3}$$

valid for $r > \frac{2GM}{c^2}$ and

$$T = \left(1 - \frac{c^2 r}{2GM}\right)^{\frac{1}{2}} e^{\frac{c^2 r}{4GM}} \cosh\left(\frac{c^3 t}{4GM}\right) \, , \tag{4}$$

$$X = \left(1 - \frac{c^2 r}{2GM}\right)^{\frac{1}{2}} e^{\frac{c^2 r}{4GM}} \sinh\left(\frac{c^3 t}{4GM}\right) \, , \tag{5}$$

valid for $r < \frac{2GM}{c^2}$. In these coordinates the line element can be written as

$$ds^2 = 32 \left(\frac{GM}{c^2}\right)^2 \frac{GM}{c^2 r} e^{-\frac{c^2 r}{2GM}} \left(-dT^2 + dX^2\right) + r^2 d\Omega^2 \, . \tag{6}$$

Find the equation for radial light rays and draw these lines in an X-T diagram.

c) The point $r = 0$ corresponds to a line in an X-T diagram. Find the equation for this line and draw it in the same X-T diagram. The surface $r = \frac{2GM}{c^2}$ also corresponds to a line in an X-T diagram. Find the equation for this line and draw it in the same X-T diagram. Use this equation and the result in **b**) to show that $r = \frac{2GM}{c^2}$ is a horizon.

d) Consider two stationary observers located outside $r = \frac{2GM}{c^2}$ at $r = r_A$ and $r = r_B$, respectively. Sketch their worldlines in the same diagram as well as the worldline of a light signal that connects the two observers.

Problem 3

a) Explain briefly an eclipsing binary. Sketch the apparent brightness as a function of time.

b) Use the formula

$$\frac{\omega}{c} = \frac{\omega'}{c} \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c}\cos\alpha} \tag{7}$$

to derive the expression for the radial velocity v_r in terms of the Doppler-shifted wavelength,

$$\frac{\Delta\lambda}{\lambda} = \frac{v_r}{c}\,. \tag{8}$$

c) Fig. 1 shows the radial velocity v_r of a spectroscopic binary as a function of time. The black dots are the actual data points, while the curve has been fitted to these points.

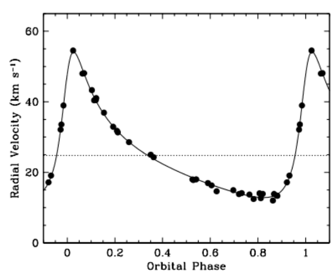

Figure 1: Radial velocities of the star HIP 50796, from Torres et al. (2006, AJ, 131, 1022).

Figure 1: Radial velocity of a spectroscopic binary.

Explain briefly how equation (8) is used to generate plots like Fig. 1. Assume that the line of sight lies in the orbital plane. Is the orbit circular?

Problem 4

a) Define a four-vector in special relativity. Give an example of a nonzero four-vector whose length is zero.

b) Explain briefly Perihelion precession in general relativity.