NTNU
Faculty of Natural Sciences

NTNU, Department of Physics

Exam FY2450 Spring 2021

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Tuesday June 8 2021
09:00-13:00

Permitted examination support material:
No restrictions.

Read carefully. Good luck! Bonne chance! Viel Glück! Veel succes! Lykke til!

You have 30 min after the exam to upload your answers in Inspera as a pdf file

Problem 1

a) The newly discovered element Gloshaugium has a characteristic line in its spectrum whose wavelength λ is 500nm. Calculate the wavelength λ' if the source is moving toward an observer with a speed of 1000000 m/s. Do you need to take into account special relativity? Explain your reasoning. Is the light blueshifted or redshifted?

b) A research team at NTNU has discovered a new spectroscopic binary consisting of S1FY2450 and S2FY2450, with mass m_1 and m_2, respectively. The atmosphere of the stars consists entirely of Gloshaugium. The period of the binary is P. The measured wavelength of the line in Gloshaugium has been measured at different times, which is tabulated in Table 1.

Time/P	$t = 0$	$t = P/4$	$t = P/2$	$t = 3P/4$
Wavelength S1FY2450	499.9nm	500.0nm	500.1nm	500.0nm
Wavelength S2FY2450	500.1nm	500.0nm	499.9nm	500.0nm

Table 1: Measured wavelengths as a function of time.

The maximum redshift (or blueshift) of the line is at $t = 0$ and $t = \frac{1}{2}P$. The orientation of the ellipses is such that the line of sight is parallel to the y-axis, see Fig. 1.

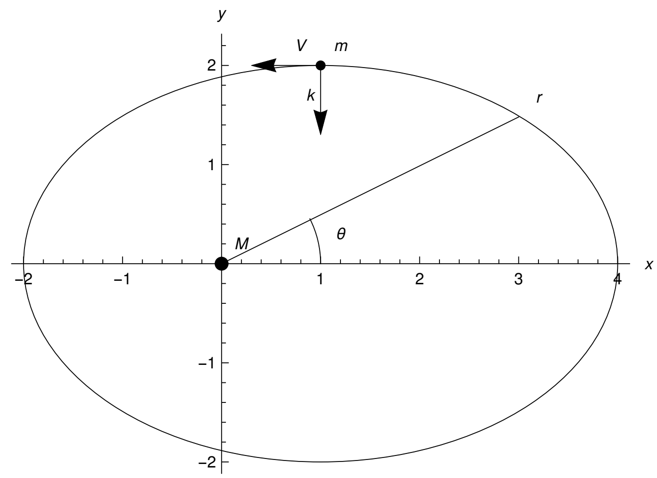

Figure 1: Sketch of orbit.

What is the radial velocity. i.e. the component of the center-of mass velocity of the binary along the line of sight? Find the ratio of the two masses m_1 and m_2. What is the eccentricity of the orbit?

c) Given the information above, can you find the the orbital speeds v_1 and v_2? Can you find the individual masses m_1 and m_2. In case the answer is no to these questions, what piece of information do you need?

d) Assume that the angle of inclination is $i = \frac{\pi}{6}$. What is the maximum redshift now? What is the eccentricity of the projected orbit?

Problem 2

Consider a spherically symmetric star of radius R and mass M, whose pressure gradient is given by

$$\frac{dP}{dr} = -\frac{4\pi}{3} G\rho_c^2 r e^{-r^2/a^2} , \tag{1}$$

where ρ_c is the central density and a is a length parameter.

a) Find $P(r)$.

b) The structure equation for a star is

$$\frac{dP}{dr} = -\frac{Gm(r)\rho_m(r)}{r^2} , \tag{2}$$

where $m(r)$ is the total mass inside a sphere of radius r and $\rho_m(r)$ is the mass density of the star. Multiplying this equation by $4\pi r^3$ and integrating yields

$$\begin{aligned} 4\pi \int_0^R r^3 \frac{dP}{dr} dr &= -\int_0^R \frac{Gm(r) 4\pi r^2 \rho_m(r)}{r} dr \\ &= -\int_0^M \frac{Gm(r)}{r} dm , \end{aligned} \tag{3}$$

where we have used $dm = 4\pi r^2 \rho_m(r) dr$. Explain why the right-hand side of Eq. (3) can be interpreted as the gravitational potential energy U of the star. Show that the left-hand side of Eq. (3) can be written as $-3V\langle P\rangle$, where V is the volume of the star and $\langle f\rangle$ denotes the average of the function $f(\mathbf{r})$ over the volume V,

$$\langle f\rangle = \frac{1}{V}\int f(\mathbf{r}) d^3 r . \tag{4}$$

c) Show that U can be written as

$$U = -\frac{2\pi^2}{3} G\rho_c^2 a^2 \left[-2e^{-R^2/a^2} R(3a^2 + 2R^2) + 3a^3\sqrt{\pi}\,\mathrm{erf}(\tfrac{R}{a})\right] , \tag{5}$$

where $\mathrm{erf}(x)$ is the so-called error function defined as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt . \tag{6}$$

d) Find the gravitational potential energy U in the limit $R \ll a$ and interpret the result. **Hint**: Use the Taylor expansion of the error function arond $x = 0$:

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\left[x - \frac{1}{3}x^3 + \frac{1}{10}x^5 + ...\right] . \tag{7}$$

Problem 3

a) A neutron star consists mainly of neutrons. Its typical radius is 10km, i.e. the size of Trondheim. Why is it stable against gravitational collapse? How can one justify statements like "Neutron stars are an example of quantum physics on a macroscopic scale".

b) It takes 24 hours for the Earth to complete one revolution around its own axis and yet high tide (and low tide) is only every 12hours and 26minutes. Explain the difference between the naive expectation of 12hours and 0minutes. Draw a figure if you find it helpful.

c) Explain why heavier elements such as Carbon and Oxygen require higher temperatures in the core of a star to fuse compared to Hydrogen. Why is the actual temperature for fusion lower than one expects from classical physics?