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Examination, course FY2450 Astrophysics
Wednesday 23rd May, 2012
Time: 09.00 – 13.00

Allowed to use: Calculator, translation dictionary, printed or hand written notes covering a maximum of one side of A5 paper.

On pages 5 and 6 you will find a table of the properties of main sequence stars, a list of physical and astronomical constants in CGS and SI units and the equations of stellar colour.

Answer all questions, each of the 15 sub-problems will be weighted equally in the grading

Please answer questions in English.

1. (a) The Kepler satellite observes variations in the light curve of a star with a spectral type of F5V. A planetary transit is observed with a repeat period of 6 months. Estimate the mean distance between the star and the planet. How does this orbital radius compare to the so-called "habitable zone" for an Earth-like planet in orbit around this F5V star?

$GMm/r^2 = mv^2/r$

$v_{orb} = (GM/r)^{1/2} = 2\pi r/P$

$r^3 = GMP^2/4\pi^2$

$r \approx 0.7$ AU

$T_p = T_*(R_*/2d)^{1/2}$

$d = RT_*^2/2T_p^2$

@273 K, $d \approx 1.6$ AU

@373 K, $d \approx 0.85$ AU

0.7 AU is closer than the inner boundary of the habitable zone – too hot.

(b) High resolution spectroscopic observations of the atomic lines in the photosphere of this star show its radial velocity changes by up to ±20 m/s over the six month orbital period. Estimate the mass of the planet.

$m_p = m_s v_s / v_p$

$v_p = 2\pi r / P$

$m_p \approx 0.7\ M_{Jupiter}$

(c) The fading time of the observed light curve as the planet both begins and ends its transit across this star's disk is measured to be 3000 seconds. Estimate the density of the planet. Compared to Solar System objects, what sort of planet is this?

$\mathbf{d_p = v_p t_f}$

$\mathbf{r_p = d_p/2}$

$\boldsymbol{\rho_p = 3m_p/4\pi r^3}$

$\boldsymbol{\rho_p \approx 1.24\ g/cm^3}$

mass, radius, and density suggest this is a gas giant – a "hot Jupiter"

(d) Photometric observations of this star reveal a B-band apparent magnitude (m_B) of +8.0 and a V-band apparent magnitude (m_V) of +7.0. Estimate the distance to the planetary system and the minimum diameter optical telescope required to spatially resolve the star and the planet at maximum separation.

$A_B - A_V = (B\text{-}V)_{obs} - (B\text{-}V)_0 = 0.55$

$A_V = 3.1 * 0.55 = 1.705$

$m_V = M_V + 5 \log_{10} (d/10pc) + A_V$

$d \approx 19.9$ pc

$\Delta\Theta = 1.22\ \lambda/d$

$\Delta\Theta \approx r/d \approx 1.6 \times 10^{-7}$ rad

e.g. @550nm

$d \approx 4.1$m

(e) Estimate the planet's mean surface temperature. At what wavelength of light does the planet's continuum spectrum reach maximum intensity? What is the ratio of the spectral luminosity of the planet to that of the star at this particular wavelength of light?

$T_p = T_*(R_*/2d)^{1/2}$

$T_p \approx 420$ K

$\lambda_{max}T = 2.898 \times 10^6$ nm K

$\lambda_{max} \approx 6.9$ μm

$I(\lambda,T) = [2hc^2/\lambda^5] / [\exp(hc/\lambda kT) -1]$

$I_p/I_s = [\exp(hc/\lambda kT_s) -1] / [\exp(hc/\lambda kT_p) -1]$

@6.9 μm

$I_p/I_s \approx 2.76 \times 10^{-3}$

$L_p/L_s = R_p^2/R_s^2 * I_p/I_s$

$\approx 1.55 \times 10^{-5}$

2. (a) Starting from the definition of the gravitational potential energy between two point masses (relative to infinity):

$U = -Gm_1m_2/r$

where m_1 and m_2 are the point masses, r is their separation and G is the gravitational constant, show how the gravitational potential energy of a star can be approximated by:

$U = -(3/5)GM^2/R$

where M is the mass of a spherical star of uniform constant density and R is the star's radius.

$M_{shell} = dM = 4\pi r^2 dr$

$M(r) = 4\pi r^3\rho/3$

$dU(r) = -G16\pi^2r^4\rho^2dr/3$

integrate between 0 and R:

$U = -G16\pi^2\rho^2R^5/15$

but, $M = 4\pi R^3\rho/3$

so, $M^2/R = 16\pi^2\rho^2R^5/9$

so, $U = -3GM^2/5R$

(b) Use this result to calculate how long the Sun could use this stored gravitational energy to maintain its current luminosity (the gravitational lifetime or Kelvin time) without any further sources of energy.

$t_g = E/L = 3GM^2/5RL$

$t_g \approx 1.8 \times 10^7$ **years**

(c) The equation of hydrostatic equilibrium can be used to relate the pressure gradient (dP/dr) and the density ($\rho(r)$) in a star:

$dP/dr = -[GM(r)/r^2]\ \rho(r)$

where M(r) is the total mass contained within a sphere of radius r. Use this, and an appropriate equation of state, to estimate the temperature in the core of the Sun assuming that the Sun has a constant uniform density and is composed entirely of ionised hydrogen.

Consider the whole star as a single shell:

$dr = \Delta r = r_{r=0} - r_R = -R$

$dP = \Delta P = P_{r=0} - P_R = P_{r=0}$ (assuming $P_R = 0$)

$P_{r=0} = GM\rho/R$

where, $\rho = 3M/4\pi R^3$

so, $P_{r=0} = 3GM^2/4\pi R^4 \approx 2.7 \times 10^{15}$ dyn/cm^2

$T_{r=0} = mP_{r=0}/\rho k = mP_{r=0}4\pi R^3/Mk$

NB, $m = 0.5 * m_p$ as there are equal numbers of protons and electrons

$T_{r=0} \approx 1.2 \times 10^7$ K

(d) Give an example of a step-by-step reaction sequence inside the Sun's core that converts protons to alpha particles.

e.g. PPI

$^{1}H + {}^{1}H \rightarrow {}^{2}D + e^{+} + \nu_e$

$^{2}D + {}^{1}H \rightarrow {}^{3}He + \gamma$

the above reactions happen twice, then:

$^{3}He + {}^{3}He \rightarrow {}^{4}He + 2{}^{1}H$

(e) As the Sun evolves during its main sequence lifetime, explain why the core slowly gets hotter. How does the photosphere of the Sun respond to this core heating – illustrate your answer with a sketch of the Sun's path as an evolutionary track relative to the zero-age main sequence on a Hertzsprung-Russell diagram during this period of evolution.

On the main sequence hydrogen in the core is slowly converted to helium

A pure hydrogen core: $m \approx m_p/2$

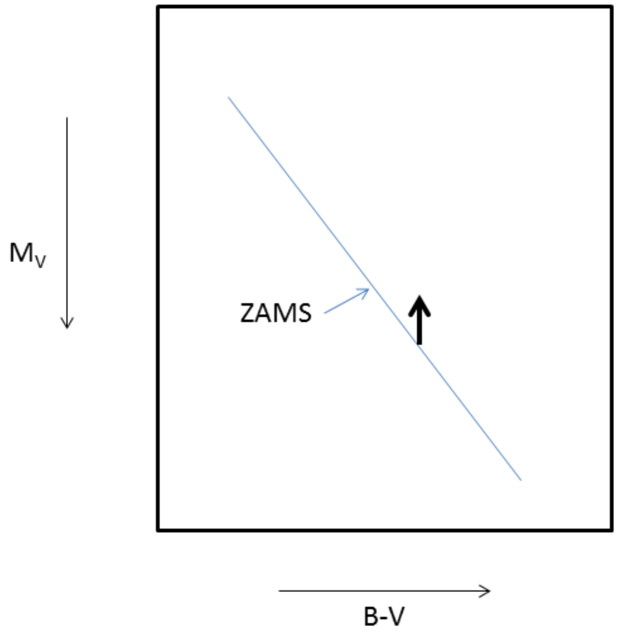
A pure helium core: $m \approx 4m_p/3$

so m gets bigger, therefore P gets smaller by the ideal gas law,

but dP/dr must support the mass of the star which is essentially unchanged,

so the core must contract to maintain dP/dr, so the core must heat up by GPE release.

the photosphere expands (with little change in T) so L increases:



3. (a) The material in a spiral galaxy is in an approximate circular orbit a distance r from the galaxy's centre of mass. The mass contained within a sphere interior to r provides the gravitational acceleration to keep the material in a circular orbit. The orbital speed of the local standard of rest (LSR) in the Milky Way is assumed to be 220 km/s at a distance 8.5 kpc from the centre of mass. Calculate the mass contained within the orbit of the LSR. Express the answer in solar masses.

$v_{orb} = (GM(r)/r)^{1/2}$

$M(r) \approx 1 \times 10^{11}\ M_{sun}$

(b) The rotation curve of The Galaxy is observed to be "flat" out to the edge of the visible disk. Thus v(r) is constant (equal to v_0). Use this information to show that the mass density at a distance r from the galactic centre is given by:

$\rho(r) = v_0^2/(4\pi G r^2)$

where G is the gravitational constant.

$M(r) = rv^2/G$

differentiate, $dM(r)/dr = v_0^2/G$ as v is not a function of r

but $dM(r)/dr = 4\pi r^2\rho(r)$ by the mass continuty relationship

so, $\rho(r) = v_0^2/G4\pi r^2$

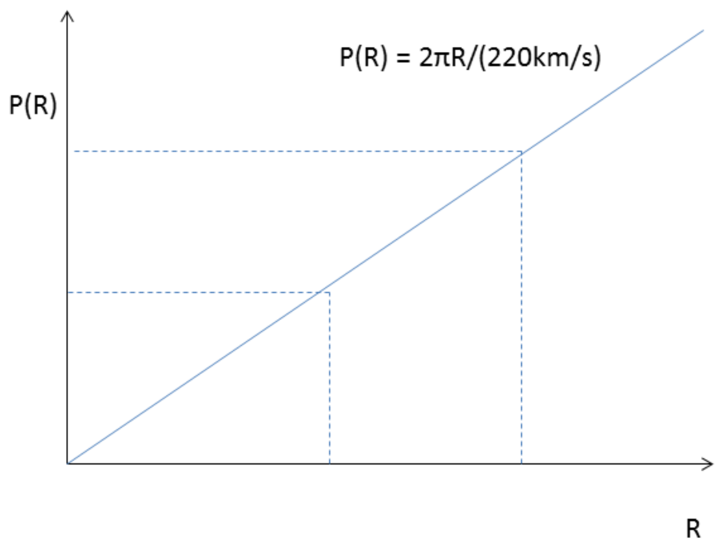
(c) Sketch the orbital period of material in the Milky Way as a function of distance from the galactic centre and explain why it is unlikely that spiral structure in the Galaxy could be carried by the visible matter in the Galaxy if the age of the Galaxy is comparable to the age of the Universe.

consider 2 points on this plot e.g.

@10kpc P ≈ 2.8 x 10^8 years ≈ 53 rotations in 15 billion years

@20kpc P ≈ 5.6 x 10^8 years ≈ 27 rotations in 15 billion years

spiral structure in the centre of the Galaxy would be wound up much faster than that in the outer reaches of the Galaxy.



(d) The spiral structure in a galaxy is believed to be due to slow-moving "density waves". Explain why you would expect to see enhanced star formation in the spiral arms of galaxies.

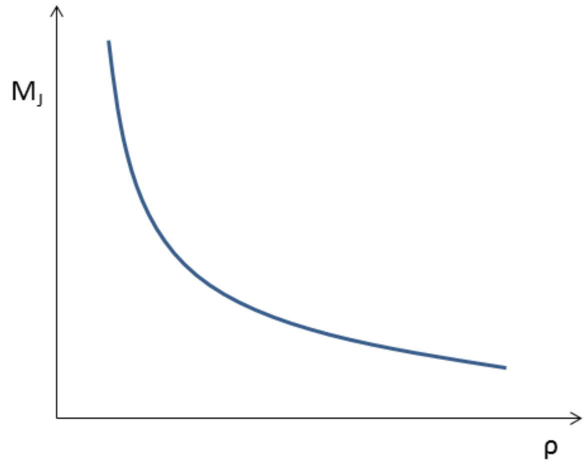
star formation occurs when a cloud becomes gravitationally bound

$M/R \geq 5kT/2Gm$

or e.g. in terms of the Jean's mass:

$M_J \approx 4[kT/Gm]^{3/2}\rho^{-1/2}$

$M_J \propto \rho^{-1/2}$



any process that compresses a cloud (e.g. a density wave) will favour star formation

(e) If a heliocentric recession velocity of +35 km/s is measured from mm observations of the CO rotational lines in a molecular cloud along a line of sight with galactic latitude = 0° and galactic longitude = +30°, estimate the kinematic distance(s) to the molecular cloud. You should get two possible answers. Describe how you might resolve this distance ambiguity observationally.

using:

$v_r = [\Omega(R) - \Omega_0]R_0\sin(l)$

$v_r/R \approx 34.1$ km/s/kpc

so, $R_{cloud} = 6.45$ kpc

using the cosine rule:

$R_{cloud}^2 = R_0^2 + d^2 - 2R_0 d\cos(l)$

$d \approx 12.2$ or 2.5 kpc

e.g. you could identify OB type stars associated with the molecular cloud, measure m_B and m_V and calculate the distance modulus as in 1d (or observe the proper motion of the cloud complex with respect to the Galactic centre)

Properties of main sequence stars

Spectral type	M_V (1)	*B-V*	$T_{eff}(K)$ (2)	M/M_{Sun}	R/R_{Sun}	L/L_{Sun}
O5	-6	-0.45	35000	39.8	17.8	3.2×10^{5}
B0	-3.7	-0.31	21000	17.0	7.6	1.3×10^{4}
B5	-0.9	-0.17	13500	7.1	4.0	6.3×10^{2}
A0	+0.7	+0.0	9700	3.6	2.6	7.9×10^{1}
A5	+2.0	+0.16	8100	2.2	1.8	2.0×10^{1}
F0	+2.8	+0.30	7200	1.8	1.4	6.3
F5	+3.8	+0.45	6500	1.4	1.2	2.5
G0	+4.6	+0.57	6000	1.1	1.05	1.3
G5	+5.2	+0.70	5400	0.9	0.93	7.9×10^{-1}
K0	+6.0	+0.81	4700	0.8	0.85	4.0×10^{-1}
K5	+7.4	+1.11	4000	0.7	0.74	1.6×10^{-1}
M0	+8.9	+1.39	3300	0.5	0.63	6.3×10^{-2}
M5	+12.0	+1.61	2600	0.2	0.32	7.9×10^{-3}

(1) Absolute V-band magnitude

(2) Effective surface temperature

Physical constants

speed of light	c	2.998×10^{10} cm s^{-1}	2.998×10^{8} m s^{-1}
gravitational constant	G	6.673×10^{-8} dyne cm^{2} g^{-2}	6.673×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k	1.381×10^{-16} erg K^{-1}	1.381×10^{-23} J K^{-1}
Planck's constant	h	6.626×10^{-27} erg s	6.626×10^{-34} J s
Stefan–Boltzmann constant	σ	5.670×10^{-5} erg cm^{-2} K^{-4} s^{-1}	5.670×10^{-8} W m^{-2} K^{-4}
Wien displacement constant	$\lambda_{max}T$	2.898×10^{-1} cm K	2.898×10^{-3} m K
Rydberg constant	R	1.097×10^{5} cm^{-1}	1.097×10^{7} m^{-1}
mass of proton	m_p	1.6726×10^{-24} g	1.6726×10^{-27} kg
mass of neutron	m_n	1.6749×10^{-24} g	1.6749×10^{-27} kg
mass of electron	m_e	9.1096×10^{-28} g	9.1096×10^{-31} kg
mass of hydrogen atom	m_H	1.6735×10^{-24} g	1.6735×10^{-27} kg

Astronomical constants

astronomical unit	*AU*	1.496×10^{13} cm	1.496×10^{11} m
parsec	*pc*	3.086×10^{18} cm	3.086×10^{16} m
solar mass	M_{Sun}	1.989×10^{33} g	1.989×10^{30} kg
solar radius (mean)	R_{Sun}	6.960×10^{10} cm	6.960×10^{8} m
solar luminosity	L_{Sun}	3.839×10^{33} erg s^{-1}	3.839×10^{26} J s^{-1}
Earth mass	M_E	5.977×10^{27} g	5.977×10^{24} kg
Earth radius (mean)	R_E	6.371×10^{8} cm	6.371×10^{6} m
Jupiter mass	M_J	1.899×10^{30} g	1.899×10^{27} kg
Jupiter radius (mean)	R_J	6.991×10^{9} cm	6.991×10^{7} m

The equations of stellar colour

Planck's empirical law: Energy per second per frequency interval per unit area

$$I(\nu,T) = [2h\nu^3/c^2] / [\exp(h\nu/kT) - 1]$$

Planck's empirical law: Energy per second per wavelength interval per unit area

$$I(\lambda,T) = [2hc^2/\lambda^5] / [\exp(hc/\lambda kT) - 1]$$

Wien's displacement law: wavelength of maximum intensity

$$\lambda_{max}T = 2.898 \times 10^6 \text{ nm K}$$

Stefan-Boltzmann law: Integrated energy per second per unit surface area

$$E = \sigma T^4$$

Integrated energy per second from a sphere: e.g. the total (bolometric) luminosity of a star

$$L = 4\pi R^2\sigma T^4$$