NTNU – Trondheim
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Department of Physics

Examination paper for FY2450 Astrophysics

Solutions

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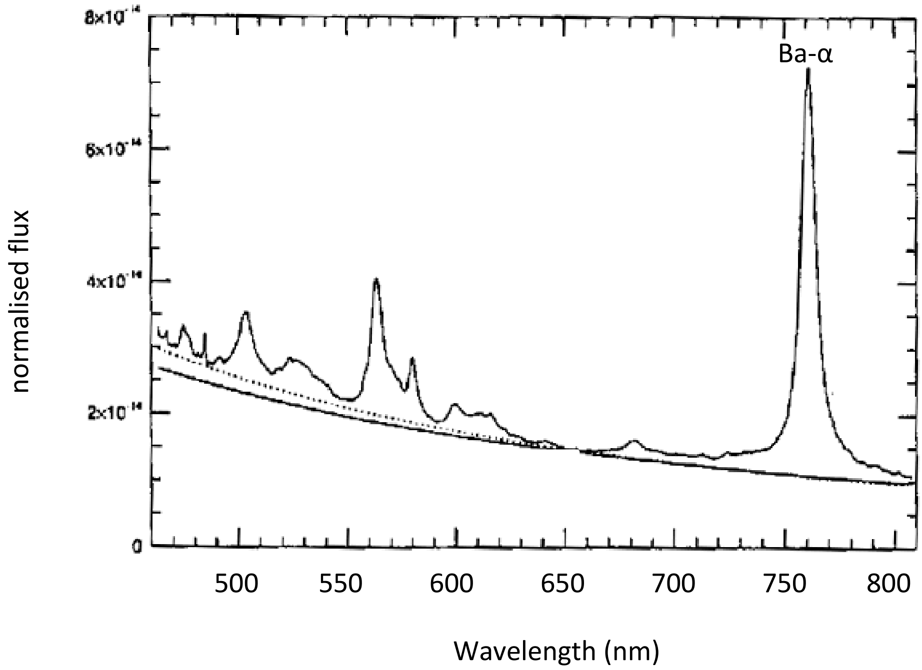
Examination date: 04-06-2013
Examination time: 09:00 – 13:00
Permitted examination support material: Calculator, translation dictionary, printed or hand-written notes covering a maximum of one side of A5 paper.

Each one of the 12 answers carried equal weight

Final grades were calculated from the 10 best answers submitted

1. The figure below is the observed optical spectrum of quasar 3C-273. The Balmer alpha line (rest $\lambda \approx 656$ nm) is labelled. The mean V-band apparent magnitude of this object is measured at $m_V = 12.9$.



1. (a). Estimate the distance, absolute V-band magnitude (M_V) and luminosity of this object. You may assume the Hubble constant (H_0) = 65 km/s/Mpc, and the absolute V-band magnitude of the Sun (M_V) = +4.8.

$v = c\Delta\lambda/\lambda_0 = H_0 d$

$\Delta\lambda = 104$ nm, $\lambda_0 = 656$nm

$d = 731$ Mpc

$m - M = 5 \log_{10} [d/10pc]$

$M = -26.4$

$M_{sun} - M_Q = 2.5 \log_{10} [L_Q/L_{sun}]$

$L_Q = 3 \times 10^{12}\ L_{sun} = 1.2 \times 10^{46}$ erg/s

1. (b). If this quasar is powered by matter falling towards a *black hole*, show that the maximum luminosity is given by:

$L_{max} = (c^2/2)(dm/dt)$

where c is the speed of light and dm/dt is the mass infall rate. Calculate the minimum mass infall rate required to produce the luminosity calculated in part (a). Express this answer in terms of solar masses per year.

$E_{max} = GMm/R_S$

$R_S = 2GM/c^2$

$E_{max} = mc^2/2$

(or $v \rightarrow c$ as $R \rightarrow R_S$, so $mv^2/2 \rightarrow mc^2/2$)

$L = dE/dt = dE/dm \,.\, dm/dt$

$L_{max} = dm/dt \,.\, c^2/2$

$dm/dt = 2L_{max}/c^2 = 0.4\ M_{sun}$ /year

1. (c). Long-term speckle imaging observations of the stars close to the object Sgr-A* at the centre of our own Galaxy show one star (S0-2) orbits the object in an approximately circular orbit with a measured radius of 0.1 arcsecs and a period of 15.8 years. If Sgr-A* is 8.5 kpc distant estimate the mass contained within the orbit of S0-2.

$M_{Sgr\text{-}A*} = 4\pi^2 r^3/P^2 G$

r is the radius in linear units (tan Θ = r/d, r = 1.3×10^{16} cm)

P is the period

$M_{Sgr\text{-}A*} = 5.2 \times 10^{39}$ g

1. (d). Another star (S0-16) orbiting Sgr-A* is observed to have a highly elliptical orbit passing within 45 AU of Sgr-A*. Estimate the minimum density of Sgr-A* and explain why Sgr-A* is unlikey to be an energetically-stable giant molecular cloud.

This mass must be contained within a volume no bigger than a sphere of radius 45 AU

$\rho = 3M_{Sgr\text{-}A*} / 4\pi r^3$

$= 4 \times 10^{-6}$ **g/cm**^{3}

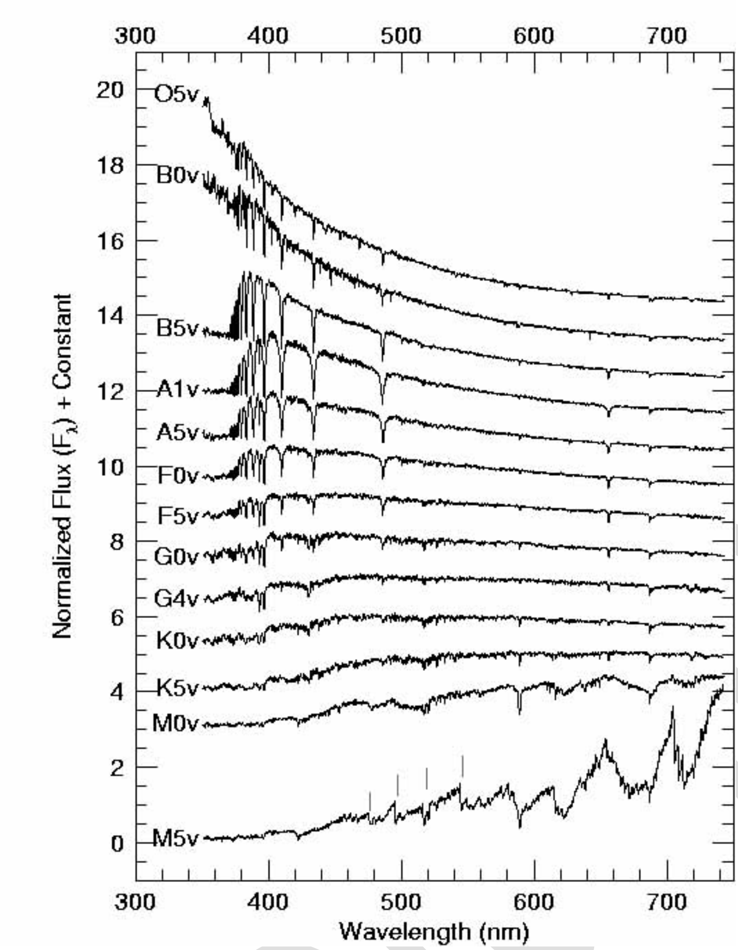
If this was a GMC, we could ask at what temperature would it be stable against collapse?

$T > 2GMm/5Rk$

(e.g. for a cloud composed of molecular hydrogen where $m = 3.3 \times 10^{-24}$ **g)**

$T > 5 \times 10^9$ **K (hotter than the core of the Sun!)**

2. (a). The following plot shows the normalized optical spectra of a series of main sequence stars labelled with spectral type. Explain why the Balmer series of hydrogen lines are weak or absent in the spectra of B-type stars and also K-type stars.



Balmer series lines are absorption lines from the n=2 ground state

B-type stars are hot – most of the photospheric hydrogen is ionised (not in the n=2 state)

K-type stars are cool – most of the photospheric atomic hydrogen is in n=1 so absorption transitions out of n=2 are weak/absent

2. (b). The equation of hydrostatic equilibrium can be used to give an expression for the approximate core pressure in a star:

$P_{r=0} = GM\rho/R$

where M is the mass of the star, ρ the constant density, R the radius of the star and G the gravitational constant. Use this to show why the luminosity of a main sequence star is observed to be approximately proportional to its mass raised to the power of 10/3. State any assumptions made.

$P \propto M\rho/R$ and $\rho \propto M/R^3$ if the star has constant density, so

$P \propto M^2/R^4$

Assume an ideal gas, we can say

$P \propto \rho/T \propto MT/R^3$ so,

$T \propto M/R$

Assume the core temperature is proportional to the photospheric temperature:

$L \propto R^2M^4/R^4 \propto M^4/R^2$ and

$M \propto R^3$ for a constant density sphere, so,

$L \propto M^4/M^{(2/3)} \propto M^{(10/3)}$

2. (c). In a gravitationally bound, dynamically relaxed cluster of stars the Virial theorem states that the time average of the kinetic energy of the stars plus half of the time average of the potential energy of the stars is zero:

$\langle K \rangle + 0.5 \langle U \rangle = 0$

Show that a binary pair of stars in a circular orbit about their common centre of mass obey the Virial theorem.

$\langle K \rangle = m_1 v_1^2/2 + m_2 v_2^2/2$

$\langle U \rangle = -Gm_1 m_2/R$ where $R = r_1 + r_2$

consider the forces on star 1:

$Gm_1 m_2/R^2 = m_1 v_1^2/r_1$

$m_1 v_1^2 = Gr_1 m_1 m_2/R^2$ and $m_2 v_2^2 = Gr_2 m_1 m_2/R^2$

$\langle K \rangle = Gm_1 m_2/2R = - 0.5\langle U \rangle$

$\langle K \rangle + 0.5\langle U \rangle = 0$

2. (d). Observations of the Doppler shift in the spectral lines of two stars in an eclipsing spectroscopic binary pair show the period of their orbit is 20 years. During this 20-year orbit star A has a maximum line-of-sight Doppler shift of 9 km/s relative to the centre of mass of the pair of stars, and star B has a maximum line-of-sight Doppler shift of 7 km/s. Estimate the individual masses of each of the two stars and their approximate spectral type assuming they are main sequence stars.

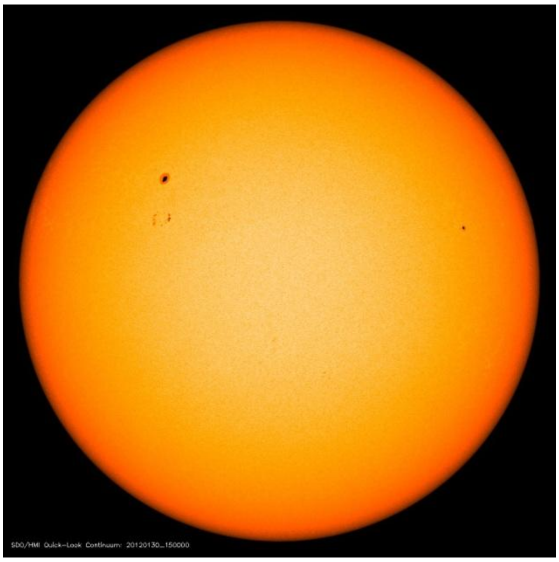
$\mathbf{m_A + m_B = P(v_A + v_B)^3/2G\pi = 6.2 \times 10^{33}\ g}$

$\mathbf{m_A/m_B = v_B/v_A = 7/9}$

$\mathbf{m_B = 3.5 \times 10^{33}\ g}$ **(F0V)**

$\mathbf{m_A = 2.7 \times 10^{33}\ g}$ **(F5V)**

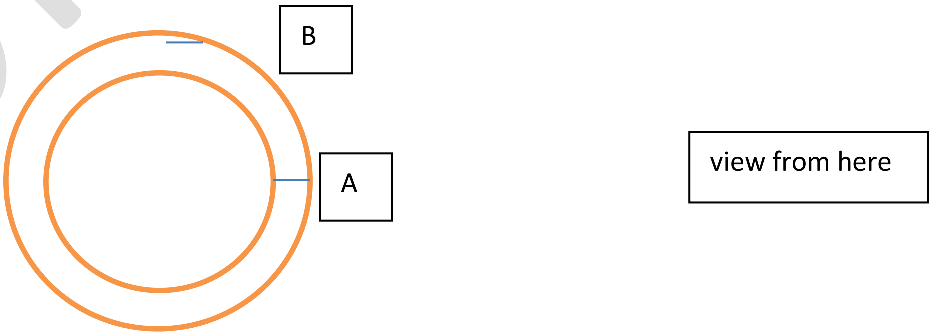
3. (a). Explain why the limb (the outer edge) of the sun appears darker and redder than the middle in a high resolution image where the solar disk is fully resolved.



Photons emerge from 1 optical depth into the sun (the same path length in a constant density, constant composition sun)

Path B sees photons from further away from the core than path A

If there is a temperature gradient of decreasing T with r in the photosphere then photons seen along path B will be from a cooler part of the sun than those from path A. If the Sun can be considered as a blackbody these photons will be redder (Wien) and less intense per unit area (Stephan Boltzmann).



3. (b). A sunspot has a measured effective temperature of 4000K compared to the surrounding photosphere at 5900K calculate the following:

- the wavelength of maximum spectral intensity for the sunspot and also the surrounding photosphere
- the ratio of total integrated energy flux per unit surface area within the sunspot and the surrounding photosphere
- the ratio of intensity of the radiation from the sunspot to that of the surrounding photosphere at a wavelength of 550 nm

λ_{max} = 2.9 x 10^6nmK/T

725 nm @ 4000K and 491 nm @ 5900 K

$E = \sigma T^4$

$E_{SS} / E_P = 0.21$

$I(\lambda,T) = [2hc^2/\lambda^5] / [\exp(hc/\lambda kT) - 1]$

$I_{SS}/I_P = [\exp(hc/\lambda kT_P) - 1] / [\exp(hc/\lambda kT_{SS}) - 1] = 0.12$ @ 550 nm

3. (c). The Sun loses mass to the solar wind. Satellites in orbit round the Earth measure typical solar wind densities of 5 protons per cubic cm with a speed of 400 km/s. Using these figures calculate the mass loss rate of the Sun through the solar wind alone and estimate the pressure of the local interstellar medium gas if the heliopause is located 100 AU from the Sun.

mass loss rate = 2 x 10^8 protons/s/cm^2 @ 1AU

A sphere of radius 1AU has a surface area of 2.8 x 10^{27} cm^2

so mass loss rate from the Sun due to solar wind is 9.4 x 10^{11} g/s

@ heliopause $P_{SW} = P_{ISM}$

Pressure = momentum/second/unit surface area

@ 1 AU = 3.34 x 10^{-16} g/s/cm^3 x 400 x 10^5 cm/s = 1.33 x 10^{-8} dyn/cm^2

@ 100 AU v is the same but the density is 10^4 less so,

P_{ISM} = 1.33 x 10^{-12} dyn/cm^2

3. (d). What is the *approximate* right ascension and declination of the Sun today (4th June)?

@ spring equinox RA_{sun} = 0 hrs, Dec_{sun} = 0°

@ midsummer RA_{sun} ≈ 6 hrs, Dec_{sun} ≈ +23.5°

RA = +04 hrs 51 min (+3 hrs to +6 hrs = full marks)

Dec = +22.5° (+15° to +23.5° = full marks)

Properties of main sequence stars

Spectral type	M_V[(1)]	*B-V*	$T_{eff}(K)$[(2)]	M/M_{Sun}	R/R_{Sun}	L/L_{Sun}
O5	-6	-0.45	35000	39.8	17.8	3.2×10^{5}
B0	-3.7	-0.31	21000	17.0	7.6	1.3×10^{4}
B5	-0.9	-0.17	13500	7.1	4.0	6.3×10^{2}
A0	+0.7	+0.0	9700	3.6	2.6	7.9×10^{1}
A5	+2.0	+0.16	8100	2.2	1.8	2.0×10^{1}
F0	+2.8	+0.30	7200	1.8	1.4	6.3
F5	+3.8	+0.45	6500	1.4	1.2	2.5
G0	+4.6	+0.57	6000	1.1	1.05	1.3
G5	+5.2	+0.70	5400	0.9	0.93	7.9×10^{-1}
K0	+6.0	+0.81	4700	0.8	0.85	4.0×10^{-1}
K5	+7.4	+1.11	4000	0.7	0.74	1.6×10^{-1}
M0	+8.9	+1.39	3300	0.5	0.63	6.3×10^{-2}
M5	+12.0	+1.61	2600	0.2	0.32	7.9×10^{-3}

(1) Absolute V-band magnitude

(2) Effective surface temperature

Physical constants

speed of light	c	2.998×10^{10} cm s^{-1}	2.998×10^{8} m s^{-1}
gravitational constant	G	6.673×10^{-8} dyne cm^{2} g^{-2}	6.673×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k	1.381×10^{-16} erg K^{-1}	1.381×10^{-23} J K^{-1}
Planck's constant	h	6.626×10^{-27} erg s	6.626×10^{-34} J s
Stefan–Boltzmann constant	σ	5.670×10^{-5} erg cm^{-2} K^{-4} s^{-1}	5.670×10^{-8} W m^{-2} K^{-4}
Wien displacement constant	$\lambda_{max}T$	2.898×10^{-1} cm K	2.898×10^{-3} m K
Rydberg constant	R	1.097×10^{5} cm^{-1}	1.097×10^{7} m^{-1}
mass of proton	m_p	1.6726×10^{-24} g	1.6726×10^{-27} kg
mass of neutron	m_n	1.6749×10^{-24} g	1.6749×10^{-27} kg
mass of electron	m_e	9.1096×10^{-28} g	9.1096×10^{-31} kg
mass of hydrogen atom	m_H	1.6735×10^{-24} g	1.6735×10^{-27} kg

Astronomical constants

astronomical unit	*AU*	1.496×10^{13} cm	1.496×10^{11} m
parsec	*pc*	3.086×10^{18} cm	3.086×10^{16} m
solar mass	M_{Sun}	1.989×10^{33} g	1.989×10^{30} kg
solar radius (mean)	R_{Sun}	6.960×10^{10} cm	6.960×10^{8} m
solar luminosity	L_{Sun}	3.839×10^{33} erg s^{-1}	3.839×10^{26} J s^{-1}
Earth mass	M_E	5.977×10^{27} g	5.977×10^{24} kg
Earth radius (mean)	R_E	6.371×10^{8} cm	6.371×10^{6} m
Jupiter mass	M_J	1.899×10^{30} g	1.899×10^{27} kg
Jupiter radius (mean)	R_J	6.991×10^{9} cm	6.991×10^{7} m

The equations of stellar colour

Planck's empirical law: Energy per second per frequency interval per unit area

$I(\nu,T) = [2h\nu^3/c^2] / [\exp(h\nu/kT) - 1]$

Planck's empirical law: Energy per second per wavelength interval per unit area

$I(\lambda,T) = [2hc^2/\lambda^5] / [\exp(hc/\lambda kT) - 1]$

Wien's displacement law: wavelength of maximum intensity

$\lambda_{max}T = 2.898 \times 10^6$ nm K

Stefan-Boltzmann law: Integrated energy per second per unit surface area

$E = \sigma T^4$

Integrated energy per second from a sphere: e.g. the total (bolometric) luminosity of a star

$L = 4\pi R^2\sigma T^4$