NTNU – Trondheim
Norwegian University of
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Department of Physics

Examination paper for FY2450 Astrophysics

Solutions

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Examination date: 31-05-2014
Examination time: 09:00 – 13:00
Permitted examination support material: Calculator, translation dictionary, printed or hand-written notes covering a maximum of one side of A5 paper.

Other information: The exam is in three parts and part 1 is multiple choice. Answer all questions in all three parts. The percentage of marks awarded for each question is shown. An Appendix of useful information is provided at the end of the question sheet.

Language: English
Number of pages: 10 (including cover)
Number of pages enclosed: 0

Checked by:

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Date Signature

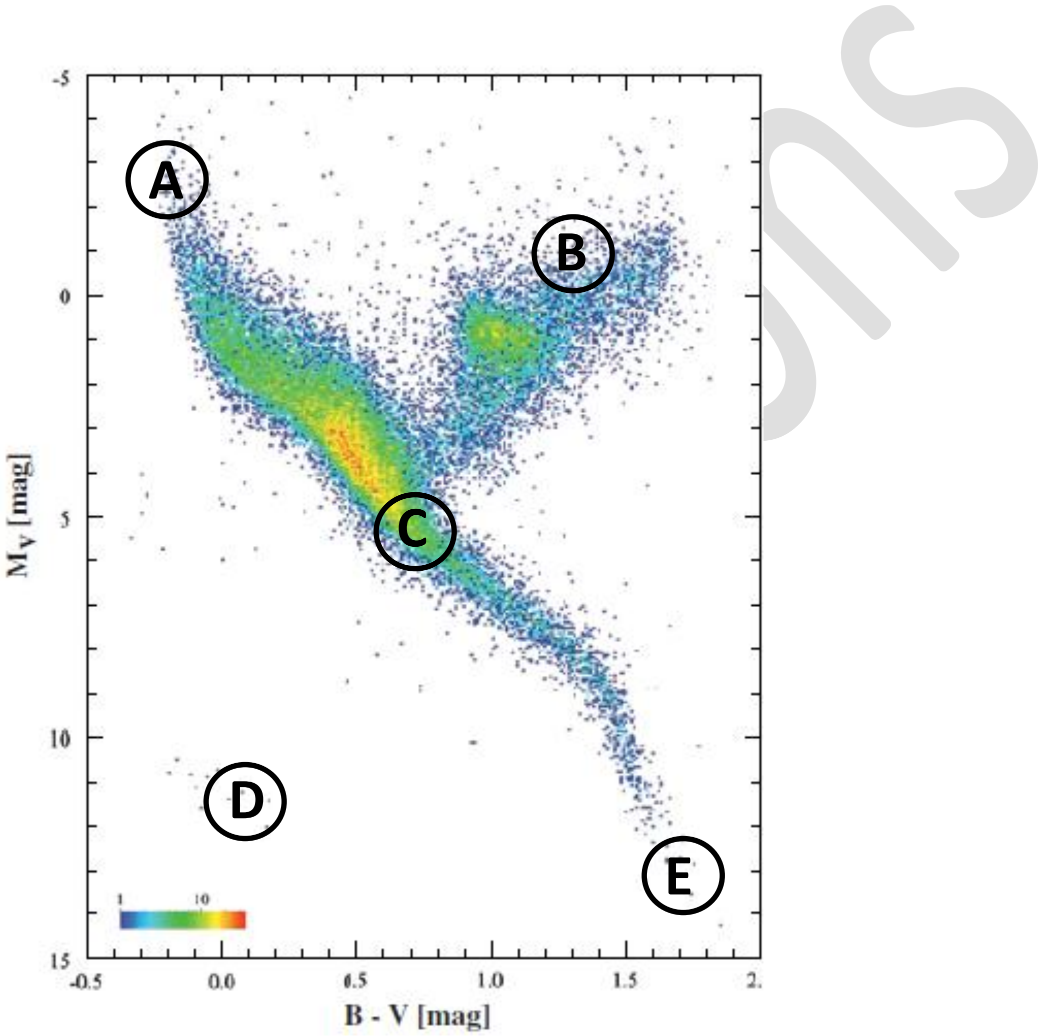
Solutions

Part 1. (total 30%)

Part 1 is multiple choice. 3 marks will be awarded for each correct answer. No marks will be awarded for an incorrect or missing answer. On your answer sheet draw a table that looks something like,

Question	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.10
Answer										

and insert your answer in the boxes. Only the answers will be marked.



The diagram above shows a colour (B-V) versus absolute visual magnitude (M_V) plot for solar neighbourhood stars compiled from observations from the Hipparcos satellite. Five regions labelled A, B, C, D and E are highlighted. Which region on the HR diagram contains:

you either know these or you don't, but $L = 4\pi R^2\sigma T^4$ (from the Appendix) helps...

(1.1) the stars with the smallest diameter **D** **(3%)**

(1.2) the stars that emit the greatest energy per second per unit surface area **A** **(3%)**

(1.3) the stars with the largest diameter **B** **(3%)**

(1.4) the stars with the lowest mass **E** **(3%)**

(1.5) the main sequence stars with the smallest mass to luminosity ratio **A** **(3%)**

		observations				
Star	Spectral type	apparent magnitude m_V (mag)	observed colour $(B\text{-}V)_{obs}$ (mag)	distance d (pc)	parallax half angle p (arcsecs)	visual extinction A_v (mag)
A	B0V	+6.2	+0.50	300	0.0033	2.5
B	A0V	+7.6	+0.32	150	0.0067	1.0
C	F0V	+10.8	+0.30	400	0.0025	0.0
D	G0V	+9.8	+1.27	40	0.0250	2.2
E	K0V	+9.8	+0.91	50	0.0200	0.3

The table above lists some observed properties of five different main sequence stars labelled A, B, C, D and E. The spectral type of each star is given. In the next five columns some observed properties of these five stars (as observed from Earth) are listed. Use the data provided and information in Appendix 1 to complete the table, and use the completed table to answer the following questions. (You may assume that the ratio of total to selective extinction (R) is 3.1 towards each of the five stars).

use some, or all, of:

$$m_v = M_v + 5 \log_{10}[d/10pc] + A_v$$

$$A_v = 3.1 ((B\text{-}V)_{obs} - (B\text{-}V))$$

$$d(pc) = p^{-1}$$

The "missing" data is highlighted in yellow in the table

(1.6) Which star appears brightest when observed from Earth (in the V band) **A** **(3%)**

(1.7) Which star is the closest to Earth **D** **(3%)**

(1.8) Which star is the furthest away from Earth **C** **(3%)**

(1.9) Which star appears reddest when observed from Earth **D** **(3%)**

solutions

(1.10) Which star is observed through the highest optical depth of interstellar extinction (in the V band) **A** **(3%)**

Part 2. (total 40%)

(2.1) Using the data in Appendix 1, explain why you would expect a G0V-type star to live longer on the main sequence than a B0V-type star. State any assumptions you make. **(10%)**

τ = Amount of fuel/rate at which fuel is used up

assume: luminosity is constant during MS lifetime
all stars end their MS life after using a similar proportion of fuel
amount of fuel is proportional to mass of star

$\tau = M/L$

$\tau_{B0V}/\tau_{G0V} = M_{B0V}L_{G0V}/M_{G0V}L_{B0V} = 1.5 \times 10^{-3}$

which is << 1

(2.2) A brown dwarf has a mass somewhere between 13 and 75 times the mass of Jupiter. It is neither a main sequence star nor a gas giant planet. Summarise the difference between a main sequence star, a brown dwarf star and a gas giant planet in terms of the nuclear physics.

MS star has a core hot enough to fuse H to He and so on... e.g. PP1,2,3 etc. It has a hot core because it requires a large P gradient to support its high mass (hydrostatic equilibrium), therefore has a high core pressure and a high core temperature (ideal gas)

BD (middle core temperature) can only fuse protons with deuterium (this has a larger reaction cross section than H + H) to form light helium, so can only use up the deuterium it was formed with, e.g...

GGP has no core nuclear fusion reactions as T is too low to overcome the electrostatic nuclear repulsion required for fusion.

solutions

(10%)

(2.3) Teide 1 was the first brown dwarf to be observationally verified. It has an effective surface temperature of approximately 2600 K (compared to the Sun at 5800 K) and a radius approximately 10 % of the Sun's. Calculate the luminosity of Teide 1. Also estimate the wavelength of light where the spectrum of Teide 1 is most intense. At a wavelength of 500 nm what is the ratio of the energy emitted per unit surface area of the Sun to that emitted per unit surface area of Teide 1. **(10%)**

$L = 4\pi R^2 \sigma T^4 = 1.6 \times 10^{30}$ erg/s

$\lambda_{max} = 2.9 \times 10^6$ nmK/T = 1.1 µm

The last part is done the hard way using Planck in lambda (see Appendix) at 2600 K and 5800 K and at 500 nm. 455 times greater.

(2.4) The gravitational potential energy of a uniform spherical cloud of interstellar material is given by:

$U = -(3GM^2) / (5R)$

where M is the mass of the spherical cloud and R is its radius. The kinetic (thermal) energy of the material in the cloud is:

$K = 3MkT / 2m$

where m is the mean mass per particle in the cloud at a temperature of T. Use this to show that the minimum mass (sometimes called the Jeans mass) of a gravitationally-bound spherical cloud of a given temperature, composition and density (ρ) is *approximately*,

$M_J \approx 4[(kT) / (Gm)]^{3/2} \rho^{-1/2}$

and calculate the smallest mass of a gravitationally-bound spherical cloud at a temperature of 60 K in a dense core in the interstellar medium composed of 10^5 hydrogen molecules per cubic cm. **(10%)**

For gravitional binding $|U| >= |K|$

$M/R >= 5kT/2Gm$

but $M = V\rho$, so $M/R = 4\pi R^2 \rho/3$

@ $R = R_J$ these are equal. Solving for R_J and assuming $(15/8\pi)^{1/2} = 1$

$R_J = [kT/Gm\rho]^{1/2}$

what mass is contained within this sphere at density ρ ?

$M_J = 4\pi R_J^3 \rho/3$

solutions

substitute for R_J and let $\pi/3 = 1$

$M_J \approx 4[(kT) / (Gm)]^{3/2} \rho^{-1/2}$

$m = 3.35 \times 10^{-24}$ g (twice the mass of a proton – it's H_2)
$\rho = 3.35 \times 10^{-19}$ g/cm^3
$T = 60$K
$k = 1.38\times10^{-16}$ erg/K
$G = 6.67 \times 10^{-8}$ dyne cm^2 g^{-2}

$M_J = 5 \times 10^{34}$ g = 25 M_{sun}

Part 3. (total 30%)

A good way to envisage the problems associated with detecting extra-solar planets is to consider how observations of the Sun-Jupiter system would appear from a distance of 10 pc viewed from within the plane of Jupiter's orbit around the Sun. Using data from the Appendix and assuming Jupiter is in a circular orbit around the Sun at a Sun-Jupiter distance of 5.2 AU and a period of 11.9 years calculate the following:

(3.1) By how many magnitudes would the Sun's apparent brightness change as Jupiter transits across the Sun's face? How long will this transit last? **(6%)**

$L_{transit} = L_{sun} [1-R_J^2/R_{sun}^2]$
$L_{transit} = 0.99\ L_{sun}$
$m_{transit} - m_{sun} = 2.5 \log_{10} [L_{sun}/L_{transit}] = +0.01$ mag
$t=d_{sun}/v_J = d_{sun}/(2*\pi*5.2\text{ AU}/11.9\text{ years}) = 29$ hours

(3.2) By how much would the *angular* position of the Sun appear to move with respect to a fixed background of stars over the course of one Jupiter orbit? **(6%)**

$a_J/a_{sun} = M_{sun}/M_J$
$a_{sun} = 7.4 \times 10^{10}$ cm
@ 10 pc
$\theta = 5 \times 10^{-4}$ arcsecs. This is the deviation about the equilibrium point so the total "wobble" over one orbit is twice this.

solutions

(3.3) By how much would the Ba-α (rest λ = 656.281 nm) spectral line in the Sun's photosphere appear to shift due to the change in radial velocity of the Sun over the course of one Jupiter orbit. **(6%)**

$v_{sun}/v_J = M_J/M_{sun}$, so $v_{sun} = 10^{-3}\ v_J$
$P(v_{sun} + v_J)^3 = 2\pi G(M_{sun} + M_J)$
$v_{sun} + v_J = 1.3 \times 10^6$ cm/s
so $v_{sun} = 1.3 \times 10^3$ cm/s
$\Delta\lambda/\lambda_0 = v_r/c = 4.3 \times 10^{-8}$ or $\pm 3 \times 10^{-5}$ nm @ Ba-α

(3.4) Repeat questions 3a, 3b and 3c, but this time assume Jupiter is only 0.52 AU from the Sun (remember the orbital period will also change). Use your answers to demonstrate why extrasolar planet detections using combined photometric and spectroscopic methods have an observational bias towards so-called "hot Jupiters". **(12%)**

(1) no change in brightness dip, transit lasts 9.3 hours ($10^{1/2}$ shorter) every 0.376 years

(2) a factor of 10 smaller (1×10^{-4} arcsec)

(3) orbital period becomes a factor of $10^{3/2}$ smaller (0.376 years) and the Doppler shift of the Sun becomes a factor of $10^{1/2}$ greater (1×10^{-4} nm)

so move a massive planet closer to the star (i.e. a "hot Jupiter") and you get a larger Doppler shift to the spectral lines of the star and a more frequent transit (but a quicker transit).

solutions

Appendix 1. Properties of main sequence stars

Spectral type	M_V [1]	*B-V*	$T_{eff}(K)$ [2]	M/M_{Sun}	R/R_{Sun}	L/L_{Sun}
O5	-6	-0.45	35000	39.8	17.8	3.2×10^5
B0	-3.7	-0.31	21000	17.0	7.6	1.3×10^4
B5	-0.9	-0.17	13500	7.1	4.0	6.3×10^2
A0	+0.7	+0.0	9700	3.6	2.6	7.9×10^1
A5	+2.0	+0.16	8100	2.2	1.8	2.0×10^1
F0	+2.8	+0.30	7200	1.8	1.4	6.3
F5	+3.8	+0.45	6500	1.4	1.2	2.5
G0	+4.6	+0.57	6000	1.1	1.05	1.3
G5	+5.2	+0.70	5400	0.9	0.93	7.9×10^{-1}
K0	+6.0	+0.81	4700	0.8	0.85	4.0×10^{-1}
K5	+7.4	+1.11	4000	0.7	0.74	1.6×10^{-1}
M0	+8.9	+1.39	3300	0.5	0.63	6.3×10^{-2}
M5	+12.0	+1.61	2600	0.2	0.32	7.9×10^{-3}

(1) Absolute V-band magnitude

(2) Effective surface temperature

Appendix 2. Physical constants

speed of light	c	2.998×10^{10} cm s^{-1}	2.998×10^{8} m s^{-1}
gravitational constant	G	6.673×10^{-8} dyne cm^2 g^{-2}	6.673×10^{-11} m^3 kg^{-1} s^{-2}
Boltzmann constant	k	1.381×10^{-16} erg K^{-1}	1.381×10^{-23} J K^{-1}
Planck's constant	h	6.626×10^{-27} erg s	6.626×10^{-34} J s
Stefan–Boltzmann constant	σ	5.670×10^{-5} erg cm^{-2} K^{-4} s^{-1}	5.670×10^{-8} W m^{-2} K^{-4}
Wien displacement constant	$\lambda_{max}T$	2.898×10^{-1} cm K	2.898×10^{-3} m K
Rydberg constant	R	1.097×10^{5} cm^{-1}	1.097×10^{7} m^{-1}
mass of proton	m_p	1.6726×10^{-24} g	1.6726×10^{-27} kg
mass of neutron	m_n	1.6749×10^{-24} g	1.6749×10^{-27} kg
mass of electron	m_e	9.1096×10^{-28} g	9.1096×10^{-31} kg
mass of hydrogen atom	m_H	1.6735×10^{-24} g	1.6735×10^{-27} kg

Appendix 3. Astronomical constants

astronomical unit	AU	1.496×10^{13} cm	1.496×10^{11} m
parsec	pc	3.086×10^{18} cm	3.086×10^{16} m
solar mass	M_{Sun}	1.989×10^{33} g	1.989×10^{30} kg
solar radius (mean)	R_{Sun}	6.960×10^{10} cm	6.960×10^{8} m
solar luminosity	L_{Sun}	3.839×10^{33} erg s^{-1}	3.839×10^{26} J s^{-1}
Earth mass	M_E	5.977×10^{27} g	5.977×10^{24} kg
Earth radius (mean)	R_E	6.371×10^{8} cm	6.371×10^{6} m
Jupiter mass	M_J	1.899×10^{30} g	1.899×10^{27} kg
Jupiter radius (mean)	R_J	6.991×10^{9} cm	6.991×10^{7} m

Solutions

Appendix 4. The equations of stellar colour

Planck's empirical law: Energy per second per frequency interval per unit area

$$I(\nu,T) = [2h\nu^3/c^2] / [\exp(h\nu/kT) -1]$$

Planck's empirical law: Energy per second per wavelength interval per unit area

$$I(\lambda,T) = [2hc^2/\lambda^5] / [\exp(hc/\lambda kT) -1]$$

Wien's displacement law: wavelength of maximum intensity

$$\lambda_{max}T = 2.898 \times 10^6 \text{ nm K}$$

Stefan-Boltzmann law: Integrated energy per second per unit surface area

$$E = \sigma T^4$$

Integrated energy per second from a sphere: e.g. the total (bolometric) luminosity of a star

$$L = 4\pi R^2\sigma T^4$$

solutions