NTNU – Trondheim
Norwegian University of
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Department of Physics

Examination paper for FY2450 Astrophysics

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Examination date: 01-06-2015
Examination time: 09:00 – 13:00
Permitted examination support material: Calculator, translation dictionary, printed or hand-written notes covering a maximum of one side of A5 paper.

Other information: The exam is in three parts. Part 1 is multiple choice. Answer all questions in all three parts. The percentage of marks awarded for each question is shown. An Appendix of useful information is provided at the end of the question sheet.

Language: English
~~Number of pages: 9 (including cover)~~
Number of pages enclosed: 0

Checked by:

Date Signature

Part 1. Total 45%

Part 1 is multiple choice. 3 marks will be awarded for each correct answer. No marks will be awarded for an incorrect or missing answer. On your answer sheet draw a table that looks something like this

Question	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	**A**	**C**	**C**	**D**	**D**	**B**	**E**	**B/C**	**A**	**A**	**B**	**D**	**A**	**E**	**B**

and insert your answer (A, B, C, D or E) in the appropriate box. Only select one answer. Select the answer closest to your answer *with the correct units*. Only the answers will be marked. You may use the data in the Appendix to answer these questions if required.

1. What is the approximate right ascension and declination of the Sun on September 23rd (the northern hemisphere autumn equinox)?

A. 12 hr, +0°
B. 0 hr, +23.5°
C. 23.5 hr, +0°
D. 0 hr, +0°
E. 12 hr, -23.5°

The opposite of the spring equinox

2. The absolute magnitude and colour of two stars (X and Y) are plotted on a Hertzprung-Russell diagram. Star X lies directly above Star Y on the diagram. Which of the following is true of the two stars?

A. Star Y is more luminous than star X.
B. The photosphere of star Y is hotter than that of star X.
C. Star Y is smaller than star X.
D. Star Y is more massive than star X.
E. All of the above.

$L = 4\pi R^2 \sigma T^4$. Lower luminosity at the same temperature, so R must be smaller

3. The absolute magnitude and colour of two stars (X and Y) are plotted on a Hertzprung-Russell diagram. Star X lies directly to the right of Star Y on the diagram. Which of the following is true of the two stars?

A. Star Y is more luminous than star X.
B. The photosphere of star Y is cooler than that of star X.
C. Star Y is smaller than star X.
D. Star Y is bigger than star X.
E. None of the above.

$L = 4\pi R^2 \sigma T^4$. Higher temperature at the same luminosity so R must be smaller

4. Estimate the distance to a star of spectral type A0V if its apparent V-band visual magnitude is measured at +11 and its observed (B-V) colour = 0.

A. 10 pc
B. 2190 pc
C. 115 pc
D. 1150 pc
E. 1.3×10^5 pc

$m_V - M_V = 5 \log_{10} [d/10\mathrm{pc}] + A_V$
$A_V = 0$ (as $(B\text{-}V)_{obs} = 0$, so no redenning), $m_V = +11$, $M_V = +0.7$

5. Estimate the distance to a star of spectral type A0V if its apparent V-band visual magnitude is measured at +11 and its observed (B-V) colour = +0.50.

A. 56 pc
B. 115 pc
C. 912 pc
D. 560 pc
E. 1150 pc

As above, but $A_V = 3.1 \times (A_B - A_V) = 3.1 \times 0.5 = 1.55$ magnitudes of extinction

6. The Ba-α (rest wavelength = 656.28 nm) lines from two stars in an eclipsing binary system are observed over the course of one complete orbit of the binary system. The maximum (heliocentric) measured Doppler shifts are ±0.01 nm and ±0.03 nm about the equilibrium position, respectively. The period of the system is 10 yr. What are the masses of the two stars?

A. 6.6 M_{Sun} and 13.3 M_{Sun}
B. 0.57 M_{Sun} and 1.72 M_{Sun}
C. 6.6 M_{Sun} and 20 M_{Sun}
D. 0.08 M_{Sun} and 0.24 M_{Sun}
E. 1 M_{Sun} and 3 M_{Sun}

$P(v_1 + v_2)^3/2\pi G = m_1 + m_2$
$v_1/v_2 = m_2/m_1$

$v = c\Delta\lambda/\lambda_0$
$P = 10 * 365 * 86400$ s
$v_1 = 4.57 \times 10^5$ cm/s
$v_2 = 1.37 \times 10^6$ cm/s
so:
$m_1 + m_2 = 4.59 \times 10^{33}$ g
and $m_1 = 3\ m_2$
so:
$m_1 = 3.44 \times 10^{33}$ g
$m_2 = 1.14 \times 10^{33}$ g

7. At what wavelength does the continuous (black-body) spectrum from a sunspot at a temperature of 3800 K peak?

A. 76.3×10^{-9} m
B. 763 µm
C. 7630 nm
D. 7.6 µm
E. 0.76 µm

$\lambda_{max} = (2.898 \times 10^6$ nm K$)/T = 762$ nm $= 0.76$ µm

8. What is the ratio of the intensity of radiation measured at a wavelength of 550 nm from a sunspot (3800K) and from the normal solar photosphere at 5800 K?

A. 0.18
B. 0.09
C. 11
D. 5.4
E. 0.66

$I(\lambda,T) = [2hc^2/\lambda^5] / [\exp(hc/\lambda kT) -1]$
Ratio = $(\exp(hc/\lambda k\ 5800) -1)/(\exp(hc/\lambda k\ 3800)-1) = 0.09$ @ 550 nm

On reflection, answer C will be accepted as well as the question didn’t specify very clearly which way round the ratio should be calculated.

9. Assume that the Sun radiates like a perfect spherical black body at a temperature of 5800 K. What is the total flux of radiation impacting on the Earth at 1 AU.

A. 1.8×10^{17} J/s
B. 7.1×10^{17} J/s
C. 3.6×10^{17} J/s
D. 2×10^{24} J/s

E. 8 x 10^{24} J/s

$L_{sun} = 4\pi R^2\sigma T^4 = 3.9 \times 10^{26}$ J/s
@ 1AU $4\pi d^2 = 2.8 \times 10^{23}$ m^2
Area the Earth intercepts = $\pi R_e^{\,2} = 1.27 \times 10^{14}$ m^2
Ratio x Luminosity = 1.8 x 10^{17} J/s

10. If the albedo of a dust grain is independent of the wavelength of radiation, calculate the temperature of a dust grain 100 AU from an F0V star.

A. 40 K
B. 75 K
C. 58 K
D. 60 °C
E. 2.7 K

$T_g = T^*(R^*/2d)^{1/2}$
T^* = 7200 K, R^* = 1.4 x 7 x 10^{10} cm, d = 100 x 1.5 x 10^{13} cm

11. The material in our Galaxy is in an approximate circular orbit around the Galaxy's centre of mass. If the rotational speed of the local standard of rest (8.5 kpc from the Galactic centre) is 220 km/s, what is the mass contained within the central 8.5 kpc of the Galaxy?

A. 10^{10} M_{Sun}
B. 10^{11} M_{Sun}
C. 10^5 M_{Sun}
D. 10^8 M_{Sun}
E. We don't know because most of the mass is dark matter

$M(r) = v_0^{\,2}R_0/G = 2 \times 10^{44}$ g

12. A uniform density, spherical core in a molecular cloud is composed of 10^5 hydrogen molecules / cm^3, estimate the minimum radius of this core if it was gravitationally bound at a temperature of 30 K.

A. 15 pc
B. 8 pc
C. 6.4 x 10^{-3} pc
D. 0.08 pc
E. 0.15 pc

Using $R_J = (kT/Gm^2n)^{1/2}$ from the textbook gives 2.4 x 10^{17} cm = 0.08 pc
Derived from first principles gives $R_J = (15kT/8\pi Gm^2n)^{1/2} = 1.84 \times 10^{17}$ cm = 0.06 pc

13. A uniform density, spherical core in a molecular cloud is composed of 10^5 hydrogen molecules / cm^3, estimate the minimum mass of this core if it was gravitationally bound at a temperature of 30 K.

A. 10 M_{Sun}
B. 18 M_{Sun}
C. 40 M_{Sun}
D. 0.06 M_{Sun}
E. 400 M_{Sun}

10^5 hydrogen *molecules* / cm^3 = 3.34×10^{-19} g/cm^3 in a sphere of volume 5.79×10^{52} cm^3 = 1.93×10^{34} g = 9.7 M_{sun}
If you used 1.84×10^{17} cm (see above) you would get 4.4 M_{sun} (still closest to A)
If you used $M_J = 4(kT/Gm)^{3/2}(nm)^{-1/2}$ from the text book you will get 1.79×10^{34} g = 9.0 M_{sun}

14. What is the escape velocity from the surface of the Sun?

A. 1.3×10^8 cm/s
B. 4.4×10^7 cm/s
C. 4.4×10^9 cm/s
D. 3×10^{10} cm/s
E. 6.2×10^7 cm/s

$v_{esc} = (2GM/r)^{1/2}$

15. What is the Schwarzschild radius (R_S) of a black hole of mass $10^8 \times M_{Sun}$?

A. 5.4×10^6 m
B. 3×10^{13} cm
C. 5.4×10^6 cm
D. 3×10^{13} m
E. 1.5×10^{13} m

$r_S = 2GM/c^2$

Part 2. Total 25%

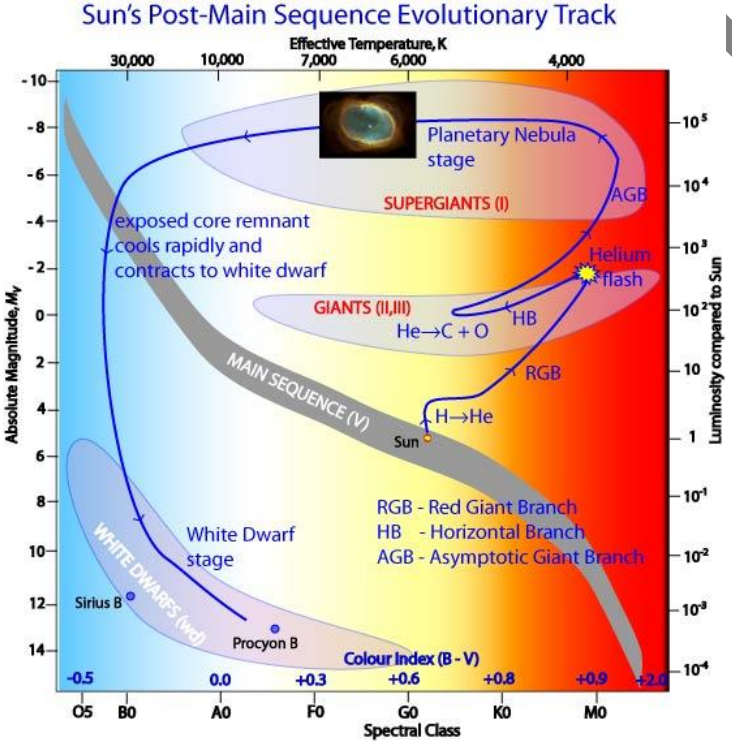
2a. Estimate the mass loss rate (dM/dt) associated with the Sun's current luminosity. Give your answer in solar mass per year. (3%)

$dm/dt = L/c^2 = 4.5 \times 10^{12}$ g/s $= 7 \times 10^{-14}$ M_{sun}/year

2b. Give a brief account of the important processes occurring during the key stages in the *post main sequence* evolution of a star with a mass equal to the Sun. Illustrate your answer with a Hertzprung-Russell diagram. (10%)

Some discussion of the following points illustrated by a (well-labelled) HR-diagram is required here:

- **Hydrogen core burning**
- **Hydrogen shell burning (inert helium core)**
- **Helium flash**
- **Helium shell burning (inert carbon core)**
- **Planetary nebula ejection when radiation pressure (F/A) > gravitational attraction**
- **Cooling to white dwarf supported by e^- degeneracy pressure**


Sun's Post-Main Sequence Evolutionary Track

2c. In a *pure carbon* white dwarf of one solar mass, what is the total number of particles? You may assume the carbon is fully ionised at a uniform temperature of 10^7 K. (3%)

^{12}C nucleus (six protons, six neutrons) + 6 electrons, total mass = 2 x 10^{-23} g per 7 particles, so 2 x 10^{33} g is 7 x 10^{56} particles .

2d. At this temperature estimate the total *thermal energy* of the white dwarf. (4%)

E_{th} = 3NkT/2 = 1.4 x 10^{48} erg

2e. A white dwarf is white because it is surrounded by a thin atmosphere of non-degenerate gas radiating at a temperature of around 10^4 K. Its radius is around 100 times smaller than the Sun. Use this information to estimate the cooling lifetime of the white dwarf. (5%)

$L = 4\pi R^2\sigma T^4$ = 3.5 x 10^{30} erg/sec
t_c = E/L = 4 x 10^{17} s = 1.3 x 10^{10} years

Part 3. Total 30%

3a. Give an example of a step-by-step reaction sequence which can convert *protons to alpha particles* in the centre of the Sun (spectral type G2V) at a temperature of around 10^7 K. Highlight the rate-determining reaction step in the sequence. (6%)

e.g. the pp I chain:
$^1_1H + {}^1_1H \rightarrow {}^2_1D + e^+ + \nu_e$ ← this is the RDS (slow)
$[e^+ + e^- \rightarrow 2\gamma]$
$^2_1D + {}^1_1H \rightarrow {}^3_2He + \gamma$
$^3_2He + {}^3_2He \rightarrow {}^4_2He + 2\,{}^1_1H$

3b. Starting from the equation for hydrostatic equilibrium,

$dP/dr = -GM(r)\rho(r)/r^2$

demonstrate that the pressure in the centre of a star is proportional to M^2/R^4, where M is the mass of the star and R is the radius. You may assume the star is a constant density uniform sphere. (10%)

$M(r) = \rho 4\pi r^3/3$ (constant density), so
$dP/dr = -G\rho^2 4\pi r/3$
$dP = -G\rho^2 4\pi r/3\ dr$, integrate between r=0 and r=R
$P_{r=0} = G\rho^2 4\pi R^2/6$
Remember $\rho = 3M/4\pi R^3$ so $\rho^2 = 9M^2/16\pi^2 R^6$ and substitute
$P_{r=0} = 3GM^2/8\pi R^4 \quad \alpha\ M^2/R^4$
You can also use the textbook approach whereby the whole star is approximated as a single shell, so dr = R and dP = $P_{r=0}$ (assuming $P_{r=R}$ =0) and get a value for $P_{r=0}$ a factor of two higher with the same M^2/R^4 dependencies.

Full credit is given for the M^2/R^4 dependency regardless of the constant of proportionality

3c. Hence, use an appropriate equation of state for a main sequence star to show that the temperature at the centre of a star is proportional to M/R. (6%)

The ideal gas equation would be appropriate here: $P = \rho kT/m$. Substituting for ρ:
$P_{r=0} = 3MkT/4\pi R^3 m = 3GM^2/8\pi R^4$ and solve for T
$T_{r=0} = GMm/2Rk \quad \alpha\ M/R$

Again, full credit for M/R – the constant term was ignored

3d. Using the answers to the questions above, together with the data for main sequence stars in Appendix 1, discuss why there are no main sequence stars with a mass less than about 0.1 M_{Sun}. Illustrate your answer with an appropriate graph. (8%)

So we know T_{core} α M/R and we are given that the Sun has a spectral type G2V with a core temperature of around 10^7 K which is (just) sufficient to support the PP chain reactions after the initial fusing of two protons (which is the RDS). We may conclude that if you have a much lower core T then two protons will not have sufficient energy to tunnel through the electrostatic repulsion barrier to fuse to deuterium and enable the PP chain to complete – i.e. we will not have a MS star. We know M and R for the Sun and a range of spectral type stars (Appendix 1). We can plot e.g. M/R vs M (or even better M/R vs log(M), or M/R vs spectral type) and see that at low mass M/R drops to values lower than that of the Sun (the appropriate graph suggested in the question). So the core T must drop lower than that of the Sun. At some point we reach a critical (low) temperature whereby the PP chain reaction cannot occur. This happens at around 10% of the mass of the Sun.

Effectively the pressure gradient required to support very low mass stars from gravitational collapse (via hydrostatic equilibrium) doesn't produce a sufficiently high core pressure to produce a sufficiently high core temperature (via the ideal gas equation) to fuse two protons.

Appendix 1. Properties of main sequence stars

Spectral type	$M_V^{(1)}$	B-V	$T_{eff}(K)^{(2)}$	M/M_{Sun}	R/R_{Sun}	L/L_{Sun}
O5	-6	-0.45	35000	39.8	17.8	3.2×10^{5}
B0	-3.7	-0.31	21000	17.0	7.6	1.3×10^{4}
B5	-0.9	-0.17	13500	7.1	4.0	6.3×10^{2}
A0	+0.7	+0.0	9700	3.6	2.6	7.9×10^{1}
A5	+2.0	+0.16	8100	2.2	1.8	2.0×10^{1}
F0	+2.8	+0.30	7200	1.8	1.4	6.3
F5	+3.8	+0.45	6500	1.4	1.2	2.5
G0	+4.6	+0.57	6000	1.1	1.05	1.3
G5	+5.2	+0.70	5400	0.9	0.93	7.9×10^{-1}
K0	+6.0	+0.81	4700	0.8	0.85	4.0×10^{-1}
K5	+7.4	+1.11	4000	0.7	0.74	1.6×10^{-1}
M0	+8.9	+1.39	3300	0.5	0.63	6.3×10^{-2}
M5	+12.0	+1.61	2600	0.2	0.32	7.9×10^{-3}

(1) Absolute V-band magnitude

(2) Effective surface temperature

Appendix 2. Physical constants

speed of light	c	2.998×10^{10} cm s^{-1}	2.998×10^{8} m s^{-1}
gravitational constant	G	6.673×10^{-8} dyne cm^{2} g^{-2}	6.673×10^{-11} m^{3} kg^{-1} s^{-2}
Boltzmann constant	k	1.381×10^{-16} erg K^{-1}	1.381×10^{-23} J K^{-1}
Planck's constant	h	6.626×10^{-27} erg s	6.626×10^{-34} J s
Stefan–Boltzmann constant	σ	5.670×10^{-5} erg cm^{-2} K^{-4} s^{-1}	5.670×10^{-8} W m^{-2} K^{-4}
Wien displacement constant	$\lambda_{max}T$	2.898×10^{-1} cm K	2.898×10^{-3} m K
Rydberg constant	R	1.097×10^{5} cm^{-1}	1.097×10^{7} m^{-1}
mass of proton	m_p	1.6726×10^{-24} g	1.6726×10^{-27} kg
mass of neutron	m_n	1.6749×10^{-24} g	1.6749×10^{-27} kg
mass of electron	m_e	9.1096×10^{-28} g	9.1096×10^{-31} kg
mass of hydrogen atom	m_H	1.6735×10^{-24} g	1.6735×10^{-27} kg

Appendix 3. Astronomical constants

astronomical unit	AU	1.496×10^{13} cm	1.496×10^{11} m
parsec	pc	3.086×10^{18} cm	3.086×10^{16} m
solar mass	M_{Sun}	1.989×10^{33} g	1.989×10^{30} kg
solar radius (mean)	R_{Sun}	6.960×10^{10} cm	6.960×10^{8} m
solar luminosity	L_{Sun}	3.839×10^{33} erg s^{-1}	3.839×10^{26} J s^{-1}
Earth mass	M_E	5.977×10^{27} g	5.977×10^{24} kg
Earth radius (mean)	R_E	6.371×10^{8} cm	6.371×10^{6} m
Jupiter mass	M_J	1.899×10^{30} g	1.899×10^{27} kg
Jupiter radius (mean)	R_J	6.991×10^{9} cm	6.991×10^{7} m

Appendix 4. The equations of stellar colour

Planck's empirical law: Energy per second per frequency interval per unit area

$I(\nu,T) = [2h\nu^3/c^2] / [\exp(h\nu/kT) - 1]$

Planck's empirical law: Energy per second per wavelength interval per unit area

$I(\lambda,T) = [2hc^2/\lambda^5] / [\exp(hc/\lambda kT) - 1]$

Wien’s displacement law: wavelength of maximum intensity

$\lambda_{max}T = 2.898 \times 10^6$ nm K

Stefan-Boltzmann law: Integrated energy per second per unit surface area

$E = \sigma T^4$

Integrated energy per second from a sphere: e.g. the total (bolometric) luminosity of a star

$L = 4\pi R^2 \sigma T^4$