NTNU – Trondheim
Norwegian University of
Science and Technology

NTNU, Department of Physics

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Lecturer: Professor Jens O. Andersen
Department of Physics, NTNU

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Problem 1

a) A tidal force is the gravitational force exerted by one body on another which is not constant across it. See Fig. 1. The attractive force from the Moon is larger at the the points A and B and

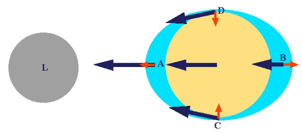

Figure 1: Tidal forces in the Moon-Earth system.

smaller at C and D. This means the oceans will look like the blue region in Fig. 1. Since Earth is rotating about its own axis, the positions A and B (correspond to high tide) and the positions C and D (corresponds to low tide) are not fixed. Since it takes approximately 24h and 50min for a full rotation of Earth with respect to the Moon, we will experience low and high tide every 12h and 25min at a specific point on Earth.

b) Since the Moon is more important for the terrestrial tides, we have low and high tide every 12h and 25min. We get spring tide when the Moon and the Sun are aligned and neap time when they are orthognal to each other, as shown in Fig. 2. Since new moon is taking place every 29.5 days, we

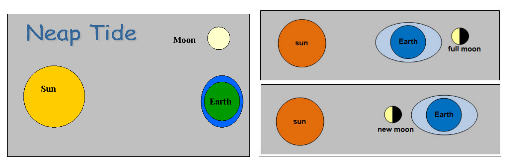

Figure 2: Neap and spring tide.

have spring tide twice every 29.5 days since Sun-Moon-Earth are all aligned.. Likewise neap tide is twice every 29.5 days since the Moon and the Sun have the counteracting gravitational effects.

Problem 2

a) In the limit $r \to \infty$, the Schwarzschild line element reduces to the usual Minkowski line element in Cartesian coordinates. The coordinate time t can therefore be interpreted as (proper) time of an observer at rest infintely far away from the mass M.

(a) A physical singularity is a singularity that is exists independently of the coordinate system used. The point $r = 0$ is an example of physical singularity

(b) A coordinate singularity is a singularity that is due to a bad coordinate system. It can be eliminated by a change of coordinates. $r = r_s = \frac{2GM}{c^2}$ is an example of coordinate singularity.

(c) A horizon is a region in space-time beyond which light cannot escape. The surface $r = r_s$ is a horizon.

b) Radial light rays satisfy ds^2 as well as $d\Omega^2 = 0$. This yields

$$\frac{32G^3M^3}{r} e^{-c^2r/2GM} \left(-dT^2 + dX^2\right) \quad = \quad 0 \,, \tag{1}$$

or $dT = \pm dX$. Integration gives

$$T \quad = \quad \underline{\underline{\pm X + C}} \,, \tag{2}$$

where C is an integration constant. These are straight lines in an X–T diagram. Three lines are shown in red, orange, and blue in Fig. 3.

c) Depending on whether we are inside or outside of the $r = \frac{2GM}{c^2}$, we find

$$X^2 - T^2 = \left(\frac{c^2r}{2GM} - 1\right) e^{\frac{c^2r}{2GM}} \,, \qquad r > r_s \,, \tag{3}$$

$$T^2 - X^2 = \left(1 - \frac{c^2r}{2GM}\right) e^{\frac{c^2r}{2GM}} \,, \qquad r < r_s \,. \tag{4}$$

The point $r = 0$ corresponds to $T^2 - X^2 = 1$, i.e. a hyperbola. This is shown as a dashed blue line in Fig. 3. The surface $r = \frac{2GM}{c^2r}$ corresponds to $X^2 - T^2 = 0$, i.e. $X = \pm T$, which are two straight lines. They are shown as two red lines in Fig. 3.

We note that $r = \frac{2GM}{c^2r}$ coincides with a lightlike curve found in **b**), namely $T = X + C$ for $C = 0$. This is the red curve in the Figure. Another lightlike curve at the $r = \frac{2GM}{c^2r}$ is the orange curve, given by $T = -X + 1$. These two lines define the lightcone at the point on $r = \frac{2GM}{c^2r}$ where they meet. Consider a massive particle crossing $r = \frac{2GM}{c^2r}$ from the outside. Since it must be inside the lightcone, it is clear that it cannot cross again once it is inside. Hence $r = \frac{2GM}{c^2r}$ is a horizon or a one-way membrane. This is the horizon of a Schwarzchild black hole in disguise.

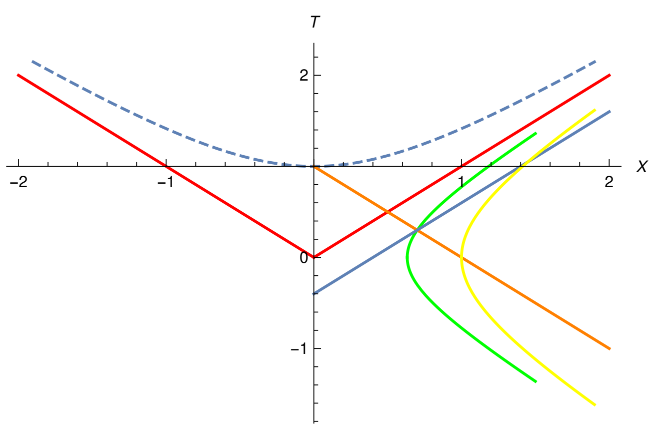

Figure 3: Various curves in a Kruskal diagram. See main text for details.

d) Since $r_A > r_s$ and $r_B > r_s$, Eq. (3) gives

$$X^2 - T^2 = \left(\frac{c^2 r_A}{2GM} - 1\right) e^{\frac{c^2 r_A}{2GM}} , \tag{5}$$

$$X^2 - T^2 = \left(\frac{c^2 r_B}{2GM} - 1\right) e^{\frac{c^2 r_B}{2GM}} , \tag{6}$$

for the worldline of the two observers. The worldlines are shown in green and yellow in Fig. 3. The orange line is a lightlike curve intersecting the two worldlines of the observers A and B. It is therefore corresponds to a light signal that connects them.

Problem 3

a) An eclipting binary consists of two stars orbiting their common center of mass. The brightness depends on the relative positive of the two stars. The brightness decreases when on of the stars blocks the light emitted from the other stars. This is shown in Fig. 4.

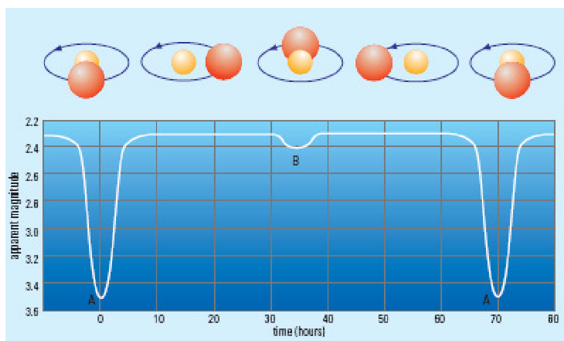

Figure 4: Eclipsing binary. Brightness varies periodically.

When the dimmer star blocks light from the brighter star, we see large drop in apparent brightness, while the drop is smaller when the brighter star blocks the light from the dimmer.

b) In the nonrelativistic limit $v \ll c$, we ignore the γ-factor Since $\omega\lambda = c$, we can write the equation as

$$\begin{aligned} \lambda' &= \frac{\lambda}{1 - \frac{v}{c}\cos\alpha} \\ &= \lambda\left(1 + \frac{v}{c}\cos\alpha\right) , \end{aligned} \tag{7}$$

where we in the last line have made a series expansion to first order in $\frac{v}{c}$ valid for $v \ll c$ Since the $v_r = v\cos\alpha$, we can finally write

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \frac{v_r}{c} . \tag{8}$$

c) The orbit is not circular. If the orbit were circular and the line of sight lies in the orbital plane, the curve would be sinusoidal.

Problem 4

a) A four-vector is a set of quantities A^μ which transforms in the same way as dx^μ under Lorentz transformations. An example of a nonzero four-vector of zero length is the four wave-vector of a photon,

$$\left(\frac{\omega}{c}, \mathbf{k}\right) , \tag{9}$$

where ω is the frequency and $\mathbf{k}$ is the usual (three) wave-vector of the photon.

b) In Newtonian physics, we know that planets in the solar system with total energy less than zero move in closed orbits, namely elliptical orbits. In general relativity, there is a small correction to the equation of motion in Newtonian physics. This gives rise to orbits that are periodic but not closed. The Perihelion is therefore precessing as shown in Fig. 5.

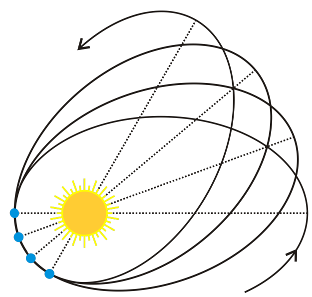

Figure 5: Perihelion preccession of a planet in the solar system. The blue dots indicate the position of the Perihelion.