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Exam in FY3106/FY8303 Phase Transitions and Critical Phenomena
Friday, May 19, 2017
09:00–13:00

Allowed help: Alternativ C

This problem set consists of 2 pages.

Problem 1

When Landau theory is used to calculate the value of critical exponents, one makes the assumption that fluctuations do not contribute to the dominating singularities at the critical point.

a) Use Landau theory to determine the exponents α and β for the heat capacity and order parameter of an isotropic ferromagnet in d spatial dimensions.

b) We now include the contribution from the fluctuations assuming that they are Gaussian. The effective Hamiltonian for the spatially fluctuating magnetization is

$$H = \frac{1}{2} \int d\vec{x} \left\{ \left(\vec{\nabla}\vec{m}(\vec{x})\right)^2 + a\tau\ \vec{m}(\vec{x})^2 \right\} , \tag{1}$$

where $\tau = (T - T_c)/T_c$ and a is a constant. Determine the thermal exponent y_τ by a suitable renormalization group transformation.

c) Determine the magnetic exponent y_h and use y_τ and y_h to determine the corrected values for α and β.

d) What does this calculation say about the fundamental assumption in Landau theory that was quoted in the introduction to the problem?

Given information: $\alpha = 2 - d/y_\tau$ (which we will derive in Problem 3), $\beta = (d - y_h)/y_\tau$ (also to be derived in Problem 3) and $C = -T(\partial^2\Omega/\partial T^2)_{H=0}$, where Ω is the free energy.

Problem 2

We study the Ising model on the triangular lattice using real-space renomalization theory with triangular cells and the Niemeijer-van Leeuwen majority rule.

a) Let the original Hamiltonian contain only the nearest neighbor coupling K_1 (with $-/k_BT$ absorbed into it). Show that the renormalization group transformation in the first cumulant approximation $\mathcal{O}(V^1)$ becomes

$$K_1' = 2\psi^2(K_1)K_1 \ , \tag{2}$$

where

$$\psi(K_1) = \frac{e^{3K_1} + e^{-K_1}}{e^{3K_1} + 3e^{-K_1}} \ . \tag{3}$$

b) Explain the dynamics of this transformation and determine the fixed points.

c) Add a term including an external magnetic field $h\sum_i s_i$ to the Hamiltonian (with $-1/k_BT$ absorbed). Will the fixed points change? Determine, in the critical fixed point, the magnetic eigenvalue to lowest order $\mathcal{O}(V^0)$.

Problem 3

As a result of a renormalization group transformation, we may write the free energy as

$$\omega(\tau, h) = \omega_0(\tau, h) + \frac{1}{l^d}\omega_s(l^{y_\tau}\tau, l^{y_h}h), \tag{4}$$

where $\tau = (T - T_c)/T_c$, h is the external magnetic field and l the scaling factor.

a) Use (4) to show that the heat capacity critical exponent $\alpha = 2 - d/y_\tau$.

b) Derive the hyperscaling relation $2 - \alpha = d\nu$.

c) Derive $\beta = (d - y_h)/y_\tau$.