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Department of Physics, NTNU

Examination paper for **FY3114 / FY8912 Functional materials**

Examination date: **December 4, 2021**
Examination time (from-to): **9-13**

Permitted examination support material:

Alternative C, Approved pocket calculator
K. Rottmann (or equivalent book): Mathematical formulas
English dictionary

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OTHER INFORMATION

Get an overview of the question set before you start answering the questions.

Read the questions carefully and make your own assumptions. If a question is unclear/vague, make your own assumptions and specify them in your answer. Only contact academic contact in case of errors or insufficiencies in the question set. Address an invigilator if you wish to contact the academic contact. Write down the question in advance.

Notifications: If there is a need to send a message to the candidates during the exam (e.g. if there is an error in the question set), this will be done by sending a notification in Inspera. A dialogue box will appear. You can re-read the notification by clicking the bell icon in the top right-hand corner of the screen.

Withdrawing from the exam: If you become ill or wish to submit a blank test/withdraw from the exam for another reason, go to the menu in the top right-hand corner and click “Submit blank”. This cannot be undone, even if the test is still open.

Access to your answers: After the exam, you can find your answers in the archive in Inspera. Be aware that it may take a working day until any hand-written material is available in the archive.

1 P1-fy3114-h2021

Carbon nanotube transistors ...
Select one alternative

- are hampered by strong electron scattering
- may provide fast switching for low electronic currents
- show very high effective mass
- are complicated since the physics is not very well understood

High-K dielectrics devices are characterized by
Select one alternative:

- that they cannot resist high electric fields
- the possibility for scaling down to obtain high circuit density
- low leakage currents
- high electrical conductivity

Which one of the following statements is correct?
Select one alternative

- the dielectric constant of a ferroelectric material is low
- a pyroelectric material is also ferroelectric
- a piezoelectric material is also ferroelectric
- a ferroelectric material is also pyroelectric and piezolectric

Which one of the following crystal systems is uniaxial?

Select one alternative

- ○ monoclinic
- ○ cubic
- ○ orthorombic
- ○ tetragonal

(1 point for each correct answer)

Maximum marks: 4

2 P2-fy3114-h2021

Piezoelectric transistors ...
Select one alternative:

- ○ may be made from centrosymmetric materials
- ○ may be used as strain sensors
- ○ are not suitable for use in touch devices
- ○ represent a mature technology

Strained Si transistors ...
Select one alternative

- ○ show increased optical scattering
- ○ cannot be used to enhance the density of charge carriers in p-doped transistors
- ○ may result in increased mobility
- ○ will not exhibit changes in the electronic band structure

Spintronic transistors ...
Select one alternative

- ○ use the spin but not the charge of the electron
- ○ are already on the market
- ○ use less power but are slower than ordinary transistors
- ○ demonstration devices exists

For a Schottky contact it holds that

Select one alternative

- ◯ $\Phi_{metal} < \Phi_{semi}\ for\ an\ n\ type\ semiconductor$
- ◯ $it\ is\ a\ minority\ carrier\ device$
- ◯ $the\ reverse\ current\ is\ small$
- x ◯ $\Phi_{metal} < \Phi_{semi}\ for\ a\ p\ type\ semiconductor$

(1 point for each correct answer)

Maximum marks: 4

3 P3-fy3114-h2021

Multiferroic materials for storage devices ...
Select one alternative:

- may be metallic, semiconducting or insulating
- represent a mature field of engineering
- cannot be used for Random Axcess memory (RAM)
- may be found in the class of perovskites

Topological transistors ...
Select one alternative

- are conducting in the bulk
- are being used in quantum computation
- are conducting at the surface
- already have a wide range of applications

Graphene transistors ...
Select one alternative

- utilize high effective mass electrons
- have electron mobilities almost as high as in Si
- are candidates for high speed computing applications
- are easy to functionalize by chemical means

Materials for phase change memories ...

Select one alternative

- require electrical power to store data in memory devices
- may be used for rewriteable memory devices
- provides a slower writing time as compared to flash memories
- are not suitable for non-volatile memory devices

(1 point for each correct answer)

Maximum marks: 4

4 P4-fy3114-h2021

Identify the stereograms (2D pointgroup projections) by placing the relevant point group notations on top of the stereograms (-4 means $\bar{4}$)

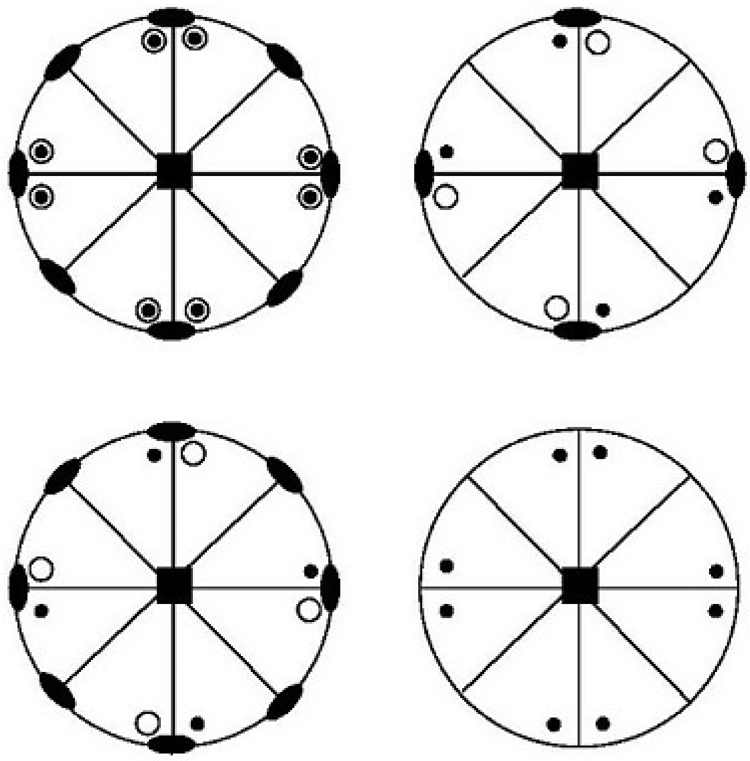
Move the point group notations listed on the left on top of the relevant stereogram

4mm

422

-42m

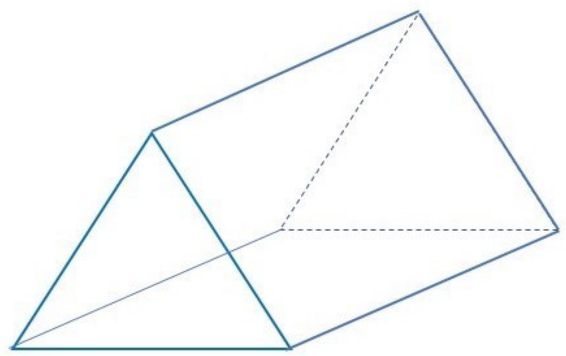
4/m 2/m 2/m



(1 point for each correct answer)

Maximum marks: 4

5 P5-fy3114-h2021



Regular triangular prism

How many symmetry elements are contained in the point group of the triangular prism as shown in the figure above?
Select one alternative:

- 18
- 9
- 16
- 6
- 12

Which one of the following symmetry elements is contained in the point group?
Select one alternative

- $\bar{6}$
- 4
- $\bar{3}$
- $\bar{1}$

(2 point for each correct answer)

Maximum marks: 4

6 P6-fy3114-h2021

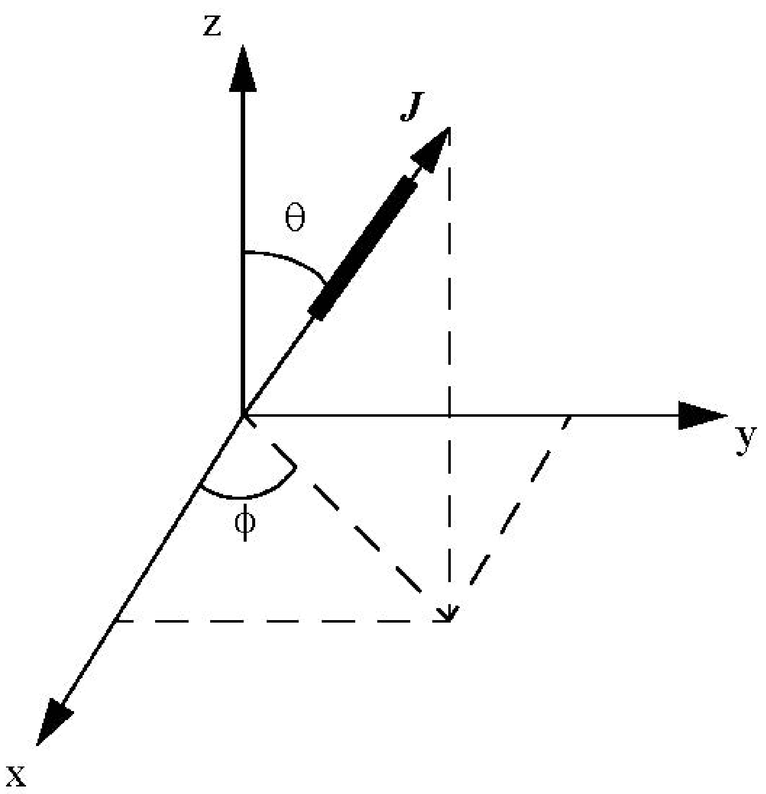
Which ones of the points groups **422**, **32**, $\mathbf{\overline{4}2m}$, $\mathbf{\overline{3}}$, **222**, **4/m**, $\mathbf{\overline{6}}$ are centrosymmetric?
Select one or more alternatives:

- $\mathbf{\overline{3}}$
- $\mathbf{\overline{4}2m}$
- $\mathbf{\overline{6}}$
- $\mathbf{32}$
- $\mathbf{222}$
- $\mathbf{4/m}$
- $\mathbf{422}$

(2 points for correct answer, -1 point for wrong answer, 0 points for no answer, minimum score is 0 points)

Maximum marks: 4

7 P7-fy3114-h2021



A trigonal conducting material is shaped as a long thin rod as shown in the figure above. The current density is ***J***. The rod is at an angle of θ = 30º with the z-axis, and the projection of the rod in the xy-plane is at an angle ϕ = 45º with the x-axis. Find the electrical resistivity along the direction of the rod in terms of the components of the resistivity tensor when ρ_{11} = $8 \cdot 10^{-8}\Omega m$ and ρ_{33} = $4 \cdot 10^{-8}\Omega m$.

Select one alternative:

- $6 \cdot 10^{-8}\Omega m$
- $5 \cdot 10^{-8}\Omega m$
- $3 \cdot 10^{-8}\Omega m$
- $4 \cdot 10^{-8}\Omega m$
- $8 \cdot 10^{-8}\Omega m$

(4 points for correct answer)

Maximum marks: 4

8 P8-fy3114-h2021

A monoclinic crystalline material has dielectric tensor elements $\epsilon_{11}, \epsilon_{12}, \epsilon_{22}, \epsilon_{33} = 2, 1, 4, 3$; respectively. What are the tensor elements for the transformed tensor after an orthogonal transformation that consists of a 45^o rotation around the z-axis?

Enter the answers below:

$\epsilon'_{11} =$ ☐

$\epsilon'_{12} =$ ☐

$\epsilon'_{22} =$ ☐

$\epsilon'_{33} =$ ☐

(1 point for each correct answer)

Maximum marks: 4

9 P9-fy3114-h2021

A second rank symmetric tensor for a monoclinic crystal has the following non-zero elements: A_{11} = 4, A_{12} = 2, A_{22} = 1, A_{33} = 2. This matrix may be diagonalized by a rotation.
Find the angle of rotation that is required to diagonalize the tensor.

Enter your answer here: ☐ ^{o}.

(4 points for correct answer)

Maximum marks: 4

10 P10-fy3114-h2021

GaAs has a band gap of 1.41 eV at room temperature (300 K). Find the energy of the electron and hole relative to the relevant band edges that results when a 1.7 eV photon is adsorbed.

The energy of the electron is:

Select one alternative

- 0.32 eV
- 0.52 eV
- 0.40 eV
- 0.15 eV
- 0.25 eV

The energy of the hole is:

Select one alternative

- -0.042 eV
- -0.15 eV
- -0.025 eV
- -0.038 eV
- -0.35 eV

(2 point for each correct answer)

Maximum marks: 4

11 P11-fy3114-h2021

A p-doped Si sample is kept at temperature 300 K. The density of holes is $5 \cdot 10^{17} cm^{-3}$. Use the law of mass action to find the conductivity due to the electrons.
The conductivity due to the electrons is:
Select one alternative

- $7.2 \cdot 10^{-14} \frac{1}{\Omega cm}$
- $28.0 \frac{1}{\Omega cm}$
- $14.0 \frac{1}{\Omega cm}$
- $7.2 \cdot 10^{-12} \frac{1}{\Omega cm}$
- $3.6 \cdot 10^{-14} \frac{1}{\Omega cm}$

The conductivity due to the holes is:
Select one alternative

- $3.6 \cdot 10^{-14} \frac{1}{\Omega cm}$
- $7.2 \cdot 10^{-14} \frac{1}{\Omega cm}$
- $28.0 \frac{1}{\Omega cm}$
- $14.0 \frac{1}{\Omega cm}$
- $3.6 \cdot 10^{-14} \frac{1}{\Omega cm}$

The conductivity of an undoped sample would be:

Select one alternative:

- $3.2 \cdot 10^{-6} \frac{1}{\Omega cm}$
- $2.4 \cdot 10^{-6} \frac{1}{\Omega cm}$
- $2.6 \cdot 10^{-6} \frac{1}{\Omega cm}$
- $1.6 \cdot 10^{-6} \frac{1}{\Omega cm}$
- $2.2 \cdot 10^{-6} \frac{1}{\Omega cm}$

(1 point for each correct answer)

Maximum marks: 3

12 P12-fy3114-h2021

By using equilibrium distributions for electron and holes the quasi Fermi levels may be defined:
$n = N_c e^{(E_{F_n} - E_c)/k_B T}$ and $p = N_v e^{(E_v - E_{F_p})/k_B T}$.
The following parameters for Si at 300 K may be assumed:
Energy gap $E_{gap} = 1.1 eV$
Densities of states at the band edges $N_c = 2.8 \cdot 10^{19} cm^{-3}$ and $N_v = 1.0 \cdot 10^{19} cm^{-3}$.

Find the energy difference between quasi Fermi levels $E_{f_n} - E_{f_p}$ when charge carrier densities $n = 1 \cdot 10^{16} cm^{-3}$ and $p = 2 \cdot 10^{17} cm^{-3}$ are injected into the semiconductor.

Write the energy difference $E_{f_n} - E_{f_p}$ here: [ ] eV.

(4 points for correct answer)

Maximum marks: 4

13 P13-fy3114-h2021

The diode equation is given by $I = I_0(e^{eV/k_BT} - 1)$.
Estimate the generating current I_0 in a silicon pn-diode, using the following parameters:
Area of diode $A = 12mm^2$
Donor density $N_D = 2 \cdot 10^{17} cm^{-3}$ (assume all states ionized)
Acceptor density $N_A = 1 \cdot 10^{16} cm^{-3}$ (assume all states filled)
Electron diffusion coefficient $D_n = 18cm^2/s$
Hole diffusion coefficient $D_p = 15cm^2/s$
Electron recombination time $\tau_n = 3 \cdot 10^{-7} s$
Hole recombination time $\tau_p = 2 \cdot 10^{-7} s$

The generating current is:
Select one alternative:

- 2.23 nA
- 1.02 nA
- 0.65 nA
- 3.53 nA
- 1,34 nA

(4 points for correct answer)

Maximum marks: 4

14 P14-fy3114-h2021

A solar cell consists of a GaAs pn-diode of area $A = 1cm^2$. The temperature is 300 K.
The total current of the diode connected to an external load is given by

$$I = I_L - I_0(e^{eV/k_BT} - 1)$$

where $I_L$is the photocurrent and $I_0$is the diffusion current of electrons and holes.
Find the open circuit $(I = 0)$ voltage V_{OC} for the solar cell.
Parameter values are:

Density of electrons and holes: $n_n = 1 \cdot 10^{16} cm^{-3}$ and $p_p = 1 \cdot 10^{17} cm^{-3}$
Electron diffusion coefficients: $D_n = 25.0cm^2/s$ and $D_p = 12.1cm^2/s$
Electron and hole recombination times: $\tau_n = 1 \cdot 10^{-8} s$ and $\tau_p = 1 \cdot 10^{-7} s$
Photocurrent: $I_L = 20mA$

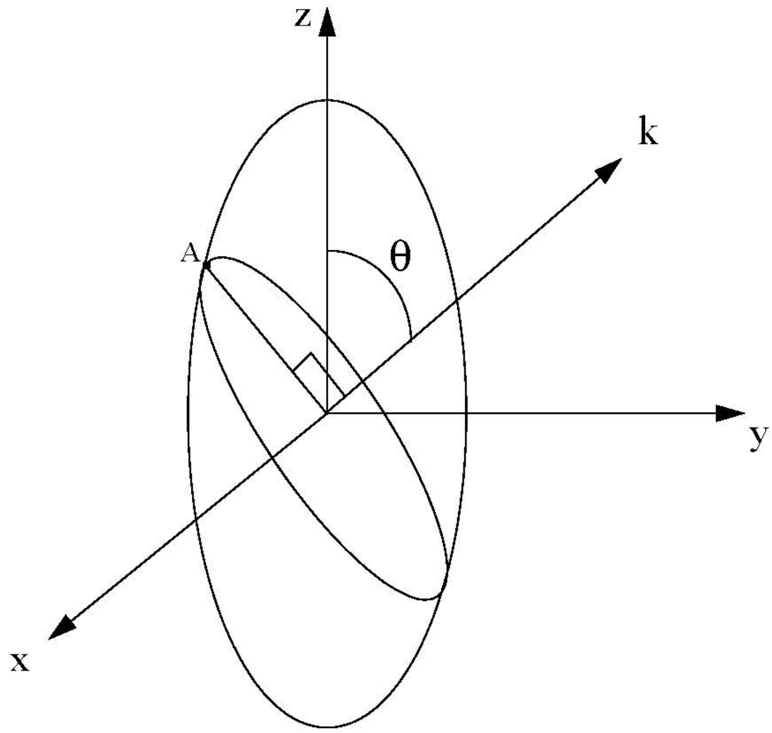
The open circuit voltage V_{OC} is:
Select one alternative:

- 0.67 V
- 0.97 V
- 0.44 V
- 0.53 V
- 0.25 V

(4 points for correct answer)

Maximum marks: 4

15 P15-fy3114-h2021



The figure above shows the index ellipsoid of a uniaxial optical active material. The propagation direction of the light is perpendicular to the ellipse as shown. The wave vector ***k*** is in the yz-plane. The point A is located in the yz-plane and on both the ellipsoid and the ellipse. The angle $\theta = 55^0$, and the ordinary and extraordinary indices of refraction are $n_o = 1.85$ and $n_e = 1.65$.

The index ellipsoid is given by $\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} = 1$

What is the refractive index $n_e(\theta = 55^0)$?

Enter the answer here: [ ] .

(4 points for correct answer)

Maximum marks: 4

16 P16-fy3114-h2021

The piezoelectric tensor of a trigonal crystal of point group 32 is given by:

$$d = \begin{bmatrix} 3 & -3 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$ in units of $10^{-12} C/N$.

Find the polarization when a normal stress $\sigma = 1.0 \cdot 10^4 N/m^2$ is applied along the y-axis.

Enter P_1 here: [ ] 10^{-8} C/m^2

Enter P_2 here: [ ] 10^{-8} C/m^2

Find the polarization when a shear stress $\sigma = 1.0 \cdot 10^4 N/m^2$ is applied around the z-axis.

Enter P_1 here: [ ] 10^{-8} C/m^2

Enter P_2 here: [ ] 10^{-8} C/m^2

(1 point for each correct answer)

Maximum marks: 4

17 P17-fy3114-h2021

Soft ferromagnetic materials may be used for:
Select one or more alternatives:

- ☐ Transformers
- ☐ Magnetic recording devices
- ☐ Motors
- ☐ Magnetic shielding
- ☐ Permanent magnets
- ☐ Induction ovens
- ☐ Flux guides

(1 point for correct answer, -1 point for wrong answer, 0 points for no answer, minimum score is 0 points)

Maximum marks: 3

18 P18-fy3114-h2021

The **Pockels** tensor elements (contracted notation) for the tetragonal electroactive material BTO are as follows:

r_{13} = r_{23} = 8 pm/V
r_{33} = 23 pm/V
r_{42} = r_{51} = 820 pm/V

other **r_{ij} = 0**

Refractive indices are **n_e = 2.18** and **n_o = 2.44**
An electric field **$E_3 = 1\cdot10^8$ V/m** is applied in the z-direction.

The impermeability tensor elements are given by $\eta_{ij}(E) = \eta_{ij}(0) + \sum_k r_{ijk}E_k$

Estimate the change in the difference in the refractive indices $\Delta n = \Delta n_o(E) - \Delta n_e(E)$ due to the electric field.

Select one alternative:

- 0.003
- 0.012
- 0.009
- 0.015
- 0.006

(4 points for correct answer)

Maximum marks: 4

Constants and parameters

$m_e = 9.1 \cdot 10^{-31} kg$	$e = 1.6 \cdot 10^{-19} C$	$k_B = 1.38 \cdot 10^{-23} \frac{J}{K}$	$h = 6.63 \cdot 10^{-34} \frac{J}{K}$
$n_i(Si) = 1.5 \cdot 10^{10} \frac{1}{cm^3}$	$\mu_n(Si) = 1000 \frac{cm^2}{Vs}$	$\mu_p(Si) = 350 \frac{cm^2}{Vs}$	(low field values)
$n_i(GaAs) = 1.84 \cdot 10^6 \frac{1}{cm^3}$	$\mu_n(GaAs) = 8000 \frac{cm^2}{Vs}$	$\mu_p(GaAs) = 400 \frac{cm^2}{Vs}$	(low field values)
$m_e^\star(GaAs) = 0.067 m_e$	$m_h^\star(GaAs) = 0.45 m_e$	$m_e^\star(Ge) = 0.56 m_e$	$m_h^\star(Ge) = 0.29 m_e$
$m_e^\star(Si) = 0.26 m_e$	$m_h^\star(Si) = 0.5 m_e$	$m_e^\star(InP) = 0.07 m_e$	$m_h^\star(InP) = 0.4 m_e$
$n_r(Si) = 3.98$	$n_r(GaAs) = 3.95$	$n_r(Ge) = 5.70$	

Formulas

Rotation matrix

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Transformation of tensors

$T'_{ij} = \sum_{kl} R_{ik} R_{jl} T_{kl}$ and $T'_{ijk} = \sum_{lmn} R_{il} R_{jm} R_{kn} T_{lmn}$

Transformation of products of coordinates

$x'_i x'_i = \sum_{kl} R_{ik} R_{jl} x_k x_l$ and $x'_i x'_j x'_k = \sum_{lmn} R_{il} R_{jm} R_{kn} x_l x_m x_n$

Dielectric permittivity tensor

$$\epsilon_{triclinic} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \epsilon_{22} & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \epsilon_{33} \end{bmatrix} \quad \epsilon_{monoclinic} = \begin{bmatrix} \epsilon_{11} & \epsilon_{12} & 0 \\ \epsilon_{12} & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \quad \epsilon_{orthorombic} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{22} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix}$$

$$\epsilon_{tetragonal} = \epsilon_{trigonal} = \epsilon_{hexagonal} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \quad \epsilon_{cubic} = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{11} \end{bmatrix}$$

Conductivity, drift velocity, mobility, diffusion

$\sigma = ne^2 \tau_{sc}/m^\star \quad v_d = \mu E \quad \mu = e\tau_{sc}/m^\star \quad D_n = \mu_n k_B T/e \quad J_{diff} = eD_n \frac{dn}{dx}$

Electrons and holes in semiconductors

$n = N_c e^{(E_c - E_F)/k_B T} \quad p = N_v e^{(E_F - E_v)/k_B T} \quad np = N_c N_v e^{-E_{gap}/k_B T} \quad N_c = 2(\frac{m_e^\star k_B T}{2\pi\hbar^2})^{3/2}$

Law of mass action

$n_n p_n = n_p p_p = n_i^2$

Diode equation (ideal diode)

$I = I_0(e^{eV/k_B T} - 1)$

Emission

$\hbar\omega = E_e - E_h = E_{gap} + \frac{\hbar^2 k^2}{2m_r^\star} \quad W_{em}^{st}(\hbar\omega) = \frac{e^2 n_r \hbar\omega}{3\pi\epsilon_0 m_e^2 c^3 \hbar^2} |p_{cv}|^2 \cdot n_{ph}(\hbar\omega) \quad \frac{2|p_{cv}|^2}{m_e} = 23eV\ (GaAs)$

Impermeablity tensor

$\sum_{ij} \eta_{ij} x_i x_j = 1 \qquad \eta = \frac{1}{\epsilon_r} = \frac{1}{n_r^2}$

Index ellipsoid

$\frac{x_1^2}{n_1^2} + \frac{x_2^2}{n_2^2} + \frac{x_3^2}{n_3^2} = 1$

Solution Exam-fy3114-h2021

1-3 multiple choice

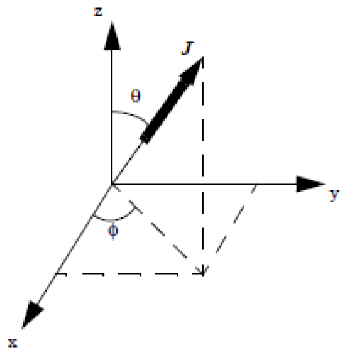
4. see table of point group stereograms (lecture notes)

5. Regular triangular prism:
12 symmetry elements in the point group: 1, 3, 3^2, 1/m, m, m, m, 2, 2, 2, -6^1, -6^5

6. see table of point groups (lecture notes)

7.

A trigonal conducting material is shaped as a long thin rod, and is directed as shown in the figure below. The current $\boldsymbol{J}$ flows in the direction of the rod. The rod is at an angle $\theta = 30°$ with the z-axis, and the projection of the rod into the xy-plane is at an angle $\phi = 45°$ with the x-axis.
Find the electrical resistivity ρ_J along the direction of the current $\boldsymbol{J}$ in terms of the components of the resistivity tensor.



The resistivity tensor of a trigonal solid is given by

$$\rho_{trigonal} = \begin{bmatrix} \rho_{11} & 0 & 0 \\ 0 & \rho_{11} & 0 \\ 0 & 0 & \rho_{33} \end{bmatrix} \quad and \quad \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = \begin{bmatrix} \rho_{11} & 0 & 0 \\ 0 & \rho_{11} & 0 \\ 0 & 0 & \rho_{33} \end{bmatrix} \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix}$$

The resistivity along the direction of the electric current is given by

$$\rho_J = E_J/J = \frac{\boldsymbol{E} \cdot \boldsymbol{J}}{J^2} = \sum_i E_i J_i / J^2 = \sum_{ij} \rho_{ij} \frac{J_j J_i}{J\,J}$$

then we get

$$\rho_J = \rho_{11}\left(\frac{J_x}{J}\right)^2 + \rho_{11}\left(\frac{J_y}{J}\right)^2 + \rho_{33}\left(\frac{J_z}{J}\right)^2$$

From the figure we see

$$J_x = J\sin\theta\cos\phi \qquad J_y = J\sin\theta\sin\phi \qquad J_z = J\cos\theta$$

which gives when $\theta = 30°$ and $\phi = 45°$

$$\rho_J = \rho_{11}(\sin\theta\cos\phi)^2 + \rho_{11}(\sin\theta\sin\phi)^2 + \rho_{33}(\cos\theta)^2$$

$$\rho_J = \rho_{11}\left(\frac{1}{2}\frac{1}{\sqrt{2}}\right)^2 + \rho_{11}\left(\frac{1}{2}\frac{1}{\sqrt{2}}\right)^2 + \rho_{33}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{\rho_{11}}{4} + \frac{3\rho_{33}}{4}$$

8.

Dielectric tensor of a monoclinic material $\varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & 0 \\ \varepsilon_{12} & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix}$

Rotation of 45° around the z-axis: $x' = \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}\,y$, and $y' = -\frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}\,y$, which gives

$$x'x' = \frac{1}{2}(x^2 + 2xy + y^2) \;\Rightarrow\; \varepsilon'_{11} = \frac{1}{2}(\varepsilon_{11} + 2\varepsilon_{12} + \varepsilon_{22})$$

$$x'y' = \frac{1}{2}(x + y)(y - x) = \frac{1}{2}(y^2 - x^2) \;\Rightarrow\; \varepsilon'_{12} = \frac{1}{2}(\varepsilon_{22} - \varepsilon_{11})$$

$$y'y' = \frac{1}{2}(x^2 - 2xy + y^2) \;\Rightarrow\; \varepsilon'_{22} = \frac{1}{2}(\varepsilon_{11} - 2\varepsilon_{12} + \varepsilon_{22})$$

answer: ε_{11}', ε_{12}', ε_{22}', ε_{33}' = 4, 1, 2 , 3

9.

Rotation around the z-axis gives

$A'_{12} = (A_{22} - A_{11})\frac{1}{2}sin2\theta + A_{12}cos2\theta$ therefore $tan2\theta = \frac{2A_{12}}{A_{11} - A_{22}}$ = 4/3

answer: θ = 26.6°

10.

GaAs has band gap of 1.41 eV at a temperature of 300 K. Find the energy of the electron and hole (relative to the respective band edge) that results when a 1.7 eV photon is adsorbed.

A photon is absorbed by GaAs and thus creates an electron in the conduction band and a hole in the valence band. By using the reduced mass we may write

$$\hbar\omega = E_e - E_h = E_{gap} + \frac{(\hbar k)^2}{2m_r^*} \quad and \quad E_e = E_c + \frac{(\hbar k)^2}{2m_e^*} \quad and \quad E_h = E_v - \frac{(\hbar k)^2}{2m_h^*}$$

The energies of the electron and hole become

$$E_e - E_c = \frac{m_r^*}{m_e^*}(\hbar\omega - E_{gap}) = \frac{0{,}0583}{0{,}067}(1{,}7 - 1{,}41) = 0{,}25eV$$

$$E_h - E_v = -\frac{m_r^*}{m_h^*}(\hbar\omega - E_{gap}) = \frac{-0{,}0583}{0{,}45}(1{,}7 - 1{,}41) = -0{,}038eV$$

11.

A p-doped Si sample is at a temperature of 300 K. The density of holes in the valence band is $p_a = 5 \cdot 10^{17}$ cm^{-3}. Use the law of mass action to find the conductivity due to the electrons. What is the conductivity due to the holes? Compare to the conductivity of an undoped sample.

The conductivity is given by

$$\sigma = \sigma_n + \sigma_p = ne\mu_n + pe\mu_p = p_a e\mu_p + \frac{n_i^2}{p_a} e\mu_n = 28\frac{1}{\Omega cm} + 7{,}2 \cdot 10^{-14}\frac{1}{\Omega cm}$$

where the law of mass action has been used:

$$n \cdot p = n_i^2$$

The conductivities are

$$\sigma_n = 7{,}2 \cdot 10^{-14} \qquad \sigma_p = 28{,}0 \qquad \sigma_{intrinsic} = n_i e\mu_n + p_i e\mu_p = 3{,}2 \cdot 10^{-6}\frac{1}{\Omega cm}$$

12.

We have $n = N_c e^{(E_{fn} - E_c)/k_B T}$ and $p = N_v e^{(E_v - E_{fp})/k_B T}$

This gives $E_{f_n} - E_{f_p} = E_c - E_v + k_B T(\ln\left(\frac{n}{N_c}\right) - \ln\left(\frac{p}{N_v}\right))$

Answer: 0.79 eV

13.

The generating current is given by $I_0 = A\left(\frac{eD_n n_p}{L_n} + \frac{eD_p p_n}{L_p}\right)$ $where\ L_n = \sqrt{D_n \tau_n}\ and\ L_P = \sqrt{D_p \tau_p}$

Using $n_n p_n = n_p p_p = n_i^2 = N_d p_n = n_p N_a$ we get

$$I_0 = Aen_i^2\left(\frac{D_n}{L_n N_a} + \frac{D_p}{L_p N_d}\right)$$

Answer: 3.53nA

14.

Consider a GaAs pn-diode that is used as a solar cell. The temperature is 300 K. The total current of the diode connected to an external load is given by I where I_L is the photocurrent and I_0 is the diffusion current of electrons and holes. Find the open circuit voltage for the solar cell.

$$I = I_L - I_0(e^{eV/k_BT} - 1)$$

Diffusion currents for electrons on the p-side and holdes on the n-side is:

$$J_n = eD_n\frac{dn}{dx} = eD_n \cdot \frac{n_p}{L_n} \qquad and \qquad J_p = eD_p\frac{dp}{dx} = eD_n \cdot \frac{p_n}{L_p}$$

Using law of mass action:

$$n_p p_p = n_i^2 = n_n p_n$$

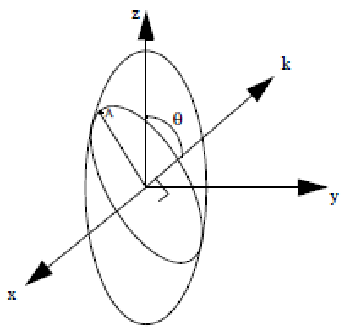
Therefore:

$$\frac{I_0}{A} = eD_n \cdot \frac{n_p}{L_n} + eD_n \cdot \frac{p_n}{L_p} \quad where \quad L_n = \sqrt{D_n\tau_n} \quad and \quad L_p = \sqrt{D_p\tau_p}$$

The open circuit voltage becomes (I =0):

$$V_{OC} = \frac{k_BT}{e}\ln\left(1 + \frac{I_L}{I_0}\right) = 0{,}97V$$

15.



A uniaxial optical active material has index ellipsoid as shown above. The propagation direction of light is perpendicular to the ellipse shown in the figure, and the wave vector k is in the yz-plane. The point A is located on the ellipsoid and the ellipse in the yz-plane. The angle $\theta=55^{o}$ and the ordinary and extraordinary indices of refraction are n_{ro} =1.85 and n_{re} =1.65. What is the the refractive index $n_{re}(\theta)$ for light traveling in the k-direction?

From figure: $x_A = 0$, $y_A = |A|\cos\theta$, $z_A = |A|\sin\theta$

From ellipse: $|A| = n_{re}(\theta)$

The point A is on the index ellipsoid:

$$\frac{x_A^2}{n_{ro}} + \frac{y_A^2}{n_{ro}} + \frac{z_A^2}{n_{re}} = 1 \Rightarrow \frac{n_{re}(\theta)\cdot(\cos\theta)^2}{n_{ro}^2} + \frac{n_{re}(\theta)\cdot(\sin\theta)^2}{n_{re}^2} = 1$$

and we get:

$$\frac{1}{n_{re}(\theta)^2} = \frac{(\cos\theta)^2}{n_{ro}^2} + \frac{(\sin\theta)^2}{n_{re}^2}$$

Inserting numbers gives: $n_{re}(\theta=55^{o}) = 1.71$

16.

The piezoelectric tensor of a trigonal material of point group 32 is given by:

$$d = \begin{bmatrix} 3 & -3 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -5 & -6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

in units of 10^{-12} C/N.

The polarization along the the x-axis for normal stress $\sigma = 10^4$ Nm^{-2} along the y-axis:

P1 = $d_{12}\,\sigma_{22} = d_{12}\,\sigma_2 = -3\ 10^8$ Cm^{-2}

The polarization along the the y-axis for shear stress $\sigma = 10^4$ Nm^{-2} around the z-axis:

P2 = $d_{26}\,\sigma_{12} = d_{26}\,\sigma_6 = -6\ 10^8$ Cm^{-2}

17.

Soft ferromagnetic materials may be used for e.g. transformers, flux guides, magnetic shielding

18.

Electroactive optic material BTO

Impermeability tensor elements $\eta_{ij} = \eta_{ij}{}^0 + r_{ij3}E_3$

Index ellipsoid given by $\sum_{ij} \eta_{ij}\, x_i x_j = 1$

This gives $\eta_{11}{}^0 x^2 + \eta_{22}{}^0 y^2 + \eta_{33}{}^0 z^2 + r_{113}E_3 x^2 + r_{223}E_3 y^2 + r_{333}E_3 z^2 = 1$

Therefore $\left(\frac{1}{n_o{}^2} + r_{13}E_3\right)x^2 + \left(\frac{1}{n_o{}^2} + r_{13}E_3\right)y^2 + \left(\frac{1}{n_e{}^2} + r_{33}E_3\right)z^2 = 1$

And change in refractive index in the E-field *is* $\Delta\left(\frac{1}{n_o{}^2}\right) = r_{13}E_3$ and $\Delta\left(\frac{1}{n_e{}^2}\right) = r_{33}E_3$

Which gives $\Delta n_o = -\frac{1}{2} n_o{}^3 r_{13} E_3$ and $\Delta n_e = -\frac{1}{2} n_e{}^3 r_{33} E_3$

And $\Delta n = \Delta n_o - \Delta n_e = -\frac{1}{2} E_3 (n_o{}^3 r_{13} - n_e{}^3 r_{33})$

answer: Δn = 0.006