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EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE

Faculty for Natural Sciences and Technology

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Time: 09:00-13:00

Number of pages: 5

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Bi-lingual dictionary permitted
Approved calculators are permitted

You may take:

Molar mass of water vapour ~ 18 kg/kmole $g=9.8$ m s⁻² and constant in z
Molar mass of dry air ~ 29 kg/kmole 1 hPa = 10^2 Pa = 10^2 N m⁻²
273 K = 0 °C Scale Height, $H=R\cdot T/g$
Values for dry air: $C_p=1004$ J·K⁻¹·kg⁻¹ $C_v=718$ J·K⁻¹·kg⁻¹ $R_d=287$ J·K⁻¹·kg⁻¹
 $\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d=C_p - C_v$ $\Gamma_{da}=9.8$ K/km

Answer all 5 questions (and good luck!):

SOLUTIONS:

1) **Atmospheric structure (20 %):**

- a) **At the beach, where both the ocean and the land can be taken to lie at a height of 0 meters, the Sun has warmed up the air-mass over the land to a uniform temperature of 25°C. The air-mass over the ocean still has a uniform temperature of 15°C. If the air pressure at the surface is 1000 hPa, determine the air pressure of the layers 2 km above the ocean and the land, respectively. Assume dry air. (15%)**

They can use the Hypsometric equation directly:

$$z_2 - z_1 = \frac{RT}{g} \ln\left(\frac{p_1}{p_2}\right)$$

They can easily derive from the Hydrostatic Equation:

$$\frac{dp}{dz} = -g\rho, \text{ and the Perfect Gas Law: } \rho = \frac{p}{RT}, \text{ to get: } \frac{dp}{p} = -\frac{g}{RT} dz, \text{ which}$$

integrates directly into the Hypsometric equation for constant T. Something we have done countless times in class! Here we have 2 temperature conditions:

*Over the water: $p_1=1000 \text{ hPa}$; $R= 287$; $g=9.8$; $z_1=0\text{m}$; $z_2=2000\text{m}$, $T=273+15=288\text{K}$
Which yields for $p_2(\text{ocean})=789 \text{ hPa}$.*

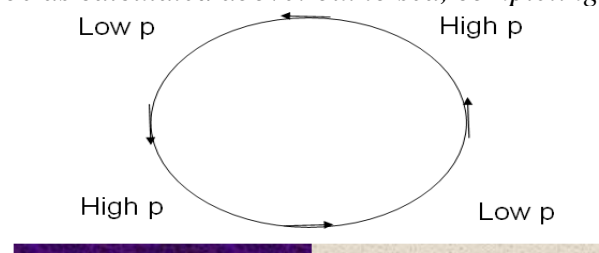
*Over the land: $p_1=1000\text{hPa}$; $R= 287$; $g=9.8$; $z_1=0\text{m}$; $z_2=2000\text{m}$; $T=273+25=298\text{K}$.
Which yields for $p_2(\text{land})=795 \text{ hPa}$*

- b) **In which direction does the wind blow at a height of 2 km? (3%)**

Since the pressure at 2000 m is greater over the land than over the water, the wind will blow from high pressure to low pressure, or towards the ocean. So, this is where the surface level sea breeze (the wind from the ocean) returns out to sea!

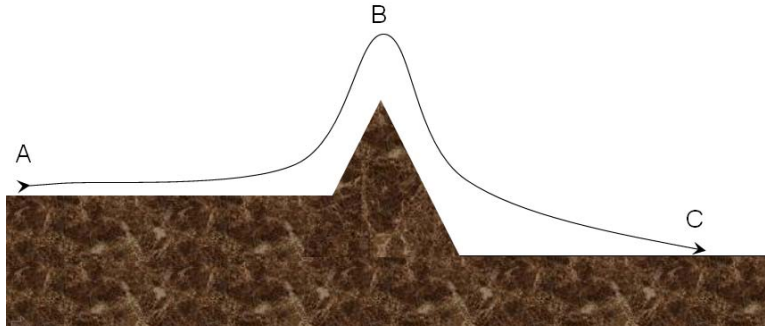
- c) **In reality a flow pattern will be set up between the land and sea at the surface. Sketch this pattern with arrows that show the local wind direction. (2%)**

In reality, the heating of the air over the land will take place mostly through long wavelength radiation from the surface, although sensible and convective heat transfer at the surface will also play a role. As we discussed in class, the heating will cause the lower atmosphere to be warmed most, and convective instabilities to develop. Thus there will be a rising of the surface air over the land and a lowering of the air pressure there. The cooler air over the sea will flow from the high pressure over the sea surface into the low-pressure area over the land. Above the boundary layer, about 2 km, the flow will be as calculated above: out to sea, completing the cycle.



2) Atmospheric thermodynamics (20 %)

An air-packet begins at a height of 500 m (point A in the figure below) where the air pressure is 940 hPa and the temperature is 15°C, and is carried up the mountain to point B where the air pressure is 800 hPa. Afterwards, the air packet comes down the other side of the mountain to point C where the air pressure is 1000 hPa. Assume dry air and that the air packet undergoes adiabatic expansion and compression.



a) Determine the potential temperature at point A. (5%)

The potential temperature at point A is calculated from:

$$\theta := T \left(\frac{P_0}{P} \right)^\kappa$$

We can choose our reference pressure anywhere we want, so I will pick 1000 hPa. Then $T = 273 + 15 = 288 \text{ K}$; $P_0 = 1000 \text{ hPa}$; $P = 940 \text{ hPa}$; $\kappa = (R/C_p) = (287./1004.) = .286$. This yields $\theta = 293 \text{ K}$. This can be different if the student has chosen another P_0 .

b) Determine the temperature at points B and C, respectively. (11%)

Since the dry parcel undergoes only adiabatic changes, its potential temperature is constant, and we can solve for its temperature at any pressure level relative to the reference level, P_0 , which I have chosen to be 1000 hPa. Solving for temperature:

$$T := \theta \left(\frac{P}{P_0} \right)^\kappa$$

Then for $P_0 = 1000 \text{ hPa}$; $P = 800 \text{ hPa}$; $\kappa = (R/C_p) = (287./1004.) = .286$, we get:
 $T_B = 275 \text{ K} = 2^\circ \text{ C}$ at point B. Changing $P = 1000 \text{ hPa}$ then gives $T_C = 293 \text{ K}$ or 20° C .

c) If the air packet had contained water vapour which just began to condense at point B, would the air packets temperature be warmer or colder than a dry air packet at points B and C, respectively. Assume that both the dry and moist air packets had the same temperature at point A. (2%)

If water vapour begins to condense, then its latent heat will **warm the air parcel at point B**. However, since it has just begun to condense at B, it will re-vapourize on its way to C and take back its latent heat. Thus, the parcel will **still be at 20°C at point C**

d) If the air packet had contained water vapour which began to rain out on the mountain top, would the air packet's temperature be warmer or colder than a dry air packet at points B and C, respectively. Assume that both the dry and moist air packets had the same temperature at point A. (2%)

Since the parcel loses water, not all the latent heat will be re-absorbed as the remaining water re-vapourizes. So it will be **warmer than dry air at both B and C**.

3) **Radiation and scattering (20 %)**

- a) **A volcanic ash cloud is 400 m thick and consists of 1 μm (10^{-6} m) diameter particles which have a uniform density of $1 \times 10^9 \text{ m}^{-3}$. A parallel beam of solar radiation with a wavelength of 1 μm hits the ash cloud at a zenith angle of 0° . The beam is scattered with Mie scattering which has a scattering cross section which is 4 times larger than the particle's cross-sectional area. Calculate the cloud layer's optical depth. (8 %)**

The optical depth is given by $\tau_z = \int_{z_{\text{bottom}}}^{\infty} k_v \cdot \rho_a \cdot dz$, and since we only want the cloud's

optical depth, the integration goes only to the top of the cloud. k_v and ρ relate to the mass, but $k_v = \sigma/m$ and $\rho = N \cdot m$, so $k_v \cdot \rho = \sigma \cdot N$. So the integral yields that $\tau_c = \sigma \cdot N \cdot 400\text{m}$, and we are told $\sigma = 4 \cdot \pi \cdot D^2/4$, $D = 1 \times 10^{-6}\text{m}$ and $N = 1 \times 10^9 \text{ m}^{-3} \Rightarrow \sigma = 3.14 \times 10^{-12}$. So, the optical depth is: $\tau = 3.14 \times 10^{-12} \text{ m}^2 \cdot 1 \times 10^9 \text{ m}^{-3} \cdot 400 \text{ m} = 1.25$

- b) **What is the cloud layer's transmission in percent? (8 %)**

The transmission coefficient is defined as:

$$T := e^{(-\tau)}$$

If one can't remember this, then it can be derived quickly from Lambert's law: $dI/I = -\sigma \cdot N \cdot dx$, which the students should remember. Substituting in for τ gives: $T = 0.28$, or 28% transmission.

- c) **Would the cloud layer's transmission be expected to increase or decrease for wavelengths at 10 μm where the Earth radiates its thermal emission? Why? (2 %)**

The cross section for scatter is a maximum when the wavelength is equal to the particle size. So as we move to different wavelengths it decreases. I would accept this if they give this explanation. They can also say that **at $\lambda = 10 \mu\text{m}$ the wavelength is 10x the particle size and we will have Rayleigh scatter whose cross section is much less than Mie scatter.** The latter is more complete, but if they know where the cross section maximizes I will be happy!

- d) **On these grounds, would the ash cloud be expected to warm or cool the Earth? (2 %)**

Well, block the solar heating and still let the Earth cool, it should cool the earth. They may mention that the ash might be dark and absorb light, directly heating the atmosphere. This is also correct. So, we can accept either of the answers with an explanation.

4) **Climate and models (20 %)**

- a) **What are the most important optical properties of a gas that allow it to create a greenhouse effect? (5 %)**

Must transmit at short wavelengths and absorb at long

- b) **Describe briefly the difference between numerical grid models and spectral models (5 %)**

In a numerical grid model, the differentials in the equations of motion are solved using finite differences between the grid points. In a spectral model, spherical harmonics are fit to the grid values, and the Fourier series differentiated.

- c) **What is meant by a radiative equilibrium temperature? (5 %)**

When the radiative energy absorbed = radiative energy emitted. If we assume that the atmosphere radiates as a grey body, the Planck formula may be used to determine the temperature at which one must be to radiate that much energy. This temperature is the radiative equilibrium temperature.

- d) **Give two examples of processes which are of such small scales that they can not be directly solved in models and must be parameterized. (5 %)**

We can't be too restrictive here, but the process must be reasonable. Things like convection, albedo, coastlines, mountain ranges, gravity waves, clouds, aerosols...

5) **Atmospheric stability (20 %)**

- a) **Under what conditions is a dry atmosphere stable against vertical motion and why? (10 %)**

*If the temperature in the atmosphere falls off less quickly than the dry adiabatic rate that an air parcel will follow, then a parcel displaced upward will expand, cool and find itself cooler, and hence denser, than the surrounding air. Since it is heavier than the surrounding air, it will sink back to its starting position. Thus the atmosphere is **stable to vertical movements** for $\Gamma_{\text{env}} < \Gamma_a$. If the atmosphere has a dry adiabatic lapse rate, $\Gamma_{\text{env}} = \Gamma_a$, then a parcel will stay where it is place, and we still say that atmosphere is neutrally stable. Either this explanation or a graph will be accepted. Ignoring neutrally stable should only loose 1/2 point.*

- b) **How does water vapour in the atmosphere influence this stability? (10%)**

*Water vapour in the atmosphere can affect the stability. If a moist parcel is displaced upwards and its water vapour **begins to condense**, the latent heat of vapourization given off will warm the parcel and keep it warmer than its dry-adiabatic counterpart. This curve is the moist-adiabatic lapse rate (or wet lapse rate), and $\Gamma_w < \Gamma_a$. Thus, the temperature in the atmosphere in the atmosphere falls off less quickly than this moist-adiabatic rate, $\Gamma_{\text{env}} < \Gamma_w$, **it will be stable for saturated air**. If the atmosphere were close to being unstable with dry air, then $\Gamma_a > \approx \Gamma_{\text{env}} > \Gamma_w$, then if a moist air parcel becomes saturated, it will follow Γ_w and become unstable.*