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EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE

Faculty for Natural Sciences and Technology

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Time: 09:00-13:00

Number of pages: 10

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Bi-lingual dictionary permitted
All calculators permitted

You may take:

Molar mass of water vapour: ~ 18 kg/kmole $g=9.8$ m s⁻² and constant in z

Molar mass of dry air: ~ 29 kg/kmole 1 hPa = 10^2 Pa = 10^2 N m⁻²

273 K = 0 °C Scale Height: $H=R\cdot T/g$

Latent heat of vaporization water: $L_v=2.5\times 10^6$ J · kg⁻¹

Gas constant for water vapour: $R_v=461$ J · K⁻¹ · kg⁻¹

Values for dry air: $C_p=1004$ J · K⁻¹ · kg⁻¹ $C_v=718$ J · K⁻¹ · kg⁻¹ $R_d=287$ J · K⁻¹ · kg⁻¹

$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d = C_p - C_v$ $\Gamma_{da} = 9.8$ K/km

$$\int \frac{dx}{a+b\cdot x} = \frac{1}{b} \ln(a+b\cdot x) \quad \int \frac{dx}{(a+b\cdot x)^2} = \frac{-1}{b\cdot(a+b\cdot x)} \quad \int \frac{dx}{(a+b\cdot x)^3} = \frac{-1}{2b\cdot(a+b\cdot x)^2}$$

Answer all questions (and good luck!):

Solutions

1) (25%) On Venus, the atmosphere consists of 95% CO₂ and 5% N₂. The gravitational acceleration 8.9 m s⁻², and the atmospheric pressure at the surface is 9000 kPa (1hPa = 0.1 kPa).

- a) What is the mean molar mass and gas constant for the Venus atmosphere? (Take the atomic weights of C, O and N to be 12, 16 and 14, respectively) (7 %)

So, to calculate the mean molar mass on Venus, we would take:

$$M_V = v_{CO_2} * M_{CO_2} + v_{N_2} * M_{N_2}, \text{ where } v_i \text{ are the respective mixing ratios.}$$

Here we have that $v_{CO_2} = 0.95$, $M_{CO_2} = 12 + 16 + 16 = 44$ g/mole, $v_{N_2} = 0.05$, $M_{N_2} = 14 + 14 = 28$ g/mole, giving a mean molar mass on Venus of **$M_V = 43.2$ g/mole.**

On earth, the gas constant for dry air is $R^*/M_{dry\ air} = 287$ J·K⁻¹·kg⁻¹, and $M_{dry\ air}$ is given as 29 g/mole, so the gas constant on Venus is

$$R_{dry\ air} * M_{dry\ air} / M_V \Rightarrow \underline{R_V = 192.66 \text{ J}\cdot\text{K}^{-1}\cdot\text{kg}^{-1}}$$

- b) If the atmospheric temperature as a function of altitude is given by the expression:

$$T = T_o - \Sigma \cdot z, \quad \text{where } \Sigma = 9.4 \text{ K}\cdot\text{km}^{-1} \text{ and } T_o = 735 \text{ K}$$

at what altitude on Venus is the pressure the same as the surface pressure on Earth (1000 hPa)? (15 %)

Here, as we have done many times in class, we start with the hydrostatic equation:

$$\frac{\partial}{\partial z} P = -\rho g$$

And the perfect gas law:

$$\rho := \frac{P}{R T}$$

To get a relation between p and T , the hypsometric equation:

$$\frac{dp}{p} = -\frac{g dz}{R T}$$

Since T is a function of z , we can put in its functional form, $T = T_o + \Sigma z$, and integrate explicitly:

$$\int_{P_o}^P \frac{1}{p} dp = \int_{Z_o}^Z -\frac{g}{R v (T_o - \Sigma z)} dz$$

And using the integral on the first page of the exam, with $a = T_o$ and $b = -\Sigma$, gives:

$$\ln(P) - \ln(P_o) = -\frac{g (-\ln(T_o - \Sigma Z) + \ln(T_o - \Sigma Z_o))}{R v \Sigma}$$

And we solve for Z to get:

$$Z := \frac{-e^{\left(\frac{\ln(P) R v \Sigma - \ln(P_o) R v \Sigma + g \ln(T_o - \Sigma Z_o)}{g}\right)} + T_o}{\Sigma} = \frac{T_o \left(1 - \left(1 - \frac{\Sigma Z_o}{T_o}\right) \left(\frac{P}{P_o}\right)^{\left(\frac{R v \Sigma}{g}\right)}\right)}{\Sigma}$$

With $Z_0=0$, this simplifies to:

$$Z = \frac{T_0 \left(1 - \left(\frac{P}{P_0} \right)^{\left(\frac{R_v \Sigma}{g} \right)} \right)}{\Sigma}$$

Substituting in:

$P_0:=90000$; $P:= 1000$; $T_0:=735$; $Z_0:=0$; $\text{Sigma}:=0.0094$;
 $R_v:=192.66$; $g:= 8.9$;

Gives: **$Z = 46894 \text{ m, or } 47 \text{ km.}$**

One could also have converted the integral to one in temperature, using the fact that $dT/dz = -\Sigma$. This would give:

$$\int_{P_0}^P \frac{1}{p} dp = \int_{T_0}^T -\frac{g \left(\frac{\partial}{\partial T} Z \right)}{R_v T} dT \quad \text{or} \quad \int_{P_0}^P \frac{1}{p} dp = \int_{T_0}^T \frac{g}{R_v T \Sigma} dT$$

Which integrates to: $\ln(P) - \ln(P_0) = \frac{g (\ln(T) - \ln(T_0))}{R_v \Sigma}$, and we can solve for the T at the upper altitude and use, $T=T_0+\Sigma z$, to get z .

Note, some chose to use the Hypsometric equation with an average T to solve this. Since an average temperature is a reasonable approximation for a linear decrease in T , I will accept this. In that case, the average temperature $[T(Z_2)+T(Z_1)]/2$ is:

$$T_{\text{avg}} = T_0 - \frac{1}{2} \Sigma Z_2 - \frac{1}{2} \Sigma Z_0$$

And the hypsometric equation is:

$$Z_2 = Z_0 + \frac{R_v T_{\text{avg}} \ln\left(\frac{P_0}{P_2}\right)}{g} = -\frac{R_v \ln\left(\frac{P_0}{P_2}\right) \Sigma Z_0 - 2 Z_0 g - 2 R_v \ln\left(\frac{P_0}{P_2}\right) T_0}{2 g + R_v \ln\left(\frac{P_0}{P_2}\right) \Sigma}$$

With $Z_0=0$, this becomes

$$Z_2 := 2 \frac{R_v \ln\left(\frac{P_0}{P_2}\right) T_0}{2 g + R_v \ln\left(\frac{P_0}{P_2}\right) \Sigma}$$

Upon substitution of the upper pressure, P_2 , and lower, P_0 , **$Z_2=49111 \text{ m}=49\text{km}$**

- c) What is the temperature at the altitude where the pressure is 1000 hPa?
 What planet has these conditions at the surface? (3%)

Here we can just use $T=T_0+\Sigma z$, and Z from above to get that **$T = 294 \text{ K} = 21 \text{ }^\circ\text{C}$** .
 $T=294\text{K}$ and $P=1000 \text{ hPa}$ is very close to what one would find at the surface of the Earth. So 50 km up in the atmosphere of Venus, we have earth like conditions!
 If the hypsometric equation was used, the temperature at 49km would be **$T=273\text{K}=0 \text{ }^\circ\text{C}$** , which certainly matches Trondheim June temperatures!

2) (25%) If there is no scattering, a parallel beam from the sun at zenith angle ϕ is absorbed in the atmosphere according to the equation:

$$-\frac{dl_\lambda}{l_\lambda} = -k_\lambda \cdot \rho(z) \cdot \sec(\phi) \cdot dz$$

- a) Given an isothermal atmosphere with constant scale height, H, what is the optical depth, τ_λ , at height, z? (8%)

Here we can borrow from problem 1 the results of the hydrostatic equation and perfect gas law:

$$\frac{\partial}{\partial z} p = -\rho g \Rightarrow \frac{dp}{p} = -\frac{g dz}{RT}$$

And the definition of the scale height $H = R \cdot T / g$ to integrate for $p = p_0 \cdot e^{-z/H}$. The perfect gas law states $p = \rho RT$, and since the scale height is constant, this means T is constant. This allows us to write $\rho = \rho_0 \cdot e^{-z/H}$.

The optical depth at any height z is defined as:

$$\tau := \int_z^\infty \rho k \sec(\phi) dz$$

Substituting in for ρ gives:

$$\tau := \int_z^\infty \rho_0 e^{\left(-\frac{z}{H}\right)} k \sec(\phi) dz$$

This integrates to:

$$\tau := \rho_0 k H \sec(\phi) e^{\left(-\frac{z}{H}\right)}$$

- b) Take the sun directly overhead ($\phi=0$), the surface density $\rho_0 = 1 \text{ kg} \cdot \text{m}^{-3}$, the scale height $H = 10 \text{ km}$ and the absorption coefficient $k_\lambda = 0.001 \text{ m}^2 \cdot \text{kg}^{-1}$. Calculate the optical depth at height levels of 40, 30, 20 and 10 km. (4%)

Just substituting into the expression from part a, remembering to convert km to m, gives:

40	$\tau = 0.18$	$T \approx 0.83$	$A \approx 0.17$
30	$\tau = 0.50$	$T \approx 0.69$	$A \approx 0.31$
20	$\tau = 1.35$	$T \approx 0.26$	$A \approx 0.74$
10	$\tau = 3.68$	$T \approx 0.03$	$A \approx 0.97$

- c) Calculate the transmission and absorption of the atmosphere at the height levels 40, 30, 20 and 10 km. (5%)

The transmission is $T = e^{-\tau}$, and the absorption is $A = 1 - T$, as given in the table above

- d) Between which two neighbouring height levels, 40, 30, 20 and 10 km, does the absorption change the most? How is this maximum change related to the optical depth? (5%)

The absorption change between 30 and 40 is $(0.31-0.17)=0.14$ (note, $\tau=0.50 \rightarrow 0.18$)

Between 20 and 30 is $(0.74-0.31)=0.43$ (note, $\tau=1.35 \rightarrow 0.50$)

Between 10 and 20 is $(0.97-0.74)=0.23$ (note, $\tau=3.68 \rightarrow 1.35$)

So, the maximum change in absorption occurs between 20 and 30 km, around the same place where $\tau=1$. This should not be a surprise. We know that the maximum heating rate, which depends upon the absorption of sunlight, occurs at $\tau=1$. In fact, if one takes $d^2A/dz^2=0$, using $A=1-\tau(z)$ and $\tau(z)$ from above, one finds this maximizes at $\tau=1$.

- e) When the sun moves off zenith ($\phi > 0$), does the maximum in the change of absorption move up or down? (3%).

Here one has to remember that as the path length increases, so will the optical depth, χ . This is just the effective optical depth, $\tau^ = \tau/\mu$, where $\mu = \cos(\phi)$. So as the sun comes in off zenith, τ^* will reach 1 sooner, at a higher altitude. This is what we saw on the heating rate, that the maximum heating rate, and hence the maximum change in absorption, will move to higher altitude as the sun comes in off zenith.*

- 3) (25 %) An air parcel at a pressure of 900 hPa has a temperature of 20°C.
- a) How do we define the lifting condensation level (LCL)? If the air parcel is lifted to the LCL and has a temperature of 13°C there, what is the atmospheric pressure there? (5%)

The lifting condensation level (LCL) is the level where an air parcel reaches saturation when lifted adiabatically. To get the atmospheric pressure at the point where the atmospheric temperature is 13°C, given the pressure and temperature at some other point, one should use the equation for potential temperature:

$$\theta := T \left(\frac{P_o}{P} \right)^k$$

Here we take $T = (273+20)K = 293K$, $P_o = 900 \text{ hPa}$ and P to be equal to P_o in order to calculate the potential temperature of the parcel at P_o . This gives $\theta_0 = T = 293K$. (One could also take a reference pressure to be $P_o = 1000 \text{ hPa}$, in which case $\theta_0 = 301.9K$. But it is a bit shorter to take the reference level to be our initial pressure and temperature. It will, however, give the same pressure at $T = 13K$ as long as we take the same reference). Now, since θ will be constant for adiabatic motion, which we have up to the LCL, the pressure at $T = (273+13) K = 286 K$ is obtained by solving for P :

$$P := P_o \left(\frac{\theta_0}{T} \right)^{\left(-\frac{1}{k} \right)}$$

*And upon substitution with $\theta_0 = 293K$ and $P_o = 900 \text{ hPa}$. This gives **$P = 827 \text{ hPa}$** . (or $\theta_0 = 301.9K$ and $P_o = 1000 \text{ hPa}$ gives the same)*

- b) The water vapour saturation pressure at temperature T is given by:

$$e_s = 6.112 \text{ hPa} \cdot \exp \left[\frac{L_v}{R_v} \left(\frac{1}{273 \text{ K}} - \frac{1}{T} \right) \right]$$

Given that the water vapour mass mixing ratio, μ , of the parcel is constant before condensation occurs, what is the relative humidity of this air parcel before lifting? (12%)

The relative humidity at the parcels initial pressure and temperature ($P_o = 900 \text{ hPa}$, $T_o = 293 \text{ K}$) is given by $RH = e(T_o)/e_s(T_o)$. Now, $e_s(T_o)$ can be calculated directly from the equation given as $e_s(T_o = 293K) = 23.7 \text{ hPa}$.

To get $e(T_o)$, the only thing we have to go on is that μ is constant right up to saturation. So, at the LCL, where $P_s = 827 \text{ hPa}$ and $T_s = 286K$, we can calculate $\mu_s(P_s, T_s)$, and this is equal to the parcel's μ at any other pressure and temperature below the LCL. We also know the relationship between μ and e is given by:

$$\mu := \frac{\varepsilon e}{P}$$

*Where ε is the ratio of the mass of water vapour to the mass of dry air = $M_v/M_d = 0.62$. Hence, at the LCL ($P_s = 827 \text{ hPa}$ and $T_s = 286K$), we have from the given equation that $e_s(T_s = 286K) = 15.1 \text{ hPa}$, and thus $\mu_s(P_s, T_s) = 0.622 * e_s(T_s)/P_s = 0.0113 = \mu(P_o, T_o)$. Then we just use the fact that $\mu(P_o, T_o) = 0.622 * e(T_o)/P_o$ to get the $e(T_o)$ that we need.*

We could also save some time and not evaluate the saturation mixing ratio by setting the mass mixing ratio, $\mu(P_o, T_o) = \mu_s(P_s, T_s)$ and solve for $e(T_o)$ (then then don't need to evaluate ε since it cancels):

$$\frac{\varepsilon e_s(T_s)}{P_s} = \frac{\varepsilon e(T_o)}{P_o} \text{ Or } e(T_o) = \frac{e_s(T_s) P_o}{P_s}$$

Where $e_s(T_s)$ can be calculated using the equation given, as we did for $e_s(T_o)$. This gives $e_s(T_s=286 \text{ K})=15.1 \text{ hPa}$, and with $P_s=827 \text{ hPa}$ and $P_o=900 \text{ hPa}$ we get $e(T_o)=16.4 \text{ hPa}$. And the relative humidity is:

$$\underline{RH=100*e(T_o)/e_s(T_o)=100*16.4/23.7= 69.2\%}$$

Quick number check:

$$L_v := .25 \cdot 10^7 \text{ J/kg}$$

$$R_v := 461 \text{ J/K/kg}$$

$$P_s := 827 \text{ hPa}, T_s := 286 \text{ K}$$

$$P_o := 900 \text{ hPa}, T_o := 293 \text{ K}$$

$$E_s_{P_o T_o} = 23.71692352 \text{ hPa}$$

$$E_s_{P_s T_s} = 15.07719551 \text{ hPa}$$

$$M_{u_s} = .01131591207$$

$$E_{T_o} = 16.40807250 \text{ hPa}$$

$$RH = 69.18297176 \text{ h\%}$$

- c) If the environmental (atmospheric) temperature decreased adiabatically with height, where would the atmosphere be unstable with respect to vertical motion? Why? (3%)

The atmosphere would become unstable with respect to vertical motion at the LCL. Up to the LCL, any vertical motion would be adiabatic as no condensation would occur, and the air surrounding the parcel would be at the same temperature as the lifted parcel (neutral stability). At the LCL, any further cooling of the parcel would result in condensation of the water vapour, and the latent heat of vaporisation would heat the parcel above the adiabatically cooled temperature. The parcel would then find itself warmer than the surrounding gas (which is at the adiabatic temperature), lighter, and would therefore continue to ascend. This would then be unstable.

- d) What mass mixing ratio of water must condense during its ascent in order to change the parcel air temperature by 10 K? (Assume the atmospheric mass is the mass of dry air.) (5%)

This should also be relatively easy. The condensing water gives off latent heat, which will be the energy source heating the parcel. The units of latent heat for water, $L_v=2.6 \times 10^6 \text{ J} \cdot \text{kg}^{-1}$, tell you how many Joules of heat will be released for every kg of water condensed. Similarly, the units of heat capacity for air, $C_p=1004 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$, tell you how many Joules of heat will raise each kg of air one K. Putting these together, we have:

$$L_v \cdot m_{\text{water}} = C_p \cdot M_{\text{air}} \cdot \Delta T$$

Note, we use C_p since the condensation is taking place with no walls, hence the volume can change but the pressure will be constant. We are told to take M_{air} as the mass of dry air, M_d . Given the temperature increase of 10 K, we can solve for $m_{\text{water}}/M_{\text{air}} = m_{\text{water}}/M_d = \Delta T \cdot C_p / L_v$, and get $\mu = m_{\text{water}}/M_d = 0.004$. This can also be expressed as 4 g/kg.

4) Short Answers (25 %)

- a) Explain how the Föhn or Chinook wind comes about. (4%)

As a particle rises over the mountain, its moisture condenses and warms with latent heat. As it comes back down its moisture re-vaporizes and cools. If it has lost moisture through rain at the top, it warms more than it can cool and you get warm down-slope winds.

If the parcel is blown up over a mountain, it drops in temperature as the pressure reduces until its moisture begins to condense, then it drops more slowly with decreasing pressure as the latent heat of condensation heats the parcel. If the parcel just comes back down, as the moisture re-vaporize it take up that excess latent heat, and it will be at the same temperature when it returns to its initial pressure. This is just due to the fact that the potential temperature (and equivalent potential temperature) for the parcel are constant.

If the parcel loses the moisture by raining or snowing, then it will not have enough moisture re-vaporizing to take up the excess latent heat, it will be warmer on the down-slope side when it returns to the same pressure at which it began. This is the Föhn or Chinook effect, a warm, down-slope wind out of the mountains.

Just as a note to the students reading these solutions, it is now thought that it is an increased westerly wind in Antarctica, caused by the decrease of ozone, which has increased the Föhn winds over the mountainous peninsula and caused the ice-sheets on the eastern side to warm and break up. So, the break-up of these ice-sheets may be more due to locally increasing winds out of the mountains than due to a general warming of Antarctica.

- b) Describe three types of optical scatter in the atmosphere with regards to wavelength, particle size and asymmetry factor. (4%)

Characterizing Rayleigh scatter by particles small compared to the wavelength of light, with equal forward and reverse scatter; Mie by scatter by particles large compared to the wavelength of light, with mostly forward scatter; and geometric as scatter by very large particles (compared to the wavelength of the light scattering) so that the light enters the particles and produces rainbows, should give full credit.

We can talk about Rayleigh scatter, which occurs for particles whose radii are much smaller than the wavelength of light that they are scattering. This will have nearly equal amounts of forward and backward scatter, giving an asymmetry factor close to zero.

If the scattering particle is of the same order or larger than the wavelength of light, then we have Mie scattering characterized by a dominance of forward scattering, giving an asymmetry factor close to 1.

If the scattering particle is much larger than the wavelength, then we have geometric scatter from the particle. That is, the ray is blocked (reflected or absorbed), over the diameter of the particle, and unaffected if it misses the particle. If the particle is a water drop or another substance that transmits light, then the light ray enters the particle, refracts and reflects internally. For a water drop, the ray emerges in back scatter, 22 degrees from the incoming beam, as a rainbow.

- c) What are cloud condensation nuclei (CCN), and why are they needed in order to condense water vapour in the atmosphere? (4%)

A CCN provides the surface with this larger radius onto which the water can deposit and begin to form drops. The larger radius reduces the drop's surface tension and allows the drop to grow at lower relative humidity.

Small particles in the atmosphere, usually with radii $< 0.1\mu\text{m}$, can act as condensation nuclei. As the curved surface of a water drop tends to increase the flow of water from its surface, a small droplet that just forms (with small radius and large curvature) will tend to evaporate when the relative humidity is less than $\sim 110\%$. Although super-saturation does occur, it leads to relative humidity less than 101% . To form a condensation drop at this humidity, one must start with a larger radius.

- d) If the greenhouse effect produces a warming in the troposphere, why is there a net 2 K/day radiative cooling in the upper troposphere? (5%)

This follows from the definition of radiative equilibrium temperature. The radiative heating or cooling rate is just the difference between these numbers, input-output, which is 0 when it is in radiative equilibrium. If it receives additional, non-radiative energy, for example from latent heat driven by convection, then its temperature will increase to radiate away this extra energy. Thus, since it is radiating more energy than it receives from radiation, there will be a net difference in the "radiative energy in" – "radiative energy out", and the negative difference is the cooling rate. Hence, a 2K/day cooling indicates that there is another, non-radiative process occurring in the upper troposphere.

Again, the better student should be able to say that the radiative temperature gradient in an atmosphere is greater than the adiabatic temperature gradient. Thus, convective overturning would be expected and the upper troposphere would have heat from below advected into it. This would also cause condensation and release of latent heat, providing another, non-radiative heat source for the upper troposphere that would increase its temperature above that of radiative equilibrium. The better student would also hasten to point out that the total energy balances, and that the displacement from radiative equilibrium is like a spring, and there is a characteristic radiative relaxation time constant.

- e) In atmospheric models, how does one treat physical or chemical processes that have a scale size smaller than the model grid size? (4 %)

One actually has to create a sub-model, called a parameterization, of the small-scale process that is in terms of the grid variables. For example, the gravity wave forcing is parameterized as the difference between the wave speed and the average wind speed. Convective heating may be parameterized as a function of temperature and dew point temperature vertical gradients to represent the probability of convection occurring and the heating due to the latent heat of vaporization

- f) If an atmospheric absorption line is saturated, describe what happens to the absorption if we continue to increase the absorber density? (4%)

A saturated line is one where the maximum absorption as a function of wavelength has reached 100%. If one continues to increase the absorber density, the 100% absorbed region will spread out in wavelength from the line centre, and the line wings will increase their absorption. The effect will be a broader absorption line whose height is limited to 100%. Note, this is why increasing CO_2 concentrations continue to have an effect even after the line centres have reached 100%. The absorption

merely spreads out to adjacent wavelengths and continues to absorb more of the Earth's outgoing long wavelength radiation. Thus, the greenhouse effect continues to increase with increasing CO₂ concentrations even after the band absorption has become saturated.