Norwegian University of Science and Technology
Department of Physics

EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE
Faculty for Natural Sciences and Technology
21 May 2014
Time: 09:00-13:00

Number of pages: 11

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Single or Bi-lingual dictionary permitted
All calculators permitted

You may take:

Molar mass of water vapour: ~18 kg/kmole $g=9.8$ m s^{-2} and constant in z
Molar mass of dry air: ~29 kg/kmole   1 hPa = 10^2 Pa = 10^2 N m^{-2}
273.15 K = 0 ^{o}C   Scale Height: $H=R \cdot T/g$
Stefan–Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ W·m^{-2}·K^{-4}
Solar constant at Earth's orbit (1 AU=150x10^6 km) = 1367 W·m^{-2}
Radius of the Earth = 6370 km
Latent heat of vaporization water: L_v=2.5x10^6 J· kg^{-1}
Gas constant for water vapour: R_v=461 J·K^{-1}·kg^{-1}
Values for dry air: C_p=1004 J·K^{-1}·kg^{-1} C_v=718 J·K^{-1}·kg^{-1} R_d=287 J·K^{-1}·kg^{-1}
$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d = C_p - C_v$ Γ_{da}=9.8 K/km

Clausius–Clapeyron relation: $e_s = 6.112\, hPa \cdot \exp\left[\dfrac{L_v}{R_v}\left(\dfrac{1}{273K} - \dfrac{1}{T} \right) \right]$

Answer all questions (English or Norwegian).
The English version of the exam is official in event of a translation difference
Good Luck!

1)   **(5%)** *Short* **answers**

   a) Can scattering of solar radiation by clouds be treated using the Rayleigh
       approximation? Why? (1%)

       No. The Rayleigh scattering approximation is valid for particles small with
       respect to the wavelengths of light. Thus, for sunlight peaking at 500 nm, the
       particle sizes should be of the order of <100 nm for the approximation to hold.
       Cloud particles are typically on the order of 1000 nm, or larger than the
       wavelengths of the incoming solar radiation.

b) Name three different types of spectral line broadening and how large of an effect they have. (1%)

Natural line broadening is the smallest, Doppler (thermal) line broadening is next largest, and Collisional line broadening (Lorentz broadening) is the largest.

Natural line broadening is much less than the other two for visible and IR wavelengths, typically on the order of 10^{-7} cm^{-1}. Doppler broadening is the next strongest, depends mostly on temperature and is on the order of 10^{-3} cm^{-1} at STP (Standard Temperature and Pressure). Lorentz broadening is that largest, dominating at higher pressures, and at STP is on the order of 10^{-2} cm^{-1}.

c) What are cloud condensation nuclei (CCN) and why are they needed in order to condense water vapour in the atmosphere? (1%)

A CCN provides the surface with this larger radius onto which the water can deposit and begin to form drops. The larger radius reduces the drop's surface tension and allows the drop to grow at lower relative humidity*.*

Small particles in the atmosphere, usually with radii < 0.1   m, can act as condensation nuclei. As the curved surface of a water drop tends to increase the flow of water from its surface, a small droplet that just forms (with small radius and large curvature) will tend to evaporate when the relative humidity is less than ~110%. Although super-saturation does occur, it leads to relative humidity less than 101%. To form a condensation drop at this humidity, one must start with a larger radius.

d) If an atmospheric absorption line is saturated, describe what happens to the absorption if the absorber density continues to increase? (1%)

A saturated line is one where the maximum absorption as a function of wavelength has reached 100%. ***If one continues to increase the absorber density, the 100% absorbed region will spread out in wavelength from the line centre, and the line wings will increase their absorption.*** *The effect will be a broader absorption line whose height is limited to 100%.*

Note, this is why increasing CO2 concentrations continue to have an effect even after the line centres have reached 100%. The absorption merely spreads out to adjacent wavelengths and continues to absorb more of the Earth's outgoing long wavelength radiation. Thus, the greenhouse effect continues to increase with increasing CO2 concentrations even after the band absorption has become saturated.

e) What are the most important optical properties of a gas that allow it to create a greenhouse effect? (1%)

Very concisely: ***It must transmit at the short, visible, wavelengths where the Sun is putting the heat into the planet, and it must absorb at long wavelengths where the planet is radiating away energy****.*

2) **(20%)** A parcel of air at a pressure of 1.6×10^{-3} hPa with a temperature of 190 K contains a minor species, A, that has a density of 6.1×10^{10} molecules·m^{-3}. The parcel is transported adiabatically to a level where the pressure is 1.8×10^{-2} hPa and the temperature is 200 K.

a) What is the total number density and volume mixing ratio of A in the parcel before it is transported? (2%)

First and foremost, we need to assume that the air parcel consists of dry air. Then we have from $P = \rho \cdot R \cdot T$, where $P = 0.16$ Pa, $R = 287$ J·K^{-1}·kg^{-1}, and $T = 190$ K, that $\rho = 2.9 \times 10^{-6}$ kg·m^{-3}. Given the mass of dry air $M_d = 29$ kg/kmole and Avagadro's number, $N_A = 6.02 \times 10^{26}$ molecules/kmole, this gives a total molecular density $[n] = \underline{\textbf{6.1} \times 10^{19} \textbf{ molecules·m}^{-3}}$, from which the mixing ratio $[n_A]/[n] = \underline{\textbf{1} \times 10^{-9}.}$

One can go back to first principles using the relationship between the number density, pressure and temperature that is given by the ideal gas law in the form:

$P = [n] \cdot k \cdot T$, where [n] is the number density in molecules/m^3 and k is Boltzmann's constant

Although not given explicitly in the formulae, k can be calculated by:

$k = R \cdot M_d/N_A$, where M_d is the mass of dry air (29 kg/kmole) and N_A is Avagadro's number $= 6.02 \times 10^{26}$ molecules/kmole

This gives $k = 1.38 \times 10^{-23}$ J/K/molecule. Here we have assumed that the minor species has not affected the total molecular mass of the air parcel.

Thus, the number density at a pressure of 1.6×10^{-3} hPa = 0.16 Pa and a temperature of 190 K of:

$[n_t] = 0.16$ Pa$/1.38 \times 10^{-23}$ J·K^{-1}·molecule^{-1} $/190$ K $= \underline{\textbf{6.1} \times 10^{19} \textbf{ molecules·m}^{-3}}$

The mixing ratio of A is then given by $v = [n_A]/[n_t] = 6.1 \times 10^{10}/6.1 \times 10^{19} = \underline{\textbf{1} \times 10^{-9}}$, or a part per billion by volume.

b) What is the temperature and total number density of the parcel after it is transported? (5%)

Here we know the parcel is transported to a higher pressure level = 1.8 Pa, and in doing so it will be compressed and heated. Since we are asked for the parcel temperature and number density, this will be $P_2 = [n_p] \cdot k \cdot T_p$ at the new pressure, P_2.

We can use the fact that the transport is adiabatic, so that the potential temperature of the parcel, $\theta = T_2 \cdot (P_o/P_2)^\kappa$, is constant in order to calculate T_2, the temperature of the parcel at P_2. We can then use that temperature to calculate the new number density in the parcel.

Taking the reference level to be the initial pressure (i.e., $P_o = P_1 = 0.16$ Pa), the potential temperature of the parcel at $P_1 = T_1 = 190$K. The temperature of the parcel at P_2 is therefore $T_2 = T_p = \theta \cdot (P_2/P_o)^\kappa = 190$ K $\cdot (1.8/0.16)^{0.286} = \underline{\textbf{379 K.}}$

The pressure of the parcel is P_2, giving a number density

$[n_p] = 1.8$ Pa$/1.38 \times 10^{-23}$ J·K^{-1}·molecule^{-1} $/379$ K $= \underline{\textbf{3.4} \times 10^{20} \textbf{ molecules/m}^3}$.

c) What is the number density and volume mixing ratio of the minor species, A, after transport? (5%)

Assuming no chemistry took place (none was mentioned), the mixing ratio of the minor species should be the same. Thus, $\underline{\nu = 1x10^{-9}}$,

and $[n_A]_2 = \nu \cdot [n_P]_2 = \underline{3.4x10^{11} \ molecules/m^3}$.

d) Approximately how many scale heights was the parcel transported? (5%)

For this we need to know the thickness of the layer between P_1 and P_2, so we use the hypsometric equation. We will assume a constant average temperature between the layers, and take this to be the average of the temperatures at P1 and P2, or $T_{avg} = (200 \ K + 190 \ K)/2 = 195 \ K$.

The hypsometric equation for a constant temperature is given by:

$\Delta z = -H \cdot [ln(P_1)-ln(P_2)]$,

and the scale height, $H = R_d \cdot T_{avg}/g = 287 \ J \cdot K^{-1} \cdot kg^{-1} \cdot 195 \ K/9.8 \ m \ s^{-2} = 5711 \ m$.

*One can calculate Δz in metres which comes out at 13822 m and divide H out to get **2.4 scale heights**, or divide out H in the hypsometric equation to calculate*

$\Delta z/H = -[ln(P_1)-ln(P_2)] = -[ln(0.16 \ Pa) - ln(1.8 \ Pa)] = \underline{\textbf{2.4 scale heights}}$,

e) Is the parcel stable or unstable with respect to vertical motion? Why? (3%)

Yes, it is stable with respect to vertical motion. When it has compressed at pressure P_2, it is now warmer than the surrounding air and should be less dense and rise. To check that it is less dense, one could calculate the number density at $P_2=1.8 \ Pa$ and $T_2=190 \ K$, finding it to be $6.5x10^{20}$ molecules/m³ as compared to $3.4x10^{20}$ molecules/m³ inside the parcel. It will therefore rise towards where it started indicating a stable atmosphere.

Marking notes:
No points deducted if an incorrect value for $N_A = 6.02x10^{26}$ molecules/k-mole was used.
 a) Leaving the answer in mass density lost half credit
 b) Using the parcel temperature as 200K lost 3 points
    Reversing the pressures lost 2 points
    Leaving the number density as mass density lost 3 points
 c) Stating volume mixing ratio ν was constant, but not multiplying by density from b) lost 3 points

3) **(25 %)** An air parcel at a pressure of 950 hPa has a temperature of 25 °C.

a) If the air parcel is lifted to the Lifting Condensation Level (LCL) and has a temperature of 15 °C there, what is the atmospheric pressure there? (5%)

The lifting condensation level (LCL) is the level where an air parcel reaches saturation when lifted adiabatically. To get the atmospheric pressure at the point where the atmospheric temperature is 15°C, given the pressure and temperature at some other point, one should use the equation for potential temperature:

$$\theta := T\left(\frac{Po}{P}\right)^{\kappa}$$

Here we take T= (273+25)K=298K, P_o = 950 hPa and P to be equal to P_o in order to calculate the potential temperature of the parcel at P_o. This gives $\theta_0 = T = 298K$. (One could also take a reference pressure to be Po=1000 hPa, in which case θ_0=302.4K. But it is a bit shorter to take the reference level to be our initial pressure and temperature. It will, however, give the same pressure at T=15°C as long as we take the same reference). Now, since θ will be constant for adiabatic motion, which we have up to the LCL, the pressure at T = (273+15) K = 288 K is obtained by solving for P:

$$P := P_o\left(\frac{\theta_o}{T}\right)^{\left(-\frac{1}{\kappa}\right)} = P_o\left(\frac{T}{\theta_o}\right)^{\left(\frac{1}{\kappa}\right)}$$

And upon substitution with T=288 K, θ_0 = 298K and P_o=950 hPa. This gives the pressure at the LCL, also known as the saturation pressure , as:,
<u>P = 843 hPa</u>. *(or θ_0=301.9K and P_o=1000hPa gives the same).*

b) What was the relative humidity of this air parcel before lifting? (12%)

The relative humidity (in %) at the parcels initial pressure and temperature (P_o=950 hPa, T_o=298 K) is given by
$$RH = 100 \cdot e(T_o)/e_s(T_o)$$
Now, $e_s(T_o)$ can be calculated directly from the Clausius-Clapyeron (C.C.) equation given as:
$e_s(T_o=298K)=32.4\ hPa$.
To get $e(T_o)$, the only thing we have to go on is that μ is constant right up to saturation. So, at the LCL, where P_s = 843 hPa and T_s = 288K, we can calculate $\mu_s(P_s,T_s)$, and this is equal to the parcel's μ at any other pressure and temperature below the LCL. In particular, it is equal to $\mu(P_o,T_o)$. We also know the relationship between μ and e is given by:

$$\mu := \frac{\varepsilon\ e}{P}$$

So, we can set the mass mixing ratios at saturation and at (P_o, T_o) to be equal,
$$\mu_s(P_s,T_s) = \mu(P_o,T_o)$$
and solve for $e(T_o)$

$$\frac{\varepsilon\ e_s(T_s)}{P_s} = \frac{\varepsilon\ e(T_o)}{P_o} \qquad \text{Or} \qquad e(T_o) = \frac{e_s(T_s)\ P_o}{P_s}$$

Where $e_s(T_s)$ can be calculated using the C.C. equation given. This gives

$e_s(T_s=288\ K)=17.2\ hPa$,

and with $P_s=843\ hPa$ and $P_o=950\ hPa$ we get $e(T_o)=19.4\ hPa$.

And finally, the relative humidity is:

<u>$RH=100*e(T_o)/e_s(T_o) = 100*19.4/32.4= 59.9\%$</u>

Not using μ = constant and taking RH= $100*e_s(288)/e_s(298)$ lost 8 points

c) After the parcel has reached the LCL, 5g/kg of water vapour suddenly condenses. What is the potential temperature change of the parcel due to condensation? (5%)

When a mass of water vapour, M_{H2O}, condenses, it releases an amount of heat $M_{H2O} \cdot L_v$, where L_v is the latent heat of vaporization that is given as 2.5×10^6 J· kg^{-1} This heat will warm the mass of dry air around it, M_d, which has a heat capacity C_p given as 1004 J·K^{-1}·kg^{-1}, raising its temperature by some ΔT. Equating these gives

$M_{H2O} \cdot L_v= M_d \cdot C_p \cdot \Delta T$, or $\Delta T = (M_{H2O} / M_d) \cdot (L_v /C_p)$

Here we note that $\mu_{condensed} = (M_{H2O} / M_d)$, which is given as 5 g/Kg, or 0.005.

So, substituting in the constants gives $\Delta T = 12.5\ K$.

The change in Potential temperature, however, will depend on the choice of reference level, P_o, as $\Delta\theta=\Delta T \cdot (P_o/P_s)^\kappa$. Hence they must state the reference. Ex:

$P_o = 950\ hPa$ $\Rightarrow$ $\Delta\theta =12.5\ K \cdot(950\ hPa/843\ hPa)^\kappa =12.9\ K$

$P_o=1000\ hPa$ $\Rightarrow$ $\Delta\theta =12.5\ K \cdot(1000\ hPa/843\ hPa)^\kappa =13.1\ K$

$P_o=843\ hPa$ $\Rightarrow$ $\Delta\theta =12.5\ K \cdot(843\ hPa/843\ hPa)^\kappa =12.5K$

In the answer was multiplied by an extra M_{H2O}/M_d, 2 points were lost.

d) What is the change in entropy as condensation occurs (3%)

Here the student must remember that the entropy is proportional to the natural log of the potential temperature relative to a reference:

$S = C_p\ \ln(\theta) + S_o$

Thus, if the potential temperature does not change, the entropy is constant. However, here the condensation has changed the potential temperature as shown above. Due to the logarithmic relationship, the change in entropy, ΔS, does not depend on the pressure reference level. Thus for a change of 12.5 K at 288 K:

$dS = C_p\ \ln(\theta_2) - C_p\ \ln(\theta_1) = C_p\ \ln(\theta_2 /\theta_1)$

Giving, $dS = C_p \cdot \ln (T_2 / T_1)$
        $= 1004$ J·K^{-1}·kg^{-1}· $\ln ((288+12.5)\ K/ 288\ K) = $ <u>**42.6 J·K^{-1}·kg^{-1}**</u>

4) **(25%)** Volcanic ash of 0.01μ ($1\mu=10^{-6}$m) radius is distributed with a constant mass mixing ratio of 3% in the lowest 3 km of an isothermal atmosphere of temperature 300 K. The atmospheric density at the surface is 1.29 kg·m^{-3}, and you may assume dry air. For 500 nm light, take the attenuation coefficient of clear air to be 0, and for ash to be 0.01 m^2·kg^{-1}.

a) At ground level, what is the optical depth of 500 nm light for the sun directly overhead? (8%)

Since it is an isothermal atmosphere with a given scale height, the density of the ash is given by μ, the ash's mass mixing ration, times the atmospheric density, or:

$$\rho := \mu\, \rho_0\; e^{\left(-\frac{z}{H}\right)}$$

This can be derived from substituting the perfect gas law into the hydrostatic equation and integrating to get P as a function of z, and then using perfect gas law and the definition of H to solve for ρ as above. However, they should be able to just write down the above expression. They will need to calculate the scale height for this atmosphere, which is constant as it is isothermal. This they can get from the equation given in the formulae:

$$H = \frac{R_d\, T_o}{g}$$

Which comes out to be H=8786 km for T_o=300 K.

Next, they just need to use the definition of optical depth, and realize they only need to integrate to 3 km, above which there is no cloud and therefore no absorption.

$$\tau = \int_0^{3000} \mu\, \rho_0\; e^{\left(-\frac{z}{H}\right)} k\; dz$$

Or $\;\tau = -\mu\, \rho_0\, k\, H\, e^{\left(-3000\frac{1}{H}\right)} + \mu\, \rho_0\, k\, H$

Substituting in the values:

$$\mu := .03$$
$$\rho_0 := 1.29$$
$$k := .01$$
$$g := 9.8$$
$$H := 8786.$$

Gives **τ =0.983**

Taking ρ to be constant gave 1 point; Using the wrong integration limits here or in part c) gave 3 points.

b) What is the atmospheric transmission in this case? (3%)
The transmission is given by:

$$T = e^{(-\tau)}$$

Or, here, with τ = 0.983, the atmospheric transmission is **T= 37%** *(62% absorption). That is why the pictures from Iceland look like it is the middle of the night!*

c) At what altitude does the absorption = 50%? (8%)

This one comes from the definition of the optical depth as above. Integrating from the altitude where A = 0.5, we will call it Z_{50}, to the top of the ash cloud:

$$\tau_{50} = \int_{Z_{50}}^{3000} \mu\,\rho_0\,e^{\left(-\frac{z}{H}\right)} k\,dz$$

Which yields:

$$\tau_{50} = -\mu\,\rho_0\,k\,H\,e^{\left(-3000\frac{1}{H}\right)} + \mu\,\rho_0\,k\,H\,e^{\left(-\frac{Z_{50}}{H}\right)}$$

And using the fact that $T = e^{(-\tau)}$, and A=1-T, we are solving for:

$$A = 1 - e^{\left(\mu\,\rho_0\,k\,H\,e^{\left(-3000\frac{1}{H}\right)} - \mu\,\rho_0\,k\,H\,e^{\left(-\frac{Z_{50}}{H}\right)}\right)}$$

For the Z_{50} which gives A=0.5. (Note, one could also solve for T=0.5 which is A+1, which makes the maths slightly easier). Solving for Z_{50} in terms of A gives:

$$Z_{50} = -\ln\left(\frac{\mu\,\rho_0\,k\,H\,e^{\left(-3000\frac{1}{H}\right)} - \ln(-A+1)}{\mu\,\rho_0\,k\,H}\right) H$$

And substituting:
$\mu = 0.03 \quad \rho_0 = 1.29 \quad H = 8786 \quad k = 0.01 \quad A = 0.5$
Gives **$Z_{50}= 784\ m$**.
Though not requested, the optical depth at this altitude is $\tau_{50\%}=0.69$

d) If the particle radius increased to 3 µ with the same mass mixing ratio, how would the extinction coefficient, transmission and asymmetry factor (ratio of forward to backward scatter) change for 500 nm light? (6%)

Here the point is to realize that we have been dealing with Rayleigh scatter since the wavelength λ=500nm = 0.5µ, is much larger than the particle radius, R=0.1µ. This is characterized by a scattering cross section much smaller than the geometric cross sectional area of the particle as well as equal forward and backward scatter, giving an asymmetry factor of 1.

If the radius increases to larger than the wavelength, then the scattering shifts to the Mie regime. In doing so, the scattering cross section will increase to beyond the geometric cross sectional area of the particle, πR^2, since the light will diffract around the edges of the particle. (In fact, over most of the Rayleigh range the cross section goes as the 6th power of the radius.)

If the mass density of particles remains constant, which it does here, one will find:
1) a larger extinction coefficient (which grows as R^3),
2) a correspondingly lower transmission, and
3) a preference for forward scattering so that the asymmetry factor becomes greater than one.
The students must still mention that the cross section becomes larger than πR^2 as one goes from Rayleigh to Mie scatter.

5) **(25%)** Mercury, with a radius of 2440 km, has an albedo of 0.16 and is 0.4 AU (Astronomical Units) from the Sun.

a) What is the radiative equilibrium temperature of Mercury? (8%)

To calculate the radiative equilibrium temperature of Mercury, we need to know the solar flux (W/m^2) incident on Mercury's surface. However, all we know is the solar flux at the radius of the earth's orbit, R_{eo} (1 au, given in the formulae), and the radius of Mercury's orbit, R_{Mo}. So, if the Sun has an intensity of L_o Watts, then the solar constant $S_o = L_o/(4 \cdot \pi \cdot R_{eo}^2)$ W/m^2.

But at Mercury, the solar constant $S_M = L_o/(4 \cdot \pi \cdot R_{Mo}^2)$ W/m^2. That is, $S_M = S_o \cdot (R_{eo}/R_{Mo})^2$. In astronomical units, this is just

$$S_M = S_o \cdot (1 / 0.4)^2 = 1367 \cdot (1/5.203)^2 \ W/m^2 = 8543 \ W/m^2.$$

Now, it is just the radiative equilibrium problem of balancing the Power Absorbed by the Power Radiated, and realizing that the amount reflected, α (the albedo), does not contribute to the heating, this is:

$$S_M \cdot (1-\alpha) \cdot \pi (R_M)^2 = 4 \cdot \pi (R_M)^2 \cdot \sigma \cdot (T_M)^4$$

Where R_M is the planet's radius and σ is the Stefan-Boltzmann constant (from the formulae). Of course the planetary radius drops out and we have the familiar form:

$$\frac{S_M (1 - \alpha)}{4} = \sigma T_M^4$$

Solving for T_M given the solar constant at Mercury(8543) and the albedo (0.16) yields a temperature of $\underline{\mathbf{T_M = 422 \ K}}$.

b) What fraction of the radiative flux emitted by the Sun does Mercury intercept? (5%)

The sun puts out its radiatve flux into 4π steradians. Mercury subtends only a fraction of that full solid angle. The solid angle Mercury subtends, Ω_M, is A_M/D^2, Where AM is the cross sectional area of Mercury, and D is the sun-Mercury distance = 0.4 AU. But 1 AU is given as 150×10^6 km. Since we are dividing km by km, I will be lazy and not convert to metres first!

Fraction of radiative flux = $\Omega_M/(4\pi)$ $= \pi \cdot R_M^2 / D_M^2 /(4\pi) = \frac{1}{4} \cdot R_M^2 / D_M^2$

$= (2440 \ km)^2 / (0.4 \cdot 150 \times 10^6 \ km)^2 /(4)$

$= \underline{\mathbf{4.1 \times 10^{-10}}}$

c) What would the radiative equilibrium temperature of the Earth be if there was no atmosphere? Assume the albedo, a=36 %. (2%)

Well, same as part A, but using S=1367 W/m^2 and a=0.36. That is:

$$\frac{S_e \ (1 - \alpha)}{4} = \sigma T_e^4$$

And solve to $T_e = [S_e \cdot (1-\alpha) / (4 \cdot \sigma)]^{1/4}$

This gives $\underline{\mathbf{T_e = 249 \ K}}$

d) If the solar irradiance, S, at Earth were to change, calculate the climate sensitivity, $\beta = \Delta T / (\Delta S/S)$. Assume there is no atmosphere. (5%)

Look at the above expression for T_e and differentiate it. One can use logs to get

$\ln(T_e) = 1/4 \cdot [\ln(S_e) + \ln(1-\alpha) - \ln(4) - \ln(\sigma)]$ and differentiate to get:

$dT_e/T_e = 1/4 \cdot [dS_e/S_e - d\alpha/(1-\alpha)]$ since 4 and σ are constants. Then taking α to be constant gives the fractional change in temperature for a fractional change in solar irradiance:

$dT_e/T_e = 1/4 \cdot dS_e/S_e$ which we can get to look like the climate sensitivity factor, β:

$$T_e/4 = dT_e /(dS_e/S_e).$$

So in our simple model, the climate sensitivity is equal to the radiative equilibrium temperature (from part c) divided by 4!

Hence $dT_e /(dS_e/S_e) = 249 \text{ K} / 4 = 62 \text{ K}/(\text{fractional change in solar irradiance})$. Since this is a dT, we can change to Celsius to get:

$\beta = dT_e /(dS_e/S_e) = 62 \text{ °C}/(\text{fractional change in solar irradiance})$.

e) Modern climate models give a climate sensitivity for solar irradiance changes of ~200 °C/(fractional change in solar irradiance). Is this different from your value, and if so, why? (5%)

Short and sweet, we have no radiative feedbacks in our simple radiative equilibrium model. No feedbacks between temperature and CO_2 changes, no feedbacks between Temperature driving albedo changes, or CO_2 changes changing the radiative transfer. Modern models have such feedbacks, and hence give very different numbers.

VEDLEGG

Bokmål oversettelse (tilnærmet) av eksamen hvis det er spørsmål om terminologi.

1) (5%) Kortbesvarelser
   a) Kan man bruker Rayleigh-tilnærming for å beskrive spredningen av solstråling av skypartikler? (1%)
   b) Navngi de tre forskjellige prosessene som utvider spektrallinjer og hvor stor effekt de har. (1%)
   c) Hva er kondensasjonskjerner (CCN) og hvorfor trenger vanndamp CCN for å kondensere? (1%)
   d) Gitt at en absorpsjonslinje blir mettet, beskriv hva skjer med absorpsjonen hvis tettheten av absorberende middel fortsetter å øke. (1%)
   e) Hva er de viktigste optiske egenskapene til en gass som skaper drivhuseffekt? (1%)

2) (20%) En luftpakke som har et lufttrykk på $1,6 \times 10^{-3}$ hPa og temperatur på 190 K inneholder en mindre bestanddel av gass A, som har en nummertetthet på $6,1 \times 10^{10}$ molekyler·m^{-3}. Luftpakken blir ført adiabatisk til punktet der lufttrykket er $1,8 \times 10^{-2}$ hPa og temperaturen er 200 K.
   a) Hva er nummertettheten til hele luftpakken og blandingsforhold for gass A før luftpakken ble transportert (2%)
   b) Hva er temperaturen og nummertettheten til luftpakken etter den blir transportert (5%)
   c) Hva er nummertettheten og blandingsforhold for gass A etter luftpakken blir transport (5%)
   d) Omtrent hvor mange skalehøyder har luftpakken blitt transportert? (5%)
   e) Er luftpakken stabile eller ustabile med hensyn til vertikale bevegelser? Hvorfor? (3%)

3) (25%) En luftpakke har lufttrykk p=950 hPa og temperatur T= 25 °C
   a) Om luftpakken blir hevet til kondensasjonsnivået (LCL) der pakken temperatur blir 15 °C, hva er lufttrykket der? (5%)
   b) Hva var den relative fuktigheten for luftpakken før heving? (12%)
   c) Etter at pakken sitter ved LCL, 5 g/kg vanndamp plutselig kondenserer. Hvor mye forandres luftpakkens potensielle temperatur på grunn av kondensasjon? (5%)
   d) Hvor mye forandres luftpakkens entropi når 5 g/kg vanndamp kondenserer? (3%)

4) (25%) En vulkanisk askesky består av partikler med radius 0,01 μm (1μ=10^{-6} m) og har et masseblandingsforhold på 3% i den laveste 3 km av atmosfæren som er isotermisk med temperatur på 300 K. Den atmosfæriske tettheten på bakken er 1.29 kg·m^{-3}, og man kan anta tørr luft. Ta klarlufts svekkingskoeffisient ved 500 nm = 0, og askens svekkingskoeffisient = 0,01 m^2·kg^{-1}.
   a) Regn ut skylagets optisk dybde ved 500 nm med solen loddrett over. (8%)
   b) Hva er i så fall atmosfærens transmisjon? (3%)
   c) Bestem høyden der absorpsjonen er lik 50%. (8%)
   d) Om partiklenes radius økte til 3 μ men blandingsforholdet var det samme, hvordan ville svekkingskoeffisienten, transmisjonen og asymmetrifaktoren (ratio forover til bakover spredning) ved 500 nm forandres.

5) Merkur har en radius på 2440 km, en albedo på 0,16 og ligger 0,4 AE (astronomiske enheter) fra Solen.
   a) Hva er Merkurs strålingsbalansetemperatur? (8%)
   b) Hvilken andel av Solens strålingsfluks blir oppfanget av Merkur? (5%)
   c) Hva ville strålingsbalansetemperaturen på jorden være om det ikke fantes noen atmosfære? Anta at albedoen er a=36%. (2%)
   d) Om solens irradians, S, ved jorden skulle endres, beregn klimasensitiviteten $\beta = \Delta T / (\Delta S/S)$. Anta at det ikke fantes noen atmosfære. (5%)
   e) Moderne klimamodeller gir klimasensitiviteten med hensyn til endringer i solens irradians $\beta = \sim 200°C/$(andelsendring i solens irradians). Har du beregnet en forskjellig verdi? I så fall, hvorfor? (5%)