Norwegian University of Science and Technology
Department of Physics

EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE
Faculty for Natural Sciences and Technology
26 May 2015
Time: 09:00-13:00

Number of pages: 13

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Single or Bi-lingual dictionary permitted
All calculators permitted

You may take:

Molar mass of water vapour: ~18 kg/kmole   Molar mass of dry air: ~29 kg/kmole

$N_A = 6.02 \times 10^{23}$ molecules/mole

273.15 K = 0 °C   1 hPa = 10^2 Pa = 10^2 N m^{-2} g=9.8 m s^{-2} and constant in z

Stefan–Boltzmann constant: $\sigma = 5.67\times10^{-8}$ W·m^{-2}·K^{-4}

Solar photospheric temperature, T_s = 5786 K   Radius of the Sun = 695800 km

Radius of the Earth = 6370 km   1 AU (Earth-Sun distance) = 150×10^6 km

Latent heat of vaporization water: $L_v = 2.5\times10^6$ J· kg^{-1}

Gas constant for water vapour: R_v=461 J·K^{-1}·kg^{-1}

Values for dry air: C_p=1004 J·K^{-1}·kg^{-1} C_v=718 J·K^{-1}·kg^{-1} R_d=287 J·K^{-1}·kg^{-1}

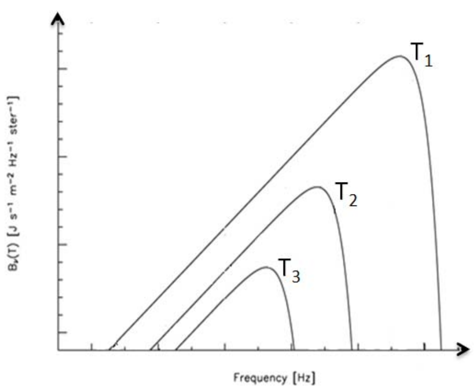
$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d = C_p - C_v$ Γ_{da}=9.8 K/km

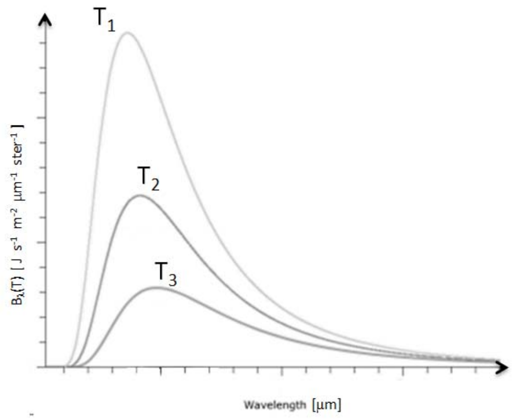
Clausius–Clapeyron relation: $e_s = 6.112\, hPa \cdot \exp\left[\frac{L_v}{R_v}\left(\frac{1}{273K} - \frac{1}{T}\right)\right]$

Answer all questions (English or Norwegian).
State all assumptions.
Good Luck!

1) **(5%) Black body radiation**

Sketch the relative spectral radiance as a function of wavelength for three blackbodies at temperatures $T_1 > T_2 > T_3$. Label the curves with their temperatures and put units on the axes.





Radiance units should be correct, although I would accept either wavelength or wavenumber units (either the left or right plot). I do not need the actual temperatures on the curves or the numbers on the axes. Curves should not cross, and the hottest temperature, T1, should peak at shorter wavelengths (higher frequencies).

2) **Multiple Choice (20%):**

There is only **one** correct answer so you must **choose the best answer**.
Answer A, B, C… (Capital letters), or leave the answer blank.
Correct answer gives +2; incorrect answers give 0.

Write the answers for the multiple choice questions **on the answer sheet you turn in** using a table similar to the following:

Question	a	b	c	d	e	f	g	h	i	j
Answer	**E**	**C**	**F**	**A**	**B**	**C**	**A**	**F**	**D/E**	**C**

a. What is the correct order of earth's atmospheric layers from bottom to top?
A) Stratosphere, Mesosphere, Troposphere, Thermosphere, Exosphere.
B) Stratosphere, Troposphere, Mesosphere, Thermosphere, Exosphere.
C) Stratosphere, Troposphere, Thermosphere, Mesosphere, Exosphere.
D) Troposphere, Mesosphere, Stratosphere, Thermosphere, Exosphere.
E) Troposphere, Stratosphere, Mesosphere, Thermosphere, Exosphere.

b. Which two atmospheric layers have temperature profiles that promote convection?

A) Mesosphere and Stratosphere.
B) Mesosphere and Thermosphere.
C) Mesosphere and Troposphere.
D) Stratosphere and Thermosphere.
E) Stratosphere and Troposphere.
F) None of the above.

The Troposphere and Mesosphere both become cooler with higher elevation which would allow a parcel moved upward to become convectively unstable.

c. In which layer of the atmosphere is ozone the major species?

A) Stratosphere
B) Mesosphere.
C) Troposphere.
D) Thermosphere.
E) Exosphere.
F) None of the above.

Definition of major and minor species! Ozone is a minor species throughout the atmosphere, with a maximum mixing ratio of only ~5 ppmv in the stratosphere.

d. If the amount of energy lost by the earth to space each year were not approximately equal to that received,

A) The atmosphere's average temperature would change.
B) The length of the year would change.
C) The sun's output would change.
D) The mass of the atmosphere would change.
E) None of the above.

In line with the Newtonian spring, if the atmosphere is driven from radiative equilibrium, it will increase/decrease its radiative rate by adjusting its temperature in order to try to get back into radiative equilibrium.

e. From what phenomenon does the Coriolis effect arise?

A) Curvature of the Earth's surface.
B) Rotation of the spherical Earth around its axis.
C) Rotation of the spherical Earth around the sun.
D) Effect of winds high in the atmosphere.
E) Motion of the oceans in their basins.
F) None of the above

True the curvature of the earth means that the rotating earth will have a higher tangential velocity at its equator, but just being curved will not create a Coriolis force. A non-rotating spherical earth would still not have the Coriolis effect.

f. Relative to the Earth's surface, what does the Coriolis effect have on masses of air or water that are changing latitude?

A) The results are unpredictable; currents can veer right or left in either hemisphere.
B) They veer to the left in the northern hemisphere and to the right in the southern hemisphere.
C) They veer to the right in the northern hemisphere and to the left in the southern hemisphere.
D) They veer to the right in both hemispheres.
E) They veer to the left in both hemispheres.

The Equator, and everything on it, has a higher tangential velocity towards the East than points at higher latitudes (North or South). Thus anything moving from the equator towards the poles will carry that higher tangential velocity with it and will, relative to the earth, turn towards the right (East) in the Northern Hemisphere (as it moves northwards), and towards the left (East) in the Southern Hemisphere (as it moves southwards). Similarly, parcels moving towards the equator with their lower tangential velocity, will turn towards the west as they cannot keep up with the higher tangential velocities of the lower latitudes. Thus, in the northern hemisphere southward moving parcels will turn right towards the west, and in the southern hemisphere norward moving parcels will turn left towards the west.

g. How can we describe light scattering by clouds?

A) Mie scattering theory.
B) Rayleigh scattering theory.
C) Tyndall scattering theory.
D) Geometric scattering theory.
E) None of the above.

Cloud particles are on the order of 1 micron in diameter, and will tend to scatter visible and near infrared light according to Mie Scatter theory.

h. What wavelengths of sunlight are absorbed by molecular nitrogen in the troposphere?

A) Infrared.
B) Ultraviolet.
C) Radio waves.
D) Microwaves.
E) Visible.
F) None of the above.

Since N_2 is homonuclear, it has not permanent dipole moment and cannot absorb the wavelengths of sunlight that make it to the troposphere (near-uv, visible and IR).

i. In an isothermal atmosphere, two air parcels, one wet and one dry, are displaced upward. What happens to the parcel temperatures?

A) They remain constant since the atmosphere is isothermal.
B) Both parcels heat at the same rate as they get nearer to the Sun.
C) The wet air parcel cools faster than the dry one due to its thermal conductivity.
D) The dry air parcel cools faster than the wet one due to latent heat effects.
E) Both parcels cool at the same rate as the pressure drops.
F) None of the above.

As the wet air parcel rises, expands and cools, the water in the parcel condenses and releases its latent heat to the air parcel. This warms the air parcel so that its temperature will be higher than an equivalent dry air parcel. Since it is not stated that the parcels are continuously displaced upward, and therefore might not condense, I will accept answer E as well.

j. The geostrophic wind results from a balance between?

A) Coriolis force and centripetal force.
B) Centripetal force, pressure gradient force, and Coriolis force.
C) Pressure gradient force and Coriolis force.
D) Pressure gradient force, Coriolis force, and friction.
E) None of the above.

These two horizontal forces balance to cause the wind to bend in a direction parallel to the lines of constant pressure (isobars). When the wind flows in this manner, it is known as a geostrophic wind. Friction will cause the winds to develop a component of velocity perpendicular to the isobars and become ageostrophic.

3) (25 %) Atmospheric thermodynamics, water vapour and structure

a. A commercial airliner suffers a sudden de-pressurization due to the loss of a cargo door. If the internal and external air pressures were 850 and 350 hPa respectively, and the internal temperature was 19°C before de-pressurization, determine the final internal temperature (assume it is an adiabatic process). (5%)

The key here is that we can assume that the process is adiabatic, and therefore the potential temperature is constant. If we take the reference pressure to be the 850 hPa in the cabin before it de-pressurizes, then the air in the cabin has a potential temperature, θ = 19+273.15 K = 292.15 K. When this air is brought adiabatically to 350 hPa by the sudden de-compression, the temperature and pressure will change to keep θ at this value according to:

$$\theta = T\left(\frac{Po}{p}\right)^{\kappa} \quad where \quad \kappa = \frac{R}{Cp}$$

Where Po =850 hPa and p = 350 hPa. This yields the final temperature to be:

T = 226.70 K = -46.45 °C

b. In practice, a fog formed in the airplane as it de-pressurized. What effect would this have on the final temperature? If the relative humidity in the cabin before it de-pressurized was 22%, how much would the final temperature change? (5%)

Given the rapid pressure fall, we can treat this as a two-step problem. That is, the pressure falls so rapidly that the condensation process does not start until the system has reached its new pressure and temperature. At this point, the fog forming tells us that the water vapour in the cabin air began to condense and release its latent heat of vaporization.

We can estimate how much latent heat was released, or an upper limit, if we assume that all the water in the cabin condensed. The latent heat of vaporization for water tells us how much energy is released when we condense a mass of water vapour. Then the specific heat of air tells us that adding a quantity of heat will raise the temperature of a mass of air by a specific amount.

One could calculate the molar mass of the wet air by summing the molar mass of water vapour multiplied by its mixing ratio ($\nu_{vapour} = e/p$) and the molar mass of dry air multiplied by its mixing ratio ($1-\nu_{vapour}$). However, at 22% RH, we can approximate the air as dry air, That is, $M_{air} = M_d$, and $M_{vapour} = M_v$. Then if we condense a mass of water m_v, the heat released is $m_v{*}L_v$. This heat then raises the temperature of an air mass, m_{air}, by ΔT according to the specific heat at constant pressure. We know this quantity, C_p, is approximately constant over the range of atmospheric pressures and temperatures, so we have the relation:

$$m_v \cdot L_v = m_{air} \cdot C_p \cdot \Delta T$$

We can re-arrange that a bit to be $m_v/m_{air} = C_p/L_v \cdot \Delta T$, and recognize immediately that the quantity m_v/m_{air} is the mass mixing ratio, $\mu = M_v/M_d \cdot e/p$, where the M's are the molar masses, e is the partial pressure of water vapour, and p is the pressure. So, we need to calculate the mass mixing ratio of water in the cabin before de-pressurization, as we assume that all of this went to vapour when the cabin pressure dropped to 350 hPa. Note that since μ is constant under an adiabatic process, the mixing ratio present at 850 hP and 19 °C will be the same when the pressure is suddenly dropped to 350 hPa and -46.45 °C. That is, even though there is less mass of water to condense and give energy, there is proportionally less mass of air to heat up! So we assume this entire mass mixing ratio condenses (since we are told that fog has formed).

First, how much water is available for condensation. The cabin humidity was 22% at 850 hPa and T = 19+273.15 K = 292.15 K. The saturation vapour pressure (in hPa) of water vapour, e_s, at 292.15 K is given by the Clausius–Clapeyron relation listed on the first page. Thus:

$$es = 6.112\, e^{\left(\frac{Lv\left(1/273 - \frac{1}{T}\right)}{Rv}\right)} \quad \text{with } T=292.15 \text{ gives } \underline{e_s = 22.47\ \text{hPa}}$$

Then the partial pressure of water vapour present in the cabin at 22% RH, e, was:

$$RH = e/e_s \Rightarrow 0.22 = e/22.47\ \text{hPa, or } \underline{e = 4.94\ \text{hPa}} \text{ of water.}$$

The mass mixing ratio is then:

$$\mu = M_v/M_d \cdot e/p = 18/29 \cdot 4.94/850$$

$$\underline{\mu = 0.0036 \text{ (or } 3.6\ \text{g/kg)}}$$

Then if all this water condenses, the final temperature will be raised by:

$$\Delta T = \mu \cdot L_v / C_p = 0.0036 \cdot 2.5\text{x}10^6\ J \cdot kg^{-1} / 1004\ J \cdot K^{-1} \cdot kg^{-1}$$

$$\underline{\mathbf{\Delta T = 9.0\ K}}$$

So, the air will warm by 9 K. Instead of -46.45 °C, it will be a balmy -37.4 °C as you struggle to put on your oxygen mask!

Of course, there could be uncondensed water left in the airplane. This is the saturated mixing ratio at T=226.7 K and p=350 hPa, and we could subtract this off from the initial mixing ratio of 3.6 g/kg. So $e_s(226.7) = 0.105$ hPa, which is that partial pressure of water that remains vapour. Its mass mixing ratio at 350 hPa is $\mu_s(226.7, 350\ \text{hPa}) = 0.2$ g/kg. Thus, only 3.4 g/kg of water condensed, and the first order correction to the temperature rise is $\underline{\mathbf{\Delta T = 8.5\ K}}$

c. In the winter hemisphere, the 500 hPa level is usually at a height of about 6000 m at a latitude of 30°, and at a height of 5600 m at a latitude of 70°. What is the mean temperature of the layer of atmosphere between 1000 hPa and 500 hPa in each case? (5%)

Here we can use the hypsometric equation directly, or if we have forgotten that, re-derive it from the hydrostatic/ideal-gas equations. That is, we start with the hydrostatic equation:

$$\frac{dp}{dz} = -\rho\, g$$

And the perfect gas law:

$$\rho = \frac{p\,T}{R}$$

To get a relation between p and T, the hypsometric equation:

$$\frac{dp}{p} = -\frac{g\,dz}{R\,T}$$

Since T, normally a function of z, is taken as the constant average temperature in the layer in this case, T_a, we can integrate this to get the hypsometric equation:

$$\ln\left(\frac{p2}{p1}\right) = -\frac{g\,(z2 - z1)}{R\,Ta}$$

Since p_1=1000 hP is the typical ground level atmospheric pressure (z_1=0), we sill assume z_1=0. Then substituting in for p_2 yields and the respective heights at the two latitudes gives:

$$\ln(500\text{ hPa}/1000\text{ hPa}) = -9.8\text{ m/s}^2 \cdot z_2 / (287\text{ J}\cdot K^{-1}\cdot kg^{-1}\cdot T_a)$$

At 30°N where Z_2 = 6000 m **$\underline{T_a = 295.58\text{ K}}$**

At 70°N where Z_2 = 5600 m **$\underline{T_a = 275.87\text{ K}}$**

d. Calculate the number density of CO_2 (365 ppmv) in the atmosphere at ground level (P = 984 hPa, T = +18° C). (5%)

Assuming you have to derive this from the front sheet, you should remember that the perfect gas law as $\underline{P=[N]\cdot k\cdot T}$. If not, then certainly $P = \rho\cdot R\cdot T$, where $\rho = [N]\cdot M_d$ and $R = R^*/M_d$, with M_d being the molar mass of dry air and R^* the universal gas constant. Then $R^*=R\cdot M_d$, and $k = R^*/N_A$, where N_A = avagadro's number. So $k = R\cdot M_d /N_A$, or:

$$k = 287\text{ J}\cdot K^{-1}\cdot kg^{-1}\cdot 29\text{ kg/kmole}/6.02\text{x}10^{26}\text{molecules/kmole, so}$$

$$\underline{k = 1.38\text{x}10^{-23}\text{J/K/molecule}}$$

Then the mixing ratio of CO2, $\nu_{CO2} = P_{CO2}/P$, so P_{CO2}= $365\text{x}10^{-6}\cdot P$=35.9 Pa. Note P must be in Pascals! With T=18+273.15=291.15, we have from gas law:

$$[N_{CO2}] = 35.9\text{ P}/(1.38\text{x}10^{-23}\text{ J}\cdot K^{-1}\text{molecule}^{-1}\cdot 291.15\text{ K}) =$$

$$\mathbf{\underline{[N_{CO2}] = 8.94\text{x}10^{21}\text{molecules/m}^3}}$$

Since pressure is force/area, a Pascal = $kg\cdot m\cdot s^{-2}\cdot m^{-2}$, giving you the density units.

Some chose to work in moles. No problem, $R^*=R/M_d=8.32\text{ J}\cdot K^{-1}\cdot mole^{-1}$(actually 8.31, but remember M_d is approximately 29kg/kmole). Thus, the molar volume would be $\nu\cdot P/(R^*\cdot T) = 1.48\text{x}10^{-2}\text{moles/m}^3$. Multiply by $N_A=6.02\text{x}10^{23}$molecules/mole to recover the number density requested.

e. An air mass of temperature +10° C and pressure 1013 hPa contains 10 g/kg water vapour. Calculate the relative humidity. (5%)

From part a), we had $\mu = M_v/M_d \cdot e/p$, but now we solve for e at a pressure of 1013 hPa given that $\mu = 0.01$. This gives:

$$e = \mu \cdot M_d/M_v \cdot p = 0.01 \cdot 29/18 \cdot 1013 \text{ hPa}$$

$$\underline{e = 16.32 \text{ hPa}}$$

and from the Clausius–Clapeyron relation listed on the first page at $T = 273.15+10$ K $= 283.15$, we get the saturation vapour pressure e_s to be:

$$\underline{e_s = 12.45 \text{ hPa}}$$

and the relative humidity is e/e_s, usually expressed in %

$$\underline{\mathbf{RH=100 \cdot 16.32/12.45 = 135\%}} \text{ !}$$

Of course this means that we would probably not see this much vapour in the air at 10 ℃, it would condense out as heavy dew or fog.

4) (20%) Radiation

In another 5 x10^9 years or so, our Sun will probably become a red giant with its photospheric temperature dropping to 4000 K and its radius swelling to 3.5 x 10^6 km. Under these conditions:

a. Derive general expressions for the solar constant, S, and the effective temperature, T_e, of a planet with an albedo, α, a distance R from the sun. (6%)

For the solar constant, we know that each unit area of a body with a temperature T will radiate a total power according to Stefan-Boltzmann law as $\sigma \cdot T^4$ Watts$\cdot m^{-2}$. Therefore, the total power coming from the sun will be:

$$\sigma \cdot (T_s)^4 \cdot 4 \cdot \pi \cdot (R_s)^2 \text{ Watts}$$

This radiation will spread out uniformly from the Sun, and when it reaches the orbit of the planet at radius R, it will be spread out over the area of the sphere $4 \cdot \pi \cdot R^2$. Thus, the solar constant, or Watts$\cdot m^{-2}$, at this radius will be:

$$S = \{\sigma \cdot (T_s)^4 \cdot 4 \cdot \pi \cdot (R_s)^2 \text{ Watts}\} / \{4 \cdot \pi \cdot R^2 \text{ m}^2\}, \text{ or:}$$

$$\underline{\mathbf{S = \sigma \cdot (T_s)^4 \cdot (R_s/R)^2}}$$

This power is incident over the cross-sectional area of the planet, $\pi \cdot (R_e)^2$, which warms to a temperature T_e. Again, each unit area of the earth will radiate to space according to the Stefan-Boltzmann law. With the albedo, α, representing the power reflected from the planet without being absorbed, the total power absorbed by the planet is $S \cdot \pi \cdot (R_e)^2 \cdot (1-\alpha)$, and the total power out of the planet is $\sigma \cdot (T_e)^4 \cdot 4 \cdot \pi \cdot (R_e)^2$. The temperature of the planet will change until these two powers are equal, and:

$$S \cdot \pi \cdot (R_e)^2 \cdot (1-\alpha) = \sigma \cdot (T_e)^4 \cdot 4 \cdot \pi \cdot (R_e)^2$$

Substituting in for S, and cancelling terms, we get the general expression:

$$\underline{\mathbf{(T_s)^4 \cdot (R_s/R)^2 \cdot (1-\alpha) = (T_e)^4 \cdot 4}}$$

b. Calculate the solar constant and effective temperature for Earth. Earth is still 1 AU from the sun, and has an albedo $\alpha = 0.3$ (6%)

Just as a check, we can apply the equations above for the standard values of solar photospheric temperature and solar radiance given on the first page and we find that these yield a solar constant of 1367 W/m^2 and an Earth temperature of 255 K = -18 ℃, which is what we should get. So I know my equations are correct. For the temperature and radius of the red dwarf Sun given in the problem, I get:

S = 7902.72 Watts·m^{-2},** giving an Earth temperature, **Te = 395.19 K = 122.04 ℃

c. What fraction of the Sun's total power output does the Earth intercept? (4%)

Just go back to basics here. Remember the total output power of the Sun is (from above) $I = \sigma \cdot (T_s)^4 \cdot 4 \cdot \pi \cdot (R_s)^2$ Watts. We don't really need a number here, instead we need the solid angle into which all this power is radiated. Since it is spreading out uniformly, this power will be radiated into $4 \cdot \pi$ steradians giving $I/(4 \cdot \pi)$ W/sr. The Earth will subtend a solid angle of $\pi \cdot (R_e)^2 / R^2$ steradians to the Sun, so the power it will receive will be $I/(4 \cdot \pi)$ W/sr $\cdot (\pi \cdot (R_e)^2 / R^2)$ sr. Therefore the fraction of the Sun's total power received by the Earth will be:

$$\textbf{Fraction} = [R_e / (2R)]^2 = 4.5\times10^{-10}$$

If you multiply this by the total power of the Sun (I above), you will duplicate the expression given in part a for $S \cdot \pi \cdot (R_e)^2$, Thus, the total power received by the Earth is the total power of the Sun/(4π) times the solid angle that the Earth subtends to the Sun.

d. Calculate the wavelength of maximum emission for both the Sun and Earth. (4%)

This is from Wien's displacement law, λ_{max} (in microns) = 2898/T, which I expect you to know. If you forget it, you can get an approximation to the constant by remembering that the normal solar temperature, given on the first page as 5786 K, results in a blackbody that peaks in the visible at about 0.5μ (500 nm). Similarly, a 288 K Earth blackbody peaks near 10μ.

For the red-dwarf case, the maximum wavelengths of the Sun and Earth (given the temperatures stated and calculated) are:

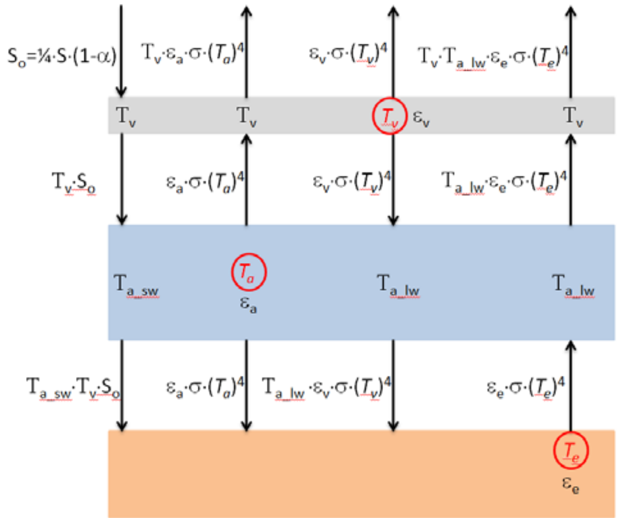
$\lambda_{max\ red\text{-}dwarf\text{-}Sun} = \mathbf{0.725\ \mu}$, and $\lambda_{max\ red\text{-}dwarf\text{-}Earth} = \mathbf{7.33\mu}$, *where* $1\mu = 10^{-6}$ m

5) (30%) Greenhouse effect

The Earth's atmosphere can be modelled as an isothermal layer with temperature T_a, above the ground, which has a temperature T_g. Under normal conditions the planetary albedo, $\alpha = 0.3$, the short wavelength absorptivity of the atmosphere is $A_a = 0.21$ and the longwave atmospheric transmission (or emissivity) is $\varepsilon_a = 0.95$.

A large volcano erupts, creating a 2 km thick ash layer on top of the atmospheric layer, increasing the planetary albedo to $\alpha = 0.35$. The density of the ash is a constant 0.03 kg·m^{-3}, and the attenuation coefficient of the volcanic ash is 0.01 m^2·kg^{-1} at all wavelengths.

a. Sketch a diagram showing the energy exchanges between the Earth and the atmosphere. (5%)



Note that this is nothing more than a two layer atmosphere. Since we are not given any information on the earth's emissivity, ε, we can assume it to be 1 and the earth to be a perfect blackbody. Thus it need not be included in the equations. It is also easy to forget terms like ε_a and ε_v. However, since these layers have a transmission, T, they will also have an emissivity ε = 1 - T. However, I will accept answers where all emissivities are 1, and only the transmission coefficients used. We can assume that the short wavelength emission of the ash cloud, like the atmosphere, is negligible.

Many people are bothered by the increased albedo: does it come from the ash or from the ground? In fact, it does not matter in the least. Albedo is the portion of the incoming energy that is ultimately sent back to space and not used in the system. It does not matter if it is sent back from the topmost or bottommost layer, it does not interact anywhere in the system and goes back to space! If I have 100 photons coming in, and 35 of them are sent back to space (a 35% albedo), I only have 65 photons that can be absorbed and turned to heat in the system. It doesn't matter if one of those 35 photons makes it all the way to the ground and back out again, it does not get absorbed by the ground, atmosphere or volcanic layer! Just get it out of the system as soon as possible and make life easy!

Note, all of the solar terms are multiplied by $\pi\cdot R_e^2$, but all of the emission terms by $4\cdot\pi\cdot R_e^2$. Cancelling these out means that the solar term is $¼\cdot S\cdot(1-\alpha)$

b. Set up the energy balance equations (9%)

i. at the top of the ash layer,

$$S_o = T_v\cdot\varepsilon_a\cdot\sigma\cdot(T_a)^4 + \varepsilon_v\cdot\sigma\cdot(T_v)^4 + T_v\cdot T_{a\_lw}\cdot\varepsilon_e\cdot\sigma\cdot(T_e)^4$$

ii. at the top of the atmospheric layer,

$$T_v\cdot S_o + \varepsilon_v\cdot\sigma\cdot(T_v)^4 = \varepsilon_a\cdot\sigma\cdot(T_a)^4 + T_{a\_lw}\cdot\varepsilon_e\cdot\sigma\cdot(T_e)^4$$

iii. at the ground.

$$T_{a\_sw}\cdot T_v\cdot S_o + \varepsilon_a\cdot\sigma\cdot(T_a)^4 + T_{a\_lw}\cdot\varepsilon_v\cdot\sigma\cdot(T_v)^4 = \varepsilon_e\cdot\sigma\cdot(T_e)^4$$

Where $S_o = ¼\cdot S\cdot(1-\alpha)$

c. Calculate the transmission of the volcanic layer (6%)

$k = 0.01\ m^2 \cdot kg^{-1}$ and is wavelength independent

$\rho_a = 0.03\ kg \cdot m^{-3}$ and is altitude independent

$\tau = \int k \cdot \rho_a \cdot dz$ on the top two km, so

$\tau = k \cdot \rho_a \cdot \Delta z = 0.01\ m^2 \cdot kg^{-1} \cdot 0.03\ kg \cdot m^{-3} \cdot 2000\ m = 0.6$

$T_v = e^{-\tau} = e^{-0.6}$

$\underline{T_v = 0.55}$

d. Solve the energy balance equations to find the temperatures of the ash layer, T_v, the atmospheric layer, T_a, and the surface, T_e, after the eruption. (10%)

Unfortunately, most people had an incorrect equation set by the time they got to this point. Thus, the solutions were correspondingly incorrect. In looking at the answers, I decided it was best to drop this last question and grade the exam as only 90 points. Doing this helped the percentage scores of everyone!

However, if you are interested in the solution:

We can assume that the earth radiates like a blackbody, so ε in the equations is just one.

Now we have

$T_v = 0.55$

$\alpha = 0.35$

$S = 1367\ W/m^2$

$T_{a\_sw} = (1- A_a) = 0.79$

$T_{a\_lw} = 0.05$ *Note there is a typo mistake in the text of the question. Emissivity should have been given as 0.95. Then transmission = 1-emissivity, so it should be 0.05! My mistake, will accept either $T_{a\_lw} = 0.05$ or $T_{a\_lw} = 0.95$*

$\varepsilon_a = 0.95$, *again with the typographical mistake, I will accept 0.95 or 0.05.*

$\varepsilon_v = 1 - T_v = 1- 0.55 = 0.45$

$\varepsilon_e = 1$

All is known except for the three temperatures, and there are 3 equations. All set to solve. Just for convenience, lets write $\varepsilon_e \cdot \sigma \cdot (T_e)^4 = te4$, $\varepsilon_a \cdot \sigma \cdot (T_a)^4 = ta4$, and $\varepsilon_v \cdot \sigma \cdot (T_v)^4 = tv4$. The equations are then

1. $S_o = T_v \cdot ta4 + tv4 + T_v \cdot T_{a\_lw} \cdot te4$
2. $T_v \cdot S_o + tv4 = ta4 + T_{a\_lw} \cdot te4$
3. $T_{a\_sw} \cdot T_v \cdot S_o + ta4 + T_{a\_lw} \cdot tv4 = te4$

If we solve 1 for tv4, we get $S_o - T_v \cdot ta4 - T_v \cdot T_{a\_lw} \cdot te4 = tv4$, which we can substitute into 2 and solve for ta4:

$ta4 = T_v \cdot S_o + (S_o - T_v \cdot ta4 - T_v \cdot T_{a\_lw} \cdot te4) = ta4 + T_{a\_lw} \cdot te4$

$S_o \cdot (1+T_v) - te4 \cdot T_{a\_lw} \cdot (1+T_v) = ta4 \cdot (1+T_v)$

$ta4 = S_o - te4 \cdot T_{a\_lw}$

We now have ta4 in terms of te4, and tv4 in terms of tev and ta4. We should go back to our expression for tv4 and eliminate ta4. Then we would have ta4 and tv4 both in terms of te4. So, substituting our expression for ta4 into that for tv4 gives:

$tv4 = S_o - T_v \cdot (S_o - te4 \cdot T_{a\_lw}) - T_v \cdot T_{a\_lw} \cdot te4$

$= S_o \cdot (1-T_v) + T_v \cdot T_{a\_lw} \cdot te4 - T_v \cdot T_{a\_lw} \cdot te4 = S_o \cdot (1-T_v)$

This makes sense as the only heat you get into the ash is whatever the sun puts in directly. All other heat from the atmosphere and the earth requires the sunlight to provide the energy, but the ash cuts that energy by the same amount, T_v. So I will expect that the ash radiating will raise the temperature of the atmosphere, *We can then put these expressions for tv4 and ta4, both in terms of te4, into the third equation and solve for te4:*

$Ta_{\_sw} \cdot T_v \cdot S_o + (S_o - te4 \cdot Ta_{\_lw}) + Ta_{\_lw} \cdot (S_o \cdot (1-T_v)) = te4$

$Ta_{\_sw} \cdot T_v \cdot S_o + S_o + Ta_{\_lw} \cdot S_o - Ta_{\_lw} \cdot T_v \cdot S_o = te4 + te4 \cdot Ta_{\_lw}$

$S_o \cdot (Ta_{\_sw} \cdot T_v + 1) + Ta_{\_lw} \cdot S_o \cdot (1-T_v) = te4 \cdot (1+Ta_{\_lw})$

Or, finally:

$te4 = \varepsilon_e \cdot \sigma \cdot (T_e)^4 = S_o \cdot [(Ta_{\_sw} \cdot T_v + 1 - Ta_{\_lw} \cdot T_v + Ta_{\_lw})]/(1+Ta_{\_lw})$

So,

$S_o = \frac{1}{4} \cdot 1367 \cdot (1-0.35) = 222.14\ W \cdot m^{-2}$

(if you don't remember $S = 1367\ W \cdot m^{-2}$*, then you can calculate it using the Sun's photospheric temperature, the radius of the sun and the earth sun distance as was done in problem 4a as* $S = \sigma \cdot (T_s)^4 \cdot (R_s/R)^2 = 1367\ W \cdot m^{-2}$*)*

$Ta_{\_sw} = 0.79$

$T_v = 0.55$

$Ta_{\_lw} = 0.05$

$\varepsilon_a = 0.95$

$\varepsilon_v = 1 - T_v = 0.45$

$\varepsilon_e = 1$

with $\sigma = 5.67 \times 10^{-8}\ W \cdot m^{-2} \cdot K^{-4}$

Gives

$T_e = 271.5$ K

$T_a = 249.9$ K

$T_v = 250.2$ K

If one used $Ta_{\_lw} = 0.95$ *and* $\varepsilon_a = 0.05$ *then the temperatures would be:*

$T_e = 247.3$ K

$T_a = 292.1$ K

$T_v = 250.2$ K *which I will also accept*

You can check these answers by letting the transmission of the volcanic ash go to 1. Then you recover the 2-layer atmosphere result. For $Ta_{\_lw} = 0.05$ *we get and earth temperature of 286 K ≈ 13 ℃, and an atmospheric temperature of 248 K, which is what we saw in class. Note the effect of the ash is to cool the earth and heat the atmosphere. So it is a balance between the blocking of the ash and the extra atmospheric radiation returning to the earth.*