Solution

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Department of Physics

Examination paper for FY3201 Atmospheric Physics and Climate Change

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Permitted examination support material:
1 side of an A5 sheet with printed or handwritten formulas permitted
Single or Bi-lingual dictionary permitted
All calculators permitted

Other information: -

Language: English

Number of pages (front page excluded): 6

Number of pages enclosed: 7

Informasjon om trykking av eksamensoppgave	
Originalen er:	
1-sidig ☐	**2-sidig** ☐
sort/hvit ☐	**farger** ☐

Checked by:

Date Signature

Solution

Additional Information

SOLUTION

You may take:

Molar mass of helium: $\sim 4\ {}^{\text{kg}}/_{\text{kmole}}$

Molar mass of water vapour: $\sim 18\ {}^{\text{kg}}/_{\text{kmole}}$

Molar mass of dry air: $\sim 29\ {}^{\text{kg}}/_{\text{kmole}}$

Molar mass of carbon dioxide: $\sim 44\ {}^{\text{kg}}/_{\text{kmole}}$

$N_A = 6.02 \times 10^{23}\ {}^{\text{molecules}}/_{\text{mole}}$

$273.15\ \text{K} = 0\ ^\circ\text{C}$ $1\ \text{hPa} = 10^2\ \text{Pa} = 10^2\ \text{Nm}^{-2}$ $g = 9.8\ \text{ms}^{-2}$ and constant in z

Stefan–Boltzmann constant: $\sigma_B = 5.67 \times 10^{-8}\ \text{Wm}^{-2}\text{K}^{-4}$

Boltzmann constant $k = 1.38 \times 10^{-23}\text{JK}^{-1}$ Speed of light $c = 3 \times 10^8$ ms^{-1}

Line broadening: approx. width for a radiative line with line centre ν_n:

Natural broadening: $\alpha_N = 3.3 \times 10^{-8}\ \text{cm}^{-1} \left(\frac{\nu_n}{1000\ \text{cm}^{-1}}\right)^3$

Pressure broadening: $\gamma_L = 0.01\ \text{cm}^{-1} \left(\frac{p}{1000\ \text{hPa}}\right) \left(\frac{273\ \text{K}}{T}\right)^{1/2}$

Doppler broadening: $\gamma_D = 0.003\ \text{cm}^{-1} \left(\frac{\nu}{1000\ \text{cm}^{-1}}\right) \left(\frac{T}{300\ \text{K}}\right)^{1/2}$

Solar photospheric temperature, $T_s = 5786\ \text{K}$ Radius of the Sun = 695800 km

1 AU (Earth-Sun distance) =150×10^6 km Radius of the Earth = 6370 km

Latent heat of vaporization water: $L_V = 2.5 \times 10^6\ \text{J kg}^{-1}$

Gas constant for water vapour: $R_V = 461\ \text{JK}^{-1}\text{kg}^{-1}$

Values for dry air: $C_p = 1004\ \text{JK}^{-1}\text{kg}^{-1}$

$C_V = 718\ \text{JK}^{-1}\text{kg}^{-1}$

$R_d = 287\ \text{JK}^{-1}\text{kg}^{-1}$

$\gamma = {}^{C_p}/_{C_V};\quad \kappa = {}^{R_d}/_{C_p};\quad R_d = C_p - C_V;\ \Gamma_{DALR} = 9.8\ {}^{\text{K}}/_{\text{km}}$

Clausius–Clapeyron relation: $e_s = 6.112\text{hPa} \times \exp\left[\frac{L_V}{R_V}\left(\frac{1}{273\text{K}} - \frac{1}{T}\right)\right]$

Answer all questions (English or Norwegian).
State all assumptions.
Good Luck!

Solution

1) Multiple Choice (10%)

There is only **one** correct answer so you must **choose the best answer**.
Answer A, B, C… (Capital letters), or leave the answer blank.
Correct answer gives +1; incorrect answer gives -0.25.
The total score for these 10 multiple choice problems together cannot be negative.

Write the answers for the multiple choice questions **on the answer sheet you turn in** using a table similar to the following:

Question	1)	2)	3)	4)	5)	6)	7)	8)	9)	10)
Answer	B	C	C	E	D	C	A	E	E	B

1) You measure an atmospheric lapse rate of 6 $^{\mathrm{K}}/_{\mathrm{km}}$. Which of the statements below is true?

A) The temperature falls with altitude and the atmosphere is absolutely unstable.
B) The temperature falls with altitude and the atmosphere is conditionally unstable
C) The temperature rises with altitude and the atmosphere is conditionally stable
D) The temperature rises with altitude and the atmosphere is conditionally unstable
E) The temperature rises with altitude and the atmosphere is absolutely stable

B) The temperature falls with altitude and the atmosphere is conditionally unstable. The lapse rate is defined with a minus sign: $\Gamma = -\frac{dT}{dz}$, so the temperature is falling. The dry adiabatic lapse rate is as stated on the front page 9.8 $^{\mathrm{K}}/_{\mathrm{km}}$, while the wet lapse rate normally lies between 3 and 6 $^{\mathrm{K}}/_{\mathrm{km}}$. Thus, if the humidity is high enough so that the parcel cools at less than 6 K/km, the atmosphere will be unstable. On the other hand, lower humidity will allow the parcel to cool in altitude faster than the atmospheric temperature drops.

2) Which of the statements below about El Niño is wrong?

A) El Niño is an interaction between the sea-surface and the atmosphere.
B) El Niño has the opposite effect on sea-surface temperature as La Niña.
C) The main effect of an El Niño is a warmer Atlantic sea-surface temperature.
D) During a typical El Niño year, air temperatures in Alaska are warmer.
E) The net integrated effect of El Niño is a slightly warmer global temperature.

C) Because el Niño is connected to the Pacific sea temperature and changes the local temperature globally. The Atlantic sea surface temperature changes only slightly, if at all.

3) At which wavelength does a solar blackbody radiation curve peak?

A) $\sim 100\mathrm{nm}$
B) $\sim 500\mathrm{nm}$
C) $\sim 1000\mathrm{nm}$
D) $\sim 2000\mathrm{nm}$

Solution

E) $\sim 3000\text{nm}$

B) $\sim 500\text{nm}$. If unsure, use Wien's displacement law to calculate with the solar temperature given on the front page.

4) In which layer of the atmosphere is ozone the major species?

A) Troposphere
B) Stratosphere
C) Mesosphere
D) Thermosphere
E) None of the above

E) None of the above. Ozone is never the major species, but always a minor one. In the well-mixed atmosphere is N_2 always the major species.

5) In the two-stream approximation, the integral over wavelength and angle can be approximated as two streams at which angles to the vertical?

A) $\theta = \pm 24°$
B) $\theta = \pm 35°$
C) $\theta = \pm 42°$
D) $\theta = \pm 53°$
E) None of the above

D) From $\mu^{-1} = \frac{5}{3}$, we find $\theta = \pm 53°$, with $\mu = \cos\theta$.

6) Which of the statements below is wrong for a neutrally stable atmosphere?

A) The atmospheric lapse rate equals the dry adiabatic lapse rate.
B) A displaced parcel will not be forced from its new altitude.
C) The Brunt Väisälä frequency is an imaginary number.
D) The temperature of the parcel equals the atmospheric temperature at every pressure level.
E) The parcel's temperature change with altitude is constant.

C) "The Brunt Väisälä frequency is an imaginary number" is wrong. It becomes zero. Imaginary frequencies indicate unstable atmospheres.

7) How much will the global mean temperature change given a solar cycle variation of the solar constant by $\pm 1\%$ if no feedbacks are included?

A) 1.5 K
B) 0.75 °C
C) Negligibly small.
D) -1 °C
E) On average 0, since it is a sinusoidal cycle.

Solution

A) Either remember from the first problem set, or use the feedback equation (here only for solar constant) $dT_e = \frac{dS}{4S} T_e$. Remember that dS=0.02, since the constant should change by plus and minus 1%. For all sensitive numbers for all earth's temperatures is A) the closest answer.

8) What does Kirchoff's law say?

A) The short wavelength heating chapman function peaks where the atmosphere becomes optically thick.

B) Long wavelength radiative transfer only cools the atmosphere.

C) The scattering from clouds can be described with Mie scattering.

D) A blackbody with a constant emissivity is a grey body.

E) A good absorber is a good emitter.

E) Kirchoff's law states only this. We introduced Kirchoff's law in one of the first lectures while talking about modelling.

9) Describe the change of the earth's visible albedo if the polar ice caps melt.

A) It will go up.

B) It will first go up, then go down.

C) It will not change.

D) It will first go down, then go up.

E) It will go down.

E) Ice has the highest visible albedo, then comes landmass, then water. So the ice surface becomes smaller and the sea surface becomes bigger. Both effects, regardless of their temporal occurrence, let the albedo go down.

10) How do you find the Lifting condensation level on a Skew-T diagram?

A) Find the water vapour mixing ratio of the dew point temperature.

B) Find the intersection between the dry adiabat and the line of constant $\mu_S = \mu$.

C) Find the region where the temperature starts to rise with altitude.

D) Find where the atmospheric lapse rate is equal to the dry adiabatic lapse rate.

E) Find where the atmospheric temperature equals the dew point temperature.

B) Again, all the answers are useful procedures for these diagrams, but the LCL is where the water vapour contained in the parcel, μ, may begin to condense. The parcel will therefore follow the dry adiabat until it reaches the point where $\mu_S = \mu$. This point can then be found by tracing the isopleth of μ_s (= μ) until it intersects the parcels dry adiabt.

2) Vertical movements (20%)

a) You stand on a cold winter day $(-13°C)$ on top of the Scandic Lerkendal hotel. The weather station there tells you that the pressure difference to the ground is 8hPa. Estimate how high the building is. (4%)

Solution

One possible answer: Let's assume $p_0 = 1000\ \mathrm{hPa}$ as ground pressure level. Furthermore, we assume an isothermal atmosphere, since we are within a city and at low altitudes. Then the pressure profile is, as calculated in class: $p(z) = p_0 \exp -\frac{z}{H_0}$, with the scale height $H_0 = \frac{RT}{g}$.

Solved for z:

$$z = -\ln\frac{p(z)}{p_0}\frac{RT}{g} \approx \sim 60\ \mathrm{m}.$$

Other pressure levels around 1000 hPa yield the approximate same result. Non-isothermal approaches are also valid, though more complicated.

b) Later on the same day, you fly on a plane. The instruments tell you that you are at an altitude of 10 km and that the pressure is 190 hPa. The thermometer tells you the atmosphere temperature is 156 K. Are your instruments working correctly? (6%)

We are still in the Troposphere, so we can assume a constant atmospheric lapse rate. This can be calculated to be $\Gamma = 10.4\ {}^{\mathrm{K}}/_{\mathrm{km}}$, which is a sensible number for these altitudes. I told you in class, that as a rule of thumb, you could take 1 degree cooler every 100 meters altitude. This instrument works correctly.

From class we know the pressure profile for such an atmosphere:

$$p(z) = p_0\left(1 - \frac{\Gamma z}{T_0}\right)^{g/R\Gamma}$$

This yields with the values stated in the task and again $p_0 = 1000\ \mathrm{hPa}$ the pressure of 190.5 hPa, so also this instrument works and you are safe in the plane.

c) Start from the general definition of the lapse rate. Derive an expression for the dry adiabatic lapse rate in terms of only approximated constants for the well-mixed atmosphere, as done in class and calculate an approximate value. (6%)

As in class:

$$\Gamma_{DALR} = -\frac{dT}{dz} = -\frac{dT}{dp}\frac{dp}{dz}$$

We know the Poisson equation:

$$\frac{p^{\gamma-1}}{T^{\gamma}} = const.$$

So we get:

$$\frac{dT}{dp} = \frac{T_p}{p}\frac{\gamma-1}{\gamma}$$

Then we use the hydrostatic equilibriums formula on the other derivation:

$$\frac{dp}{dz} = -\rho g$$

And the perfect gas law:

Solution

$$\rho = \frac{p}{RT_{atm}}$$

Together we find:

$$\Gamma_{DALR} = -\frac{dT}{dz} = -\frac{dT}{dp}\frac{dp}{dz} = \frac{T_p}{p}\kappa\frac{pg}{RT_{atm}}$$

Assuming that $T_p = T_{atm}$, which holds for small changes, gives then finally:

$$\Gamma_{DALR} = \kappa\frac{g}{R}$$

d) Starting from the Poisson relations, derive and define the potential temperature θ. (4%)

Again as in class, from Poisson:

$$Tp^{-\kappa} = T'p'^{-\kappa}$$

Now we define θ to be the temperature T' of the parcel if transported adiabatically to a reference pressure $p' = p_0$ and solve for θ:

$$\theta = T\left(\frac{p_0}{p}\right)^{\kappa}$$

3) Radiation Absorption (20%)

If there is no scattering, a parallel beam from the sun at zenith angle ϕ is absorbed in the atmosphere according to the equation:

$$-\frac{dI_\lambda}{I_\lambda} = -k_\lambda\rho(z)\sec(\phi)dz$$

a) Given an isothermal atmosphere, what is the optical depth τ_λ, at height z? (8%)

Here we can borrow from problem 2 the results of the hydrostatic equation and perfect gas law:

$$\frac{\partial}{\partial z}p = -\rho g \Rightarrow \frac{dp}{p} = -\frac{gdz}{RT}$$

And the definition of the scale height $H = {}^{RT}/_{g}$ to integrate for $p = p_0 e^{-z/H}$. The perfect gas law states $p = \rho RT$, and since the scale height is constant, this means T is constant. This allows us to write $\rho = \rho_0 e^{-z/H}$

The optical depth at any height z is defined as:

$$\tau := \int_z^\infty \rho k \sec\phi \, dz$$

Substituting for ρ gives:

$$\tau := \int_z^\infty \rho_0 e^{-z/H} k \sec\phi \, dz$$

This integrates to:

$$\tau = \rho_0 k H \sec\phi \, e^{-z/H}$$

b) Take the sun directly overhead ($\phi = 0$), the surface density $\rho_0 = 1\ \mathrm{kg\ m^{-3}}$, the scale height $H = 10\ \mathrm{km}$ and the absorption coefficient $k_\lambda = 0.001\ \mathrm{m^2kg^{-1}}$. Calculate the optical depth at height levels of 35, 25, 15 and 5 km. (4%)

Solution

Using the result from a), we get:

altitude	τ	T	A
35	0.30	0.74	0.26
25	0.82	0.44	0.56
15	2.23	0.11	0.89
5	6.07	0.002	0.998

c) Calculate the transmission and absorption of the atmosphere at the height levels 35, 25, 15 and 5 km. (4%)

The transmission is $T = e^{-\tau}$ and the absorption is A=1-T, as given in the table above.

d) Between which two neighbouring height levels, 35, 25, 15 and 5 km does the absorption change the most? How is this maximum change related to the optical depth? (4%)

The absorption change between 25 and 35 is (0.56-0.26)=0.30
Between 15 and 25 is (0.89-0.56)=0.43
Between 5 and 15 is (1.00-0.89)=0.11

So, the maximum change in absorption occurs between 15 and 25 km, around the same place where τ =1. This should not be a surprise. We know that the maximum heating rate, which depends upon the absorption of sunlight, occurs at τ =1. In fact, if one takes $\frac{d^2A}{dz^2} = 0$, using A=1-τ(z) and τ(z) from above, one finds this maximizes at τ=1.

4) Observation of a highly simplified planet (30%)

You observe a highly simplified planet "p", which orbits our sun at 1.5 times the distance of the earth from the sun. You measure a visible albedo of 20%. This planet only emits pure black body radiation.

a) Create a simple no-atmosphere model and calculate the radiative equilibrium temperature of this highly simplified planet "p". (6%)

The radiation per unit area of the sun is given by $\sigma_B T_S^4$. Then the radiation from all the sun is integrated over the sun's surface: $\sigma_B T_S^4 4\pi R_S^2$. Though at the planet arriving radiation per unit area is only this divided by the thought sphere at the distance d: $\sigma_B T_S^4 \frac{R_S^2}{d^2}$.

The planet intercepts a surface of a thought disk of the radius of the planet. Integrating over this and accounting for the albedo, we get the total energy into the system as:

$$\sigma_B T_S^4 \frac{R_S^2}{d^2} \pi R_p^2 (1-a)$$

The energy lost is the blackbody radiation over the planet's surface:

$$\sigma_B T_p^4 4\pi R_p^2$$

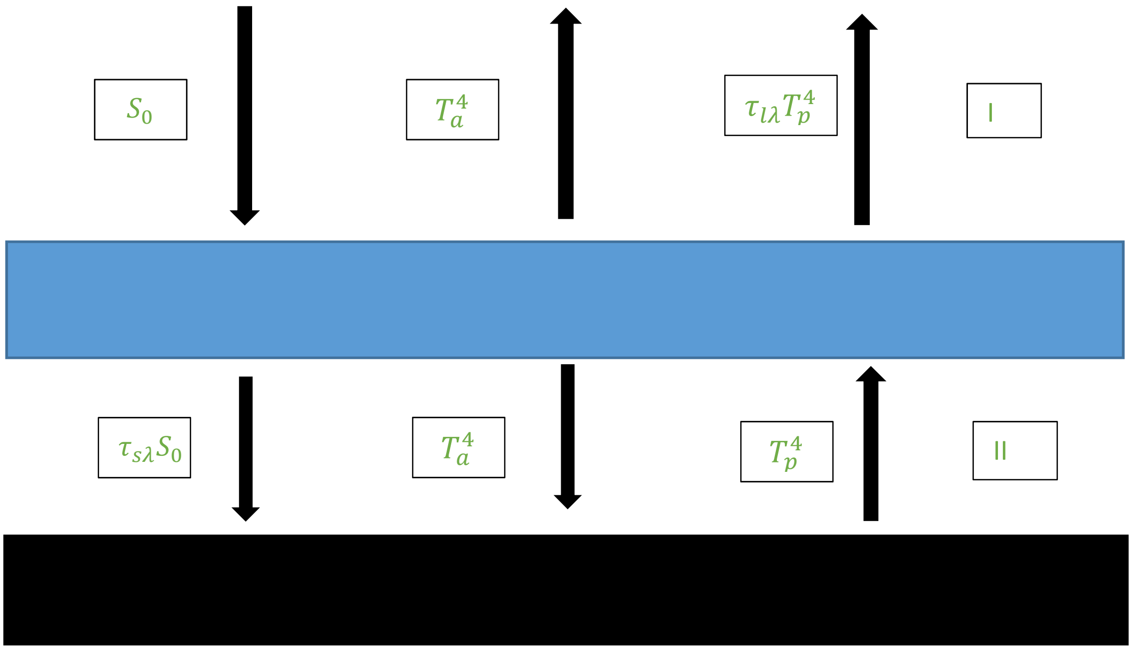
The energy in is the same as the energy out in radiative equilibrium. Solve for the planet's temperature and plug in the numbers:

$$T_p = \left(\frac{T_S^4 R_S^2}{4d^2}(1-a)\right)^{1/4} = 215.17\ K$$

Solution

b) More measurements with a crashing satellite show that the planet's surface temperature is actually 273 K on average. You can also determine that the atmosphere absorbs on average only 15% of the incoming visible light.
Create a single layer atmosphere model to calculate the temperature for the planet's atmosphere. Calculate the transmission coefficient for long wavelengths of this atmosphere. (6%)
Tip: Drawing a diagram may help you account for all beams.

The radiation is as seen in the figure below, with $\tau_{s\lambda} = 0.85 = 1 - 0.15$, since the visible light is short wavelengths.



Where $S_0 = \frac{1}{4}(1-a)T_S^4 \frac{R_S^2}{d^2}$, and with σ_B divided out already. This is just a constant.
We get two equations:

$$I: S_0 = T_a^4 + \tau_{l\lambda} T_p^4$$
$$II: \tau_{s\lambda} S_0 + T_a^4 = T_p^4$$

Which can be solved to give:

$$T_a = \left(T_p^4 - \tau_{s\lambda} S_0\right)^{1/4} = 196.6\ K$$
$$\tau_{l\lambda} = \frac{S_0 - T_a^4}{T_p^4} = 0.1953$$

A spectroscopic measurement from an orbiting satellite shows strong CO_2 lines, which were neglected until now. As the crashing satellite passed through the atmosphere, it observed a CO_2 layer in this planet's atmosphere that is narrowly centred at a pressure level of 1 hPa.

c) You observe an unsaturated CO_2 absorption line centred on $\nu_n = 2360\ \mathrm{cm}^{-1}$. For this line, find the natural, pressure and Doppler broadening line widths. First, assume the atmosphere at this pressure level has the same temperature as calculated in part b). Which type of broadening has the largest line width? Which broadening process would be the largest on earth at this same pressure level? (2%)

Solution

Setting the values given into the approximate formulas from the front page yields directly:

Natural: $\alpha_N = 4.34e^{-7}cm^{-1}$

Pressure: $\gamma_L = 1.18e^{-5}cm^{-1}$

Doppler: $\gamma_D = 0.0057cm^{-1}$

Doppler broadening is the largest, as it would be on earth. The latter one can either remember from class or think about that there is no difference when you use the formulas given above.

d) Which mechanism would produce the greatest line-width at a pressure level of 1000 hPa? Explain the difference. (2%)

We only have to calculate the pressure broadening anew, since the others are independent of the pressure:

$$\gamma_L = 0.118cm^{-1}$$

We see, that here, the pressure broadening is the dominant one. This is because at higher pressure levels, more collisions between the excited CO_2 and other molecules happen, inducing an early radiative de-excitation.

e) The actual line width is 0.005 cm^{-1}. Assuming only the dominant broadening for the CO_2 in this atmosphere at 1 hPa, calculate the temperature of the CO_2 layer. (1%)

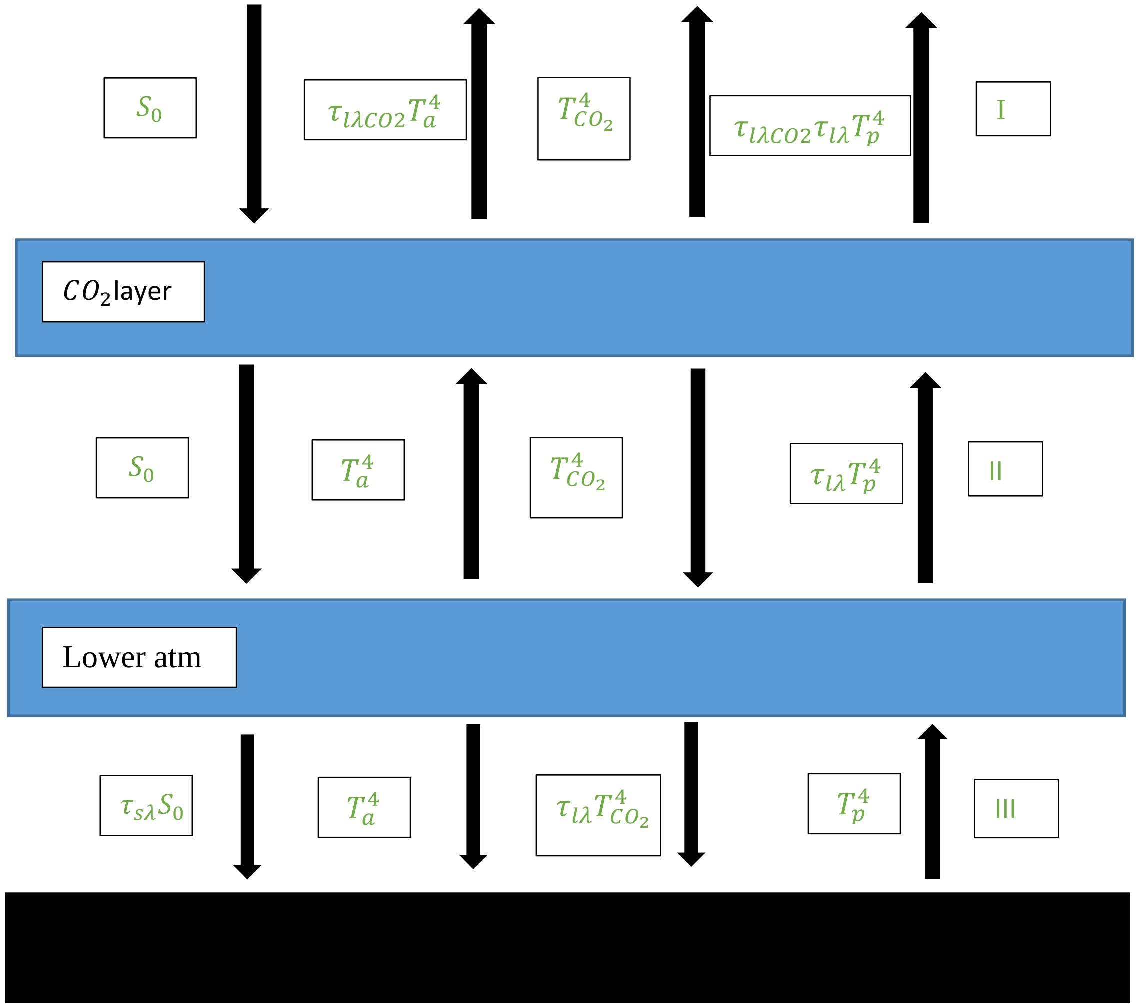
Solving the Doppler formula for the temperature yields:

$$T = \frac{\gamma_D^2}{3\nu_n^2} \times 10^{14}\ \mathrm{K} = 150\ \mathrm{K}$$

f) (If you could not find an answer to the previous part, use 140 K for the temperature of the CO_2 layer)
Now assume a two-layer atmosphere, with one normal atmosphere layer and the CO_2 layer from above, at higher altitudes. You can assume that the absorption for visible light of the CO_2 layer is 0. The long wavelength absorption may (and will) be different from the absorption from the first layer.
Find the transmission coefficients for short wavelengths for both layers and the temperature of the lower atmosphere layer. (13%)

Let's make a figure as in b), with $\tau_{s\lambda}$ of the CO_2=1 and not shown.

Solution



We get three equations, where only one is dependent on $\tau_{l\lambda CO_2}$:

$$I: S_0 = \tau_{l\lambda CO_2}T_a^4 + T_{CO_2}^4 + \tau_{l\lambda CO_2}\tau_{l\lambda}T_p^4$$
$$II: S_0 + T_{CO_2}^4 = T_a^4 + \tau_{l\lambda}T_p^4$$
$$III: \tau_{s\lambda}S_0 + T_a^4 + \tau_{l\lambda}T_{CO_2}^4 = T_p^4$$

Solving this yields:

$$\tau_{l\lambda} = \frac{(1+\tau_{s\lambda})S_0 + T_{CO_2}^4 - T_p^4}{T_p^4 - T_{CO_2}^4} = 0.4105$$
$$T_a = \left(T_p^4 - \tau_{l\lambda}S_0 - \tau_{l\lambda}T_{CO_2}^4\right)^{1/4} = 189.44\ K$$
$$\tau_{l\lambda CO_2} = \frac{S_0 - T_{CO_2^4}}{T_a^4 + \tau_{l\lambda}T_p^4} = 0.6179$$

5) Balloon sounding (20%)

A light helium balloon is released at ground level $p_0 = 1000$ hPa, where the temperature was 0°C. It rises quickly stops rising at a pressure level of $p = 0.1$ hPa in dry air. You may assume the weight of the balloon is only the weight of the gas inside. The specific heat constant for helium is ${C_p}_{He} = 5190\ \mathrm{JK^{-1}kg^{-1}}$.

a) Calculate the temperature of the atmosphere at the altitude, where the balloon stops rising. (8%)

Solution

The balloon stops rising when the weight of the air displaced by the balloons volume is equal to the balloons weight, which in this case is only the weight of the helium.

$$m_{air} = m_{He} \Leftrightarrow m_{He} = V_{balloon}\rho_{air} = \frac{m_{He}}{\rho_{He}}\rho_{air}$$

Here we see that this is equal to the statement, that the densities of the dry air is the same as the density of the helium. Now we can use the perfect gas law. Where we have to think about the specific gas constant of the helium. For this, we can use: $R_{He} = \frac{R^*}{M_{m_{air}}}$.

$$\frac{p}{R_{He}T_{He}} = \frac{p}{R_{air}T_{air}}$$

$$\Leftrightarrow \frac{T_{He}}{M_{m_{He}}} = \frac{T_{air}}{M_{m_{air}}}$$

The temperature of the helium can be calculated employing the potential temperature of the light gas. We need to recognize that the potential temperature already is given as $\theta_{He} = 273.15$ K at $p_0 = 1000$ hPa. We then get:

$$T_{air} = \theta_{He}\left(\frac{p}{p_0}\right)^{\kappa}\frac{M_{m_{air}}}{M_{m_{He}}} = 142.15\ \mathrm{K}$$

The balloon's instrument measure a different temperature of the atmosphere of $T_1 = 200$ K.

b) You assume the balloon could be leaking, such that a part of the Helium left the balloon. What percentage of the helium mass is still left in the balloon, if the atmosphere really is $T_1 = 200$ K at $p = 0.1$ hPa? (4%)

We redo the calculation from above, but with only the mass $x \cdot m_{He}$ left in the balloon. Solving for x yields then:

$$x = \frac{\theta_{He}}{T_{air}}\left(\frac{p}{p_0}\right)^{\kappa}\frac{M_{m_{air}}}{M_{m_{He}}} = 71.07\%$$

c) Assume again a non-leaking balloon. Another explanation would be that there was water vapour in the helium. The condensing water vapour would change the helium's temperature and the result from part a) was due to a wrong value of the temperature this gas.
Calculate how much water vapour condensed in the balloon. State your answer in grams water vapour condensed per kg helium in the balloon. You may assume for this that the condensation is the only change in heat. (8%)

When a mass of water vapour, m_{H_2O}, condenses, it releases an amount of heat $m_{H_2O}L_V$, where L_V is the latent heat of vaporization. This heat will warm the helium around it by some ΔT. No other heat exchange is considered here. The mass of the helium around it is m_{He}. This gives:

$$m_{H_2O}L_V = m_{He}{C_p}_{He}\Delta T$$

We see that with the mass mixing ration of the condensed water $\mu_{condensed} = m_{H_2O}/\ m_{He}$, we get:

$$\Delta T = \mu_{condensed}\frac{L_V}{{C_p}_{He}}$$

Solution

Now the real temperature of the helium is $T'_{He} = T_{He} + \Delta T$. Employing the formula derived in a), we get:

$$T_{He} + \Delta T = T_1 \frac{M_{m_{He}}}{M_{m_{air}}}$$

$$\mu_{condensed} = \frac{C_{p_{He}}}{L_V}\left(T_1 \frac{M_{m_{He}}}{M_{m_{air}}} - T_{He}\right) = \frac{C_{p_{He}}}{L_V}\left(T_1 \frac{M_{m_{He}}}{M_{m_{air}}} - \theta_{He}\left(\frac{p}{p_0}\right)^{\kappa}\right) = 3.2\,\frac{\mathrm{g}}{\mathrm{kg}}$$