Norwegian University of Science and Technology
Department of Physics

EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE
Faculty for Natural Sciences and Technology
1 Jun 2017
Time: 09:00-13:00

Number of pages: 14

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Single or Bi-lingual dictionary permitted
All calculators permitted

You may take:

Molar mass of dry air: ~29 kg/kmole
Molar mass of helium: ~4 kg/kmole
Molar mass of H_2O: ~18 kg/kmole

$N_A = 6.02\text{x}10^{23}$ molecules/mole Boltzmann's constant $k = 1.38\text{x}10^{-23}$ J/K

273.15 K = 0 °C 1 hPa = 10^2 Pa = 10^2 N m^{-2} g=9.8 m s^{-2} and constant in z

Stefan–Boltzmann constant: $\sigma = 5.67\times10^{-8}$ W·m^{-2}·K^{-4}

Solar photospheric temperature, T_s = 5786 K Radius of the Sun = 695800 km

Radius of the Earth = 6370 km 1 AU (Earth-Sun distance) =150x10^6 km

Latent heat of vaporization water: L_v=2.5x10^6 J· kg^{-1}

Gas constant for water vapour: R_v=461 J·K^{-1}·kg^{-1}

Values for dry air: C_p=1004 J·K^{-1}·kg^{-1} C_v=718 J·K^{-1}·kg^{-1} R_d=287 J·K^{-1}·kg^{-1}

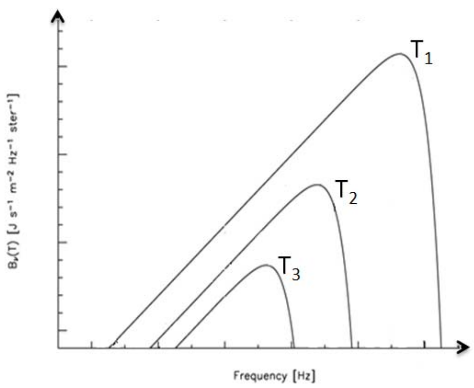
$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d = C_p - C_v$ Γ_{da}=9.8 K/km

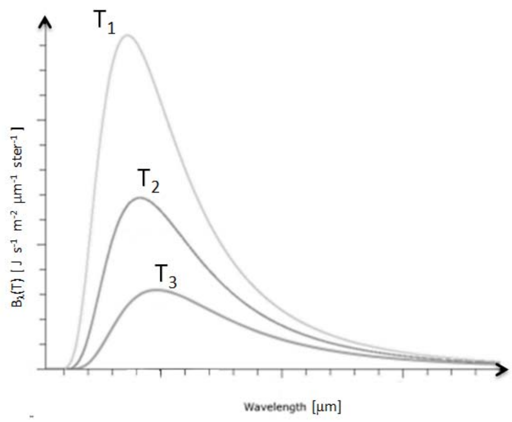
Clausius–Clapeyron relation: $e_s = 6.112\,hPa \cdot \exp\left[\frac{L_v}{R_v}\left(\frac{1}{273K} - \frac{1}{T}\right)\right]$

Answer all questions (English, Norwegian, or Swedish).
State all assumptions.
Good Luck!

1) (5%) Black body radiation

Sketch the relative spectral radiance as a function of wavelength or frequency for three blackbodies at temperatures $T_1 > T_2 > T_3$. Label the curves with their temperatures and give the units used for the axes.

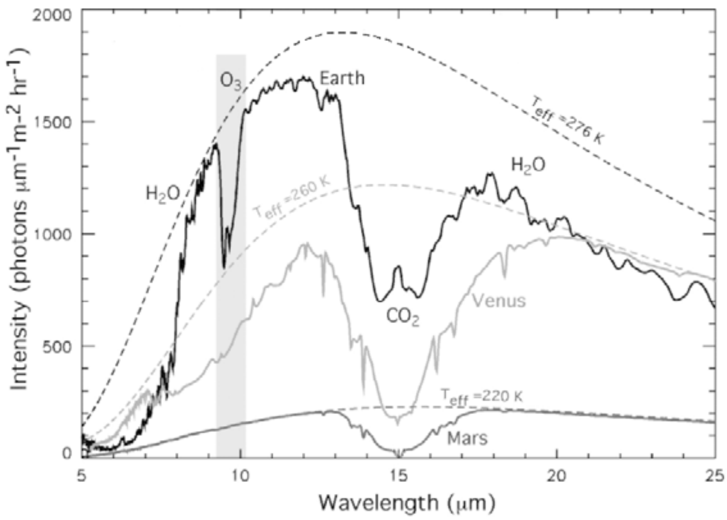




Radiance units should be correct, although I would accept either wavelength or wavenumber units (either the left or right plot). I do not need the actual temperatures on the curves or the numbers on the axes. Curves should not cross, and the hottest temperature, T1, should peak at shorter wavelengths (higher frequencies).

2) (15%) Remote sensing and radiation

An inhabited planet in another solar system is excited at the prospect of finding life on other planets. They observe Earth, Venus, and Mars in the infrared and take the following spectra:



a) Estimate the surface temperatures of the three planets from their spectra (3%)

From areas clear of absorption, one can read off the graph that T_{Earth} = 276K, T_{Venus} = 260K and T_{Mars}=220K. One should NOT use Weins displacement (-1) because the radiance is in photon units, not Watts (1 photon = $h \cdot c/\lambda$ Watts).

b) Using the CO_2 absorption feature, estimate the atmospheric temperatures (3%)

Again, from the graph the minimum of the CO_2 absorption gives T_{atm_Earth} between 220 and 260 ≈ 240 K. T_{atm_Venus}<220K or ≈218 K. Given the printing fault, 220 K is acceptable. T_{atm_Mars}< 220 K, ≈200 K

c) From the behaviour of the absorption near the centre of the CO_2 band, estimate whether the atmospheric temperature increases or decreases with height in the three atmospheres? (5%)

The rotational lines near the centre of the band are closer to the ground state, and therefore have high population densities (Boltzmann→$N_J \propto N_{total} \cdot e^{-E_J/(kT)}$). Hence they reach optical depth τ=1 higher in the atmosphere. On earth, the band centre has a brightness temperature warmer than the lines further from the ground state, which have lower population density and reach τ=1 lower in the atmosphere. This would indicate that there is a warmer atmospheric layer above a cooler one.

The same logic applied to Venus and Mars would say that the atmosphere cools with altitude since there is a dip in the brightness at the band centre. But will take into account the poor printing by the exam office when grading!

d) The surface temperature of Venus is actually 735 K. If your estimate from part a) is different, can you explain why? (4%)

It means that the atmosphere of Venus is optically thick at all wavelengths. That is, there is always a point in the atmosphere where τ=1 that is above the actual surface. So we never see the surface, only the point where τ=1. This could be due to molecular absorption or clouds in the atmosphere emitting radiation.

3) Multiple Choice

There is only **one** correct answer so you must **choose the best answer**.
Answer A, B, C… (Capital letters).
Correct answer gives +2; incorrect or blank answers give 0.

Write the answers for the multiple choice questions **on the answer sheet you turn in** using a table similar to the following:

Question	a	b	c	d	e	f	g	h	i	j	K
Answer	D	A	C	D	F	B	C	A	D	B	C

Note, the students can earn an extra 2 points here as there are 11 questions!

a. If the atmospheric pressure at the surface of the Earth is 1000 hPa, what is the weight of the atmosphere in kg?

A) $5\text{x}10^{15}$ B) $5\text{x}10^{16}$ C) $5\text{x}10^{17}$ D) $5\text{x}10^{18}$ E) $5\text{x}10^{19}$

Pressure in Pa is force/unit area, or Nt/m2. So the force is $P_{surface}\cdot(4\pi\cdot R_e^2)$. In SI units this is 100000 Nt/m^2 $\cdot 4\pi\cdot(6370\text{x}10^3)^2$ m^2 $\approx 5\text{x}10^{19}$ Nt. And since F=M·g, we customarily give the weight in kg as F/g $\approx 5\text{x}10^{18}$ kg.

b. The dry adiabatic ***lapse rate*** is __________ than the moist adiabatic lapse rate.

A) Always greater
B) Always smaller
C) Never greater
D) Sometimes greater
E) Sometimes smaller

Since the lapse rate, $\Gamma=-dT/dZ$, our dry adiabatic lapse rate will mean that a parcel going up in the atmosphere will drop in temperature ≈ 10 K for each km it rises, or $\Gamma a \approx 10$ K/km. For a parcel that is undergoing condensation, it would have dropped in temperature this far, but the latent heat released by condensation warms it so that it does not get as cold, maybe only dropping 5 K for each km it rises. Thus the $\Gamma moist < \Gamma a$, or the dry lapse rate is always greater than the moist.

c. A small cloud droplet will evaporate ______ a large cloud droplet?

A) At the same rate as
B) More slowly than
C) Faster than
D) It will not evaporate

Because of the curvature of a droplet's surface, the molecules on that surface are more exposed to their environment and are less tightly bound by surface tension forces. Therefore, they evaporate more readily than molecules on a plane surface. The smaller the drop, the tighter the radius of curvature and the higher the relative humidity needs to be to keep the drop in equilibrium. Thus, a small drop will evaporate faster than a larger drop sitting beside it in the same humidity conditions. Note that a cloud condensation nuclei will give the drop a larger radius and reduce the relative humidity required to keep the drop in equilibrium.

d. Which two atmospheric layers would the mean temperature profiles be stable against convection?

A) Mesosphere and Stratosphere.
B) Mesosphere and Thermosphere.
C) Mesosphere and Troposphere.
D) Stratosphere and Thermosphere.
E) Stratosphere and Troposphere.
F) None of the above.

The stratosphere and thermosphere both have temperature gradients that increase with altitude. Thus, any parcel of air moving upward into a lower pressure regime will become cooler, and therefore will be cooler than the surrounding atmosphere. It will therefore sink back down.

e. In which layer of the atmosphere is ozone the major species?

A) Stratosphere
B) Mesosphere.
C) Troposphere.
D) Thermosphere.
E) Exosphere.
F) None of the above.

Definition of major and minor species! Ozone is a minor species throughout the atmosphere, with a maximum mixing ratio of only ~5 ppmv in the stratosphere.

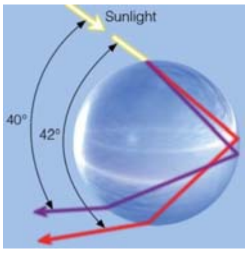
f. If the greenhouse effect produces a temperature warming in the troposphere, why do we find a net 2 K/day radiative cooling there?

A) The greenhouse heating is offset by this cooling, resulting in a steady temperature.
B) It is offsetting non-radiative processes that heat the atmosphere in this region
C) That cooling is only affecting the radiation and not the temperature.
D) There is, in fact, a net cooling of the troposphere
E) None of the above

The key is that the radiative cooling/heating rate does not drive temperature. Rather, the temperature drives the radiative cooling/heating rate (σT^4). If there is a net radiative cooling, it is because another, non-radiative process is heating the atmosphere and driving away from radiative equilibrium. Since it is hotter, it radiates away energy at a faster rate in order to balance the total (radiate plus non-radiative) energy input.

g. At sunset, in which direction would you look to find a rainbow?

A) North.
B) South.
C) East.
D) West.



A rainbow forms when the light enters the droplet. As it does, the red and blue light refract at different angles (red less than blue). They reflect off the backside of the drop, which reverses the red and blue rays. They then exit the drop in the direction from which they came (toward the sun). To see the rainbow, you have to be looking away from the sun, or at sunset, toward the east.

h. At sunset, which type of scattering causes the Sun to appear orange?

A) Rayleigh scattering.
B) geometric scattering.
C) Mie scattering.
D) total internal reflection.
E) Refraction

At sunset, the sun has a long path through the atmosphere. Since Rayleigh scattering is more effective for blue light than red, and since it tends to scatter in all directions, most of the blue light has been scattered out of the beam by the time the sunlight has traversed the long path to you. As a result, the light you see at sunset is more red/orange than white/yellow.

i. If the atmospheric absorption of ozone at 9.6µm becomes saturated, what happens if the concentration of ozone continues to increase?

A) The absorption remains the same because it is saturated.
B) The absorption begins to decrease near the band centre.
C) The absorption continues to increase near the band centre.
D) The absorption increases as lines farther from the band centre begin to saturate.

The higher rotational lines, farther from the band centre, have higher rotational energies (higher J values) and therefore lower population densities (Boltzmann→$N_J \propto N_{total} \cdot e^{-E_J/(kT)}$). As a result of their low densities, they will have smaller (but non-zero) optical depths. However, if you increase the total amount of the molecule (N_{total}), then the number of molecules in the high-J rotational levels will increase, and the optical depth, and thus the absorption, will continue to increase.

j. What two sets of conditions, working together, will make the atmosphere the most unstable?

A) Warm the surface and warm the air above.
B) Warm the surface and cool the air above.
C) Cool the surface and cool the air above.
D) Cool the surface and warm the air above.
E) None of the above.

The faster the temperature falls with altitude, the greater the chance that a rising particle that adiabatically cools will be warmer than the surrounding air, and thus continue to rise. Hence, making the atmosphere's fall-off in temperature steeper by warming the surface and cooling the air above will make the atmosphere more unstable.

k. Under geostrophic conditions, in which direction will the wind blow?

A) In the direction of the Coriolis force.
B) In the direction of the pressure gradient force.
C) Parallel to the isobars or contours of pressure.
D) At an angle between 10° and 30° to the contours and toward the low pressure.
E) At an angle between 10° and 30° to the contours and toward the high pressure
F) None of the above.

Under geostrophic conditions, the wind that is trying to move in the direction of the pressure gradient force is turned by the Coriolis force, which always acts at 90° to the velocity vector. In equilibrium, the pressure gradient force is balanced by the Coriolis force, and thus the wind vector is parallel to the lines of constant pressure.

4) (20%) Atmospheric thermodynamics and structure

A helium balloon, weighing 50 kg when empty, must carry an instrument payload weighing 100 kg to an altitude where the pressure is 40 hPa and the temperature is 230 K.

a) Assuming this spherical balloon has been floating at this altitude, and the temperature of the helium has equilibrated to the surrounding air temperature, what is the approximate radius of the balloon? (6%)

The balloon will float because the gravitational force on the He and the balloon plus payload (= m_b) is equal to the gravitational force on the air the balloon displaces. At 40 hPa we can safely assume that the air is dry. Thus: $m_{air} \cdot g = (m_{He} + m_b) \cdot g$, or

$$m_{air} - m_{He} = m_b.$$

Now, the $m_{air} / V_b = \rho_{air}$ and $m_{He} / V_b = \rho_{He}$ where V_b is the volume of the balloon. The quick way is to just say

$$V_b \cdot (\rho_{air} - \rho_{He}) = m_b.$$

and remember that at its float altitude, the pressure inside and outside of the balloon is that same. Thus:

$P_{outside} = \rho_{air} \cdot R_{air} \cdot T_{air}$ which is equal to $P_{inside} = \rho_{He} \cdot R_{He} \cdot T_{He}$, and we have:

$$P_{inside} = P_{outside} = P$$

Therefore, we can just calculate the density of He and Air and difference them. But, remember that the gas constants are specific gas constants, so $R_{He} = R_{air} \cdot M_{air}/M_{He}$.

We could also write:

$m_{air} = \rho_{air} \cdot V_b = P \cdot V_b / (R_{air} \cdot T_{air})$, and $m_{He} = \rho_{He} \cdot V_b = P \cdot V_b / (R_{He} \cdot T_{He})$

and to get everything into molar masses (i.e. atomic weights), we can replace the specific gas constants R_{air} and R_{He} with

$$R_{air} = N_A \cdot k / M_{air} \text{ and } R_{He} = N_A \cdot k / M_{He},$$

with k = Boltzmann's constant and N_A = Avagadro's number, given on page 1. After all of this, our equation $m_{air} - m_{He} = m_b$ can be written as:

$$m_b = \frac{P \cdot V_b}{N_A \cdot k} \cdot \left(\frac{M_{air}}{T_{air}} - \frac{M_{He}}{T_{He}} \right)$$

Here, we are told that $T_{air} = T_{He} = T$, so we can solve the above equation for V_b as:

$$V_b = \frac{N_A \cdot k \cdot T \cdot m_b}{P \cdot (M_{air} - M_{He})},$$

Giving **$V_b = 2866\ m^3$**, or a radius of **$r = 8.8\ m$** for a spherical balloon ($V=4/3 \cdot \pi \cdot r^3$)

b) A similar balloon and payload is launched from a ground station where the pressure was 1000 hPa and the temperature was 300 K. If it ascended quickly to 40 hPa such that the helium in the balloon behaved adiabatically and did not have time to equilibrate with the surrounding air, what would the temperature of the helium inside the balloon be when it reached 40 hPa? The specific heat for helium is Cp = 5190 $J \cdot K^{-1} \cdot kg^{-1}$. (6%)

So here we have a different balloon where the temperature of the He inside the balloon is not equal to the outside air temperature at 40 hPa. That is, we cannot assume that the balloon has the same amount of He as the first one. We are told that the balloon ascended adiabatically, so we can use the potential temperature equation to calculate the temperature of the helium after the ascent to 40 hPa. The only difference is the value for $\kappa = R / Cp$, where we now have to use the values for He instead of dry air. Since R_i, the specific gas constant for gas i, is R^*/M_i, where R^* is the universal gas constant, to convert R_{air} to R_{He} use:

$$R_{He} = R_{air} \cdot M_{air}/M_{He} = 287\ J \cdot K^{-1} \cdot kg^{-1} \cdot 29 / 4 = 2081\ J \cdot K^{-1} \cdot kg^{-1}$$

$$\text{Giving us a } \kappa' = R_{He}/Cp_{He} = 0.4$$

Now we will take the reference level to be at 1000 hPa, and the potential temperature, θ, will be equal to the temperature at the reference level, θ =300 K. As it ascends adiabatically, this potential temperature will remain constant, and the temperature of the helium in the balloon will drop as:

$$T_{He} = \theta \left(\frac{P}{P_o}\right)^{\kappa'}$$

Substituting in for $\kappa' = 0.4$, $\theta = 300$ K, Po = 100000 Pa, and P = 4000 Pa, we get that the temperature of the He as the balloon reaches 40 hPa is **$T_{He} = 82.5$ K**.

c) In the case of the adiabatic ascent of part (b), what is the approximate radius of the balloon when it reaches 40 hPa? (6%)

Here we can use the same equation as in part a before we assumed that the helium and the surrounding atmosphere were at the same temperature. That is:

$$m_b = \frac{P \cdot V}{N_A \cdot k}\left(\frac{M_{air}}{T_{air}} - \frac{M_{He}}{T_{He}}\right)$$

At a pressure of 4000 Pa, **$V = 4013\ m^3$**, or a radius ($V=4/3 \cdot \pi \cdot r^3$) of **r = 9.9 m**

d) What happens to the balloon that has adiabatically ascended to 40 hPa when the helium inside begins to equilibrate with the surrounding air? (2%)

This balloon was designed to zip up to 40 hPa and stop there, allowing for the adiabatic cooling of the helium. However, once there, the helium will begin to equilibrate its temperature with the surrounding gas. **As it does so, it will warm and expand, causing the balloon to displace a larger volume of gas and rise to a higher altitude.**

This is a problem with scientific ballooning. You either stop well short of your desired float altitude and then slowly rise to it as the gas inside the balloon equilibrates, or you zip up to your altitude and then slowly rise above it as the gas warms and the balloon volume expands, reducing the density of the gas inside. You will also rise during the day as the sun warms the balloon, and sink at night as it cools off. One solution is to use a constant volume balloon, also known as a super-pressure balloon. This is a balloon in bondage, where a constraining net around the balloon prevents it from expanding further. Thus, the balloon will rise until reaching its maximum volume, and the warming of the gas will not increase the volume further (which would displace a larger mass of air).

5) (20%) Atmospheric water vapour and thermodynamics

Our now infamous air parcel starts at a pressure of 950 hPa and a temperature of 17 °C.

a. How do you define the Lifting Condensation Level (LCL)? If our parcel is lifted to the LCL at a pressure level of 800 hPa, what would its temperature be? (5%)

The LCL is the level at which the water vapour inside the parcel reaches its saturation vapour pressure in a rising, expanding, and therefore cooling parcel.

If the LCL is given as being at 80000 Pa, what would the temperature of a parcel lifted to this level be? Well, below the LCL, we know that no condensation will take place and the motion will be adiabatic. Therefore, the potential temperature of the parcel will remain constant. If we take the reference pressure level, P_o, to be 95000 Pa, then the parcel's potential temperature will be the same as its temperature at the reference level, $\theta_p = 17.0 + 275.15$ K = 290.15K. At any other point below the LCL, if it undergoes only adiabatic motion, it will keep this potential temperature, and the actual temperature of the parcel will be:

$$T = \theta_p / (P_o/P)^{\kappa}, \text{ where } \kappa = R/C_p, \text{ and we will take } R \approx R_d$$

The temperature at the LCL can be found by setting $P = P_{LCL} = 80000$ Pa. This gives a value of **T_{LCL} = 276.34 K, or 3.1 °C**

b. What was the dew point temperature and relative humidity of the parcel before it was lifted? (5%)

All we know is that at the LCL, the water vapour in the parcel was at saturation. Below this level, the mass mixing ratio of water will be constant. We can use the Clausius–Claperyon relation from the first page and the T_{LCL} from part a) to calculate the saturated water vapour pressure $e_s(T_{LCL})$, and the pressure there, P_{LCL}, to calculate $\mu_s = \varepsilon \cdot e_s(T_{LCL})/P_{LCL}$, where ε is the ratio of the mass of water vapour to dry air. This gives a value of $\mu_s(T_{LCL}, P_{LCL}) = 0.006$, or 6 g/kg. At the LCL, the mass mixing ratio of water in the parcel is equal to the saturated mixing ratio, and below the LCL the mass mixing ratio of water in the parcel is constant. Thus, the parcel contains $\mu = 6$ g/kg of water at the starting point, which becomes saturated at the LCL.

The dew point temperature of the parcel at 95000 Pa would be the temperature at which 6 g/kg of water would be saturated. Thus,

$$\mu(T_o,P_o) = 0.006 = \mu_s(T_D, P_o) = \varepsilon \cdot e_s(T_D)/P_o.$$

Since we know Po, we have to invert the Clausius-Clapeyron equation to solve for T_D. This can be inverted as:

$$T_D = 273.\frac{Lv}{Lv - 273.\ln\left(0.163612565\frac{\mu_s P_o}{\varepsilon}\right)Rv}$$

And gives a value for the dew point temperature of **T_D = 279 K = 5.6 °C**

The relative humidity requires we know $e(T_o)$ and $e_s(To)$. The first we can calculate from the mixing ratio $\mu(T_o,P_o) = \varepsilon \cdot e(T_o)/P_o = 0.006$. This give us $e(T_o) = 9.1$ hPa. Of course $e_s(T_o)$ is from the Clausius-Clapeyron equation evaluated at T_o, and this gives $e_s(T_o) = 19.1$ hPa. The relative humidity is therefore **RH = $e(T_o)/e_s(T_o)$ = 0.477, or 47.7%.** You can also skip a step and not actually calculate $\mu_s(T_{LCL}, P_{LCL})$, just equate: $\mu_s(T_{LCL},P_{LCL}) = \varepsilon \cdot e_s(T_{LCL})/P_{LCL} = \varepsilon \cdot e(T_o)/P_o, = \mu(T_o,P_o)$, and get $e(T_o) = e_s(T_{LCL}) \cdot P_o/P_{LCL}$, where you calculate $e_s(T_{LCL})$ using Clausius-Calperyon.

c. What mass mixing ratio of water must condense during its ascent in order to change the parcel air temperature by 10 K? (Assume the atmospheric mass is the mass of dry air.) (5%)

When a mass of water vapour, M_{H2O}, condenses, it releases an amount of heat $M_{H2O} \cdot L_v$, where L_v is the latent heat of vaporization that is given as 2.5×10^6 J· kg^{-1}
This heat will warm the mass of dry air around it, M_d, which has a heat capacity C_p given as 1004 J·K^{-1}·kg^{-1}, raising its temperature by some ΔT. Equating these gives

$$M_{H2O} \cdot L_v = M_d \cdot C_p \cdot \Delta T, \text{ or } \Delta T = (M_{H2O} / M_d) \cdot (L_v / C_p)$$

Here we note that $\mu_{condensed} = (M_{H2O} / M_d)$, so that if we raise the temperature by 10 K, the mixing ratio of water that must be condensed is: **$\mu_{condensed} = \Delta T \cdot (C_p / L_v) = 0.004$, or 4 g/kg of water must condense.**

d. If the environmental (atmospheric) temperature decreased adiabatically with height, where would the atmosphere be unstable with respect to vertical motion? Why? (3%)

The atmosphere would be unstable with respect to vertical motion at or above the LCL. Below the LCL a parcel undergoing vertical motion would cool adiabatically and be at the same temperature as the surrounding gas at its new level. **Above the LCL, the water vapour within a vertically moving parcel would condense as the parcel cooled, and the release of latent heat would warm the parcel causing it to be warmer than the surrounding air whose temperature is decreasing adiabatically.**

e. On the bus, an air-filled balloon is hanging from the ceiling by a string. A helium-filled balloon floats while tied to the floor by a string. Assuming the string and balloon material itself has negligible mass, how do the balloons move as the bus accelerates forward, and why? (2%)

An interesting thought problem that I recently saw demonstrated on the friendly number 5 bus. There was a helium balloon tied to a child's pram that was floating, and another child had an air-filled balloon dangling from a string. When the bus started, the inertia of the air caused the air molecules, from my point of view, to rush toward the back of the bus. This caused a pressure gradient within the bus, with a build-up of pressure at the back and a decrease of pressure toward the front of the bus. The air balloon, like the air around it, moved toward the back of the bus. The helium balloon, however, floated toward the lower pressure and moved forward on the bus. As the pressure in the bus equilibrated, the balloons returned to their equilibrium positions.
Since this was an AtB bus with a particularly aggressive driver, I got to see this demonstration repeated over, and over, and over…!

6) (20%) Radiation

In another 5 x10^9 years or so, our Sun will probably become a red giant with its photospheric temperature dropping to 4000 K and its radius swelling to 3.5 x 10^6 km. Under these conditions:

a. Construct a general expressions for the solar constant, S, and the effective temperature, T_e, of a planet with no atmosphere but with an albedo, α, a distance R from the sun. (6%)

For the solar constant, we know that each unit area of a body with a temperature T will radiate a total power according to Stefan-Boltzmann law as $\sigma \cdot T^4$ Watts·m^{-2}. Therefore, the total radiative power coming from the Sun will be:

$$\sigma \cdot (T_S)^4 \cdot 4 \cdot \pi \cdot (R_S)^2 \text{ Watts}$$

We can take this radiative flux to be emitted uniformly in all directions (4π steradians) from the centre of the Sun. After all, if we want the surface brightness of the sun, we can take these Watts and divide by the surface area of a sphere that lies on the surface of the Sun, and we recover the solar surface brightness.

This radiative flux will spread out uniformly from the centre of the Sun, and when it reaches the orbit of the planet at radius R from the Sun's centre, it will have spread out over the area of a sphere $4 \cdot \pi \cdot R^2$. Thus, the solar constant, or Watts· m^{-2}, at this radius will be:

$$S = \{\sigma \cdot (T_S)^4 \cdot 4 \cdot \pi \cdot (R_S)^2 \text{ Watts}\} / \{4 \cdot \pi \cdot R^2 \text{ m}^2\}, \text{ or:}$$

$$\underline{S = \sigma \cdot (T_S)^4 \cdot (R_S / R)^2 \text{ in Watts} \cdot m^{-2}}$$

Note that since we have taken all the total radiative power to be at the centre of the Sun, we use R instead of R-R_s in our equation. A fraction of the power passing though this sphere or radius R will fall upon the cross-sectional area of the planet, $\pi \cdot (R_p)^2$, which warms to a temperature T_e. Again, each unit area of the planet will radiate to space according to the Stefan-Boltzmann law. With the albedo, α, representing the power reflected from the planet without being absorbed, the total power absorbed by the planet is $S \cdot \pi \cdot (R_p)^2 \cdot (1-\alpha)$, and the total power out of the planet is $\sigma \cdot (T_e)^4 \cdot 4 \cdot \pi \cdot (R_p)^2$. The temperature of the planet will change until these two powers are equal, and:

$$S \cdot \pi \cdot (R_p)^2 \cdot (1-\alpha) = \sigma \cdot (T_e)^4 \cdot 4 \cdot \pi \cdot (R_p)^2$$

Substituting in for S, and cancelling terms, we get the general expression:

$$\underline{(T_S)^4 \cdot (R_S / R)^2 \cdot (1-\alpha) = (T_e)^4 \cdot 4}$$

b. Calculate the solar constant and effective temperature for Venus under these conditions. Venus is 0.72 AU from the sun, has a radius of 6052 km, and has an albedo α = 0.71 (6%)

We can use the expression above to calculate the equilibrium temperature of Venus, and all we need to do is to substitute in numbers. Note that the equilibrium temperature only depends on the distance from the Sun and its temperature. In this case, the equilibrium temperature of Venus without any atmosphere would be:

$$\tfrac{1}{4} \cdot 4000^4 \cdot [3.5 \text{ x } 10^6 / (0.72 * 150 \text{x} 10^6)]^2 \cdot (1-0.71) = T_p^4$$

Or **$\underline{T_p = 373.7 \text{ K or } 100.5\ °C}$.** Similarly, the equation derived above gives the irradiance as $S = 5.67 \times 10^{-8} \cdot 4000^4 \cdot [3.5 \text{ x } 10^6 / (0.72 * 150 \text{x} 10^6)]^2$, or **$\underline{S = 15244 \text{ Watts} \cdot m^{-2}}$**

c. What fraction of the Sun's total power output does the Venus intercept? (4%)

The sun puts out its total radiatve flux into 4π steradians. Venus subtends only a fraction of that full solid angle. The solid angle Venus subtends, Ω_V, is A_V/R^2, Where A_V is the cross sectional area of Venus, and R is the sun-Venus distance = 0.72 AU. But 1 AU is given as 150×10^6 km. Since we are dividing km by km, I will be lazy and not convert to metres first!

Fraction of radiative flux = $\Omega_V/(4\pi)$ $= \pi \cdot R_V^2 / R^2 /(4\pi) = \frac{1}{4} \cdot R_V^2 / R^2$

$= \frac{1}{4} \cdot (6052 \text{ km})^2 / (0.72 \cdot 150 \times 10^6 \text{ km})^2$

$= \underline{\mathbf{7.85 \times 10^{-10}}}$

Some were worried about the Solar radius, but there is no need. If you think about the total power coming from each m^2 of the Sun, it is $\sigma \cdot T^4$, and the total power, in Watts, coming from the Sun is $\sigma \cdot T^4 \cdot 4\pi R_s^2$. At the orbit of Venus, the total flux is distributed over $4\pi R_{s\text{-}v}^2$, so the total power per m^2 is going to be $\sigma \cdot T^4 \cdot 4\pi R_s^2 / 4\pi R_{s\text{-}v}^2$. Now we have:

The Irradiance at Venus orbit = $I = \sigma \cdot T^4 \cdot R_s^2 / R_{s\text{-}v}^2$ in Watts/m^2.

The total power at Venus orbit = $I \cdot 4\pi R_{s\text{-}v}^2$, in Watts,

And, the total power on Venus = $I \cdot \pi R_v^2$ also in Watts.

So the power on Venus/total power at Venus's orbit = $\frac{1}{4} \cdot R_v^2 / R_{s\text{-}v}^2$

That is, the solar radius drops out since I cancels in the division.

d. Calculate the wavelength of maximum emission for both the Sun and Venus. (4%)

This is from Wien's displacement law, λ_{max} (in microns) = 2898/T, which I expect you to know. If you forget it, you can get an approximation to the constant by remembering that the normal solar temperature, given on the first page as 5786 K, results in a blackbody that peaks in the visible at about 0.5μ (500 nm). Similarly, a 288 K Earth blackbody peaks near 10μ as shown in problem 2. Note too that this constant is ONLY for black body radiance in W (not photons)

For the red-dwarf case, the maximum wavelengths of the Sun and Earth (given the temperatures stated and calculated) are:

$\underline{\lambda_{max\_red\text{-}dwarf\text{-}Sun} = 0.725\ \mu}$, and $\underline{\lambda_{max\_red\text{-}dwarf\text{-}Earth} = 7.75\mu}$, where $1\mu = 10^{-6}$ m