Norwegian University of Science and Technology
Department of Physics

EXAMINATION IN FY3201 ATMOSPHERIC PHYSICS AND CLIMATE CHANGE

Faculty for Natural Sciences and Technology
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Time: 09:00-13:00

Number of pages: 12

Permitted help sources: 1 side of an A5 sheet with printed or handwritten formulas permitted
Single or Bi-lingual dictionary permitted
All calculators permitted

You may take:

Molar mass of dry air:	~29 kg/kmole
Molar mass of helium:	~4 kg/kmole
Molar mass of H_2O:	~18 kg/kmole

$N_A = 6.02 \times 10^{23}$ molecules/mole   Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

273.15 K = 0 °C   1 hPa = 10^2 Pa = 10^2 N m^{-2} g=9.8 m s^{-2} and constant in z

Stefan–Boltzmann constant: $\sigma = 5.67\times10^{-8}$ W·m^{-2}·K^{-4}

Solar photospheric temperature, T_s = 5786 K   Radius of the Sun = 695800 km

Radius of the Earth = 6370 km   1 AU (Earth-Sun distance) = 150×10^6 km

Latent heat of vaporization water: $L_v = 2.5 \times 10^6$ J· kg^{-1}

Gas constant for water vapour: R_v=461 J·K^{-1}·kg^{-1}

Values for dry air: C_p=1004 J·K^{-1}·kg^{-1} C_v=718 J·K^{-1}·kg^{-1} R_d=287 J·K^{-1}·kg^{-1}

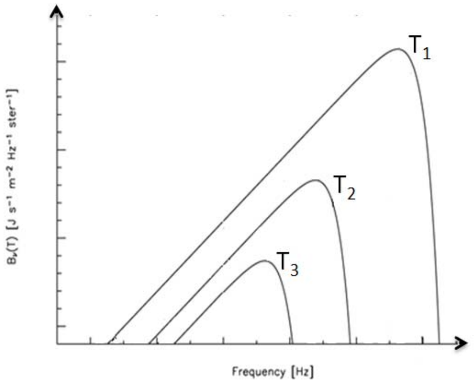
$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d = C_p - C_v$ Γ_{da}=9.8 K/km

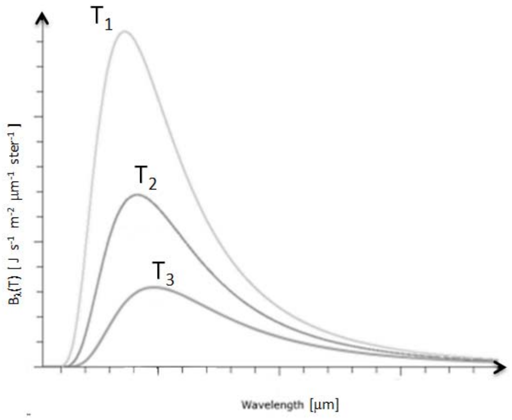
Clausius–Clapeyron relation: $e_s = 6.112\,hPa \cdot \exp\left[\frac{L_v}{R_v}\left(\frac{1}{273K} - \frac{1}{T}\right)\right]$

Answer all questions (English, Norwegian, or Swedish).
State all assumptions.
Good Luck!

1) (5%) Black body radiation

Sketch the relative spectral radiance as a function of wavelength or frequency for three blackbodies at temperatures $T_1 > T_2 > T_3$. Label the curves with their temperatures and give the units used for the axes.





Radiance units should be correct, although I would accept either wavelength or wavenumber units (either the left or right plot). I do not need the actual temperatures on the curves or the numbers on the axes. Curves should not cross, and the hottest temperature, T1, should peak at shorter wavelengths (higher frequencies).

2) (20%) Atmospheric Stability

The Voyager I spacecraft observed Titan, a moon of Saturn. It discovered a hydrocarbon aerosol layer at a pressure of 1000 hPa, where the atmospheric temperature was measured to be 88 K. Even though the surface temperature could not be measured, the surface pressure was measured to be 1500 hPa. For the dry Titan atmosphere, which is 80% nitrogen, the gas constant is R_T=290 J·K^{-1}·kg^{-1}, C_{PT}=1044 J·K^{-1}·kg^{-1}, and $g_T << g_{Earth}$.

a) Estimate the maximum surface temperature assuming the atmosphere is stable with respect to vertical motions. (7%)

The maximum temperature at the surface for a stable atmosphere would be if it followed the adiabatic lapse rate from the known temperature point (if it were higher, convective motions would develop and rapidly redistribute heat vertically until the lapse rate would adjust to the adiabatic lapse rate). Since we do not have P(z), we cannot integrate the perfect gas law and we have to use the potential temperature equation. Easiest is to use the reference pressure, P_o = 1000 hPa where the temperature is 88 K. That means the potential temperature of a parcel at 1000 hPa is θ = 88 K. Then at a surface pressure P=1500 hPa, we have:

$$T = \theta \left(\frac{P}{P_o}\right)^{\kappa}$$

Where $\kappa = R_T / C_{pT}$. For the values of R_T and C_{pT} given we calculate κ = 0.278, and at P = 1500 hPa we get a temperature of ***T_s = 98.5 K****.*

b) The true surface temperature was measured later by Voyager II to be 94 K. Do you expect convection in Titan's atmosphere? Why or why not? (6%)

We would not expect convective motions on Titan. *From part a) we have shown that a parcel starting at the surface at a pressure of 98.5 K would cool to 88 K when it reached 1000 hPa. However,* ***the air at the surface is only 94 K, so if it ascends to 1000 hPa it will be proportionally cooler than 88 K. Since it will be cooler and heavier than the surrounding air, it will tend to fall back to its starting position, and convection will cease****. In fact, a parcel of air at the surface (T=94 K and Po=1500 hPa) has a potential temperature,* $\theta = 94$ *K (taking the reference pressure to be 1500 hPa). It will keep this* θ *as it ascends to P = 1000 hPa, and its temperature there will decrease to* $T_p = 84$ *K, cooler than the surrounding 88 K air. This latter calculation is not required, but won't be penalized. The real key is to realize the atmospheric temperature gradient is constant, and the parcel changes its temperature as it moves vertically. So if you bring a parcel at T = 88 K down to the surface and it heats to 98.5 K, then if you start at the surface and bring a parcel of cooler surface air up, it will be cooler than 88 K. Since it is cooler than the surrounding air, we have stability.*

c) Assuming isothermal conditions, how many scale heights above the surface was the hydrocarbon aerosol layer? (7%)

Under isothermal conditions, we know that the scale height, H = R·T/g, is constant. Therefore the hydrostatic equation can be integrated to give:

$$P(z) = Po\, e^{\left(-\frac{z}{H}\right)}$$

Solving this for Z_d/H *in terms of the pressure of the dust layer,* $P_d = 1000$ *hPa and the surface pressure,* $P_o = 1500$ *hPa, we get:* $\frac{Zd}{H} = -\ln\left(\frac{Pd}{Po}\right)$, *or* ***$Z_d/H = 0.41$***. *That is,* ***$Z_d = 0.41$ scale heights***.

3) (25%) Atmospheric thermodynamics, water vapour and structure

a. A commercial airliner suffers a sudden de-pressurization due to the loss of a cargo door. If the internal and external air pressures were 700 and 300 hPa respectively, and the internal temperature was 19°C before de-pressurization, determine the final internal temperature (assume it is an adiabatic process). (5%)

As the airliner depressurizes the density, pressure and temperature are all changing, so we cannot use the ideal gas law. The key here is that we can assume that the process is adiabatic, and therefore the potential temperature is constant. If we take the reference pressure to be the 700 hPa in the cabin before it de-pressurizes, then the air in the cabin has a potential temperature, $\theta = 19+273.15$ *K = 292.15 K. When this air is brought adiabatically to 300 hPa by the sudden de-compression, the temperature and pressure will change to keep* θ *at this value according to:*

$$\theta = T\left(\frac{Po}{p}\right)^{\kappa} \qquad \text{where } \kappa = \frac{R}{Cp}$$

Where Po =700 hPa and p = 300 hPa. This yields the final temperature to be:

$T = 229.3$ K = -43.8 ℃

b. In practice, a fog formed in the airplane as it de-pressurized. What effect would this have on the final temperature? If the relative humidity in the cabin before it de-pressurized was 25%, how much would the final temperature change? (5%)

Given the rapid pressure fall, we can treat this as a two-step problem. That is, the pressure falls so rapidly that the condensation process does not start until the system has reached its new pressure and temperature. At this point, the fog forming tells us that the water vapour in the cabin air began to condense and release its latent heat of vaporization.

We can estimate an upper limit as to how much latent heat was released if we assume that all the water in the cabin condensed. The latent heat of vaporization for water tells us how much energy is released when we condense a mass of water vapour. Then the specific heat of air tells us that adding a quantity of heat will raise the temperature of a mass of air by a specific amount.

One could calculate the molar mass of the wet air by summing the molar mass of water vapour multiplied by its mixing ratio ($\nu_{vapour} = e/p$*) and the molar mass of dry air multiplied by its mixing ratio (*$1-\nu_{vapour}$*). However, at 25% RH, we can approximate the air as dry air, That is,* $M_{air} = M_d$*, and* $M_{vapour} = M_v$*. Then if we condense a mass of water* m_v*, the heat released is* $m_v * L_v$*. This heat then raises the temperature of an air mass,* m_{air}*, by* ΔT *according to the specific heat at constant pressure. We know this quantity,* C_p*, is approximately constant over the range of atmospheric pressures and temperatures, so we have the relation:*

$$m_v \cdot L_v = m_{air} \cdot C_p \cdot \Delta T$$

We can re-arrange that a bit to be $m_v/m_{air} = C_p/L_v \cdot \Delta T$*, and recognize immediately that the quantity* m_v/m_{air} *is the mass mixing ratio,* $\mu = M_v/M_d \cdot e/p$*, where the M's are the molar masses, e is the partial pressure of water vapour, and p is the pressure. So, we need to calculate the mass mixing ratio of water in the cabin before de-pressurization, as we are going to assume that all of this went to vapour when the cabin pressure dropped to 300 hPa. Note that since* μ *is constant under an adiabatic process, the mixing ratio present at 700 hP and 19 °C will be the same when the pressure is suddenly dropped to 300 hPa and -43.8 °C. Then we assume this entire mass mixing ratio condenses (since we are told that fog has formed).*

First, how much water is available for condensation. The cabin humidity was 25% at 700 hPa and T = 19+273.15 K = 292.15 K. The saturation vapour pressure (in hPa) of water vapour, e_s*, at 292.15 K is given by the Clausius–Clapeyron relation listed on the first page. Thus:*

$$es = 6.112\, e^{\left(\frac{Lv\left(1/273 - \frac{1}{T}\right)}{Rv}\right)} \quad \text{with } T=292.15 \text{ gives } \underline{e_s = 22.47 \text{ hPa}}$$

Then the partial pressure of water vapour present in the cabin at 25% RH, e, was:

$$RH=e/e_s \Rightarrow 0.25 = e/22.47 \text{ hPa, or } \underline{e = 5.62 \text{ hPa}} \text{ of water.}$$

The mass mixing ratio is then:

$$\mu = M_v/M_d \cdot e/p = 18/29 \cdot 5.62/800$$

$$\underline{\mu = 0.00498 \text{ (or 4.98 g/kg)}}$$

Then if all this water condenses, the final temperature will be raised by:

$$\Delta T = \mu \cdot L_v / C_p = 0.00498 \cdot 2.5\text{x}10^6\ J \cdot kg^{-1} / 1004\ J \cdot K^{-1} \cdot kg^{-1}$$

$\Delta T = 12.4\ K$

So, the air will warm by 12.4 K. Instead of -43.8℃, it will be a balmy -31.4℃ as you struggle to put on your oxygen mask!

*Of course, there could be uncondensed water left in the airplane. This is the saturated mixing ratio at T=229.3 K and p=300 hPa, and we could subtract this off from the initial mixing ratio of 4.98 g/kg. So $e_s(229.3) = 0.139$ hPa, which is that partial pressure of water that remains vapour. Its mass mixing ratio at 300 hPa is $\mu_s(229.3, 300\ hPa) = 0.287$ g/kg. Thus, only 4.69 g/kg of water condensed, and the first order correction to the temperature rise is **$\Delta T = 11.7\ K$**. Note that this correction need not be calculated for full credit.*

c. In the winter hemisphere, the 500 hPa level is usually at a height of about 6000 m at a latitude of 30°, and at a height of 5600 m at a latitude of 70°. What is the mean temperature of the layer of atmosphere between 1000 hPa and 500 hPa in each case? (5%)

Here we can use the hypsometric equation directly, or if we have forgotten that, re-derive it from the hydrostatic/ideal-gas equations. That is, we start with the hydrostatic equation:

$$\frac{dp}{dz} = -\rho\, g$$

and the perfect gas law:

$$\rho = \frac{p\, T}{R}$$

To get a relation between p and T, the hypsometric equation:

$$\frac{dp}{p} = -\frac{g\, dz}{R\, T}$$

Since T, normally a function of z, is taken as the constant average temperature in the layer in this case, T_a, we can integrate this to get the hypsometric equation:

$$\ln\left(\frac{p2}{p1}\right) = -\frac{g\,(z2 - z1)}{R\, Ta}$$

Since p_1=1000 hP is the typical ground level atmospheric pressure (z_1=0), we sill assume z_1=0. Then substituting in for p_2 yields and the respective heights at the two latitudes gives:

$$\ln(500\ hPa/1000\ hPa) = -9.8\ m/s^2 \cdot z_2 / (287\ J \cdot K^{-1} \cdot kg^{-1} \cdot T_a)$$

At 30°N where $Z_2 = 6000$ m **$T_a = 295.58\ K$**

At 70°N where $Z_2 = 5600$ m **$T_a = 275.87\ K$**

d. Calculate the number density of CO_2 (425 ppmv) in the atmosphere at ground level (P = 1000 hPa, T = +20° C). (5%)

Assuming you have to derive this from the front sheet, you should remember that the perfect gas law as $P=[N] \cdot k \cdot T$. If not, then certainly $P = \rho \cdot R \cdot T$, where $\rho = [N] \cdot M_d$ and $R = R^*/M_d$, with M_d being the molar mass of dry air and R^* the universal gas *constant. Then $R^*=R\cdot M_d$, and $k = R^*/N_A$, where N_A = avagadro's number. So $k = R\cdot M_d/N_A$, or:*

$$k = 287\ J\cdot K^{-1}\cdot kg^{-1}\cdot 29\ kg/kmole/6.02x10^{26}\ molecules/kmole,\ so$$

$$\underline{k = 1.38x10^{-23}\ J/K/molecule}$$

Then the mixing ratio of CO2, $\nu_{CO2} = P_{CO2}/P$, so $P_{CO2}= 425x10^{-6}\cdot P=42.5$ Pa. Note P must be in Pascals! With T=20+273.15=293.15, we have from gas law:

$$[N_{CO2}] = 42.5\ P/(1.38x10^{-23}\ J\cdot K^{-1}molecule^{-1}\ \cdot 293.15\ K) =$$

$$\underline{\mathbf{[N_{CO2}] = 1.05x10^{22} molecules/m^3}}$$

Since pressure is force/area, a Pascal = $kg\cdot m\cdot s^{-2}\cdot m^{-2}$, giving you the density units. Some chose to work in moles. No problem, $R^*=R/M_d=8.32\ J\cdot K^{-1}\cdot mole^{-1}$(actually 8.31, but remember M_d is approximately 29kg/kmole). Thus, the molar volume would be $\nu\cdot P/(R^*\cdot T) = 1.74x10^{-2}moles/m^3$. Multiply by $N_A=6.02x10^{23}$molecules/mole to recover the number density requested.

e. An air mass of temperature +10° C and pressure 1013 hPa contains 10 g/kg water vapour. Calculate the relative humidity. (5%)

From part a), we had $\mu = M_v/M_d\cdot e/p$, but now we solve for e at a pressure of 1013 hPa given that $\mu = 0.01$. This gives:

$$e = \mu\cdot M_d/M_v\cdot p = 0.01\cdot 29/18\cdot 1013\ hPa$$

$$\underline{e = 16.32\ hPa}$$

and from the Clausius–Clapeyron relation listed on the first page at T = 273.15+10 K = 283.15, we get the saturation vapour pressure e_s to be:

$$\underline{e_s = 12.45\ hPa}$$

and the relative humidity is e/e_s, usually expressed in %

$$\underline{\mathbf{RH=100\cdot 16.32/12.45 = 131\%}}\ !$$

Of course this means that we would probably not see this much vapour in the air at 10°C, it would condense out as heavy dew or fog.

4) Multiple Choice (30%):

There is only **one** correct answer so you must **choose the best answer**.
Answer A, B, C… (Capital letters).
Correct answer gives +3; incorrect or blank answers give 0.

Write the answers for the multiple choice questions **on the answer sheet you turn in** using a table similar to the following:

Question	a	b	c	d	e	f	g	h	i	j
Answer	**D**	**C/D**	**D**	**C**	**F**	**B**	**E**	**A**	**F**	**D/E**

a. If the atmospheric pressure at the surface of the Earth is 1000 hPa, what is the weight of the atmosphere in kg?

A) $5x10^{15}$ B) $5x10^{16}$ C) $5x10^{17}$ D) $5x10^{18}$ E) $5x10^{19}$

Pressure in Pa is force/unit area, or Nt/m2. So the force is $P_{surface}\cdot(4\pi\cdot R_e^2)$*. In SI units this is* $100000\ Nt/m^2\cdot 4\pi\cdot(6370x10^3)^2\ m^2 \approx 5x10^{19}$ *Nt. And since* $F=M\cdot g$*, we customarily give the weight in kg as* $F/g \approx 5x10^{18}$ *kg.*

b. If an atmospheric absorption line is saturated, describe what happens to the total absorption in the line if the absorber density continues to increase?

A) The total absorption stays the same
B) The total absorption begins to decrease
C) The total absorption continues to increase
D) The line becomes pressure broadened and absorption increases
E) The line becomes Doppler broadened and absorption increases
F) None of the above

The absorption in the wings continues to increase whether or not the line is Doppler, pressure, or naturally broadened. If the major species is the absorbing species, there is the possibility that the line will become pressure broadened as the density increases. Thus, either C or D will be accepted

c. In the two-stream approximation, the integral over wavelength and angle can be approximated as two streams at which angles to the vertical?

A) $\theta = \pm 24°$
B) $\theta = \pm 35°$
C) $\theta = \pm 42°$
D) $\theta = \pm 53°$
E) None of the above

The two-stream approximation is the result of taking the point where $\Delta\tau = \pm 1$*, the maximum contribution to the integral over wavelength and angle, as the total contribution. Thus, integrating the 2 * (third exponential integral) over angle with* $\Delta\tau = \pm 1$*, and setting the result equal to* $e^{-1/\mu}$*, we get* $\mu = \cos(\theta) = \pm 3/5$*, or* $\theta = \pm 53°$

d. Which of the statements below is wrong for a neutrally stable atmosphere?

A) The atmospheric lapse rate equals the dry adiabatic lapse rate.
B) A displaced parcel will not be forced from its new altitude.
C) The Brunt Väisälä frequency is an imaginary number.
D) The temperature of the parcel equals the atmospheric temperature at every pressure level.
E) The parcel's temperature change with altitude is constant.
F) None of the above.

"The Brunt Väisälä frequency is an imaginary number" is wrong. It becomes zero. Since the Brunt Väisälä frequency is defined as:

$$N \quad \sqrt{\frac{g}{\theta}\frac{d\theta}{dz}}$$

For it to be imaginary, the potential temperature of the atmosphere would have to decrease with altitude, indicating an unstable atmosphere.

e. In which layer of the atmosphere is ozone the major species?

A) Stratosphere
B) Mesosphere.
C) Troposphere.
D) Thermosphere.
E) Exosphere.
F) None of the above.

Ozone is a minor species throughout the atmosphere, with a maximum mixing ratio of 5-6 ppmv.

f. How do you find the Lifting condensation level on a Skew-T diagram?

A) Find the water vapour mixing ratio of the dew point temperature.
B) Find the intersection between the dry adiabat and the line of constant $\mu_s = \mu$
C) Find the region where the temperature starts to rise with altitude.
D) Find where the atmospheric lapse rate is equal to the dry adiabatic lapse rate
E) Find where the atmospheric temperature equals the dew point temperature
F) None of the above

All the answers are useful procedures for these diagrams, but the LCL is defined as the point where the water vapour contained in the parcel, μ, may begin to condense as it is transported upward. The procedure is to find the isopleth where $\mu = \mu_s$, and trace this upward until it crosses the dry adiabat that the parcel will follow as it is moved upward.

g. In the northern hemisphere, in which direction will the Coriolis force direct air moving northward or southward?

A) North.
B) South.
C) East.
D) West.
E) Toward the right of the trajectory of the air
F) Toward the left of the trajectory of the air

The direction in which the Coriolis force operates is $V \times 2\Omega$, where V is the velocity (in the direction of the trajectory) and Ω is the angular velocity vector (pointing upward at the north pole). For a southward moving parcel in the northern hemisphere this is toward the west (or to the right of its trajectory). For a northward moving parcel in the northern hemisphere this is toward the east (or, again to the right of its trajectory).

h. How can we describe light scattering by clouds?

A. Mie scattering theory.
B. Rayleigh scattering theory.
C. Tyndall scattering theory.
D. Geometric scattering theory.
E. None of the above.

Cloud droplets have radii that are close to the size of visible light (500 nm, giving a diameter of about 1 μ). Therefore they scatter in the Mie portion of the curve (forward scatter, very little change in the scattering cross section relative to the particle cross-sectional area, and in the forward direction).

i. What wavelengths of sunlight are absorbed by molecular nitrogen in the troposphere?

A) Infrared.
B) Ultraviolet.
C) Radio waves.
D) Microwaves.
E) Visible.
F) None of the above.

Since N_2 is homonuclear, it has not permanent dipole moment and cannot absorb the wavelengths of sunlight that make it to the troposphere (near-uv, visible and IR).

j. In an isothermal atmosphere, two air parcels, one wet and one dry, are displaced upward. What happens to the parcel temperatures?

A) They remain constant since the atmosphere is isothermal.
B) Both parcels heat at the same rate as they get nearer to the Sun.
C) The wet air parcel cools faster than the dry one due to its thermal conductivity.
D) The dry air parcel cools faster than the wet one due to latent heat effects.
E) Both parcels cool at the same rate as the pressure drops.
F) None of the above.

As the wet air parcel rises, expands and cools, the water in the parcel condenses and releases its latent heat to the air parcel. This warms the air parcel so that its temperature will be higher than an equivalent dry air parcel. Since it is not stated that the parcels are continuously displaced upward, and therefore might not condense, I will accept answer E as well.

5) (20%) Radiation

The optical depth for incoming short wavelength solar radiation (averaged over all wavelengths) is defined by:

$$\tau(z) = \int_z^\infty \rho\ \mu\ k\ dz$$

Where $\rho(z)$ is the air density, $\mu(z)$ is the mass mixing ratio of the absorbing species i, and k is the attenuation coefficient.
Assume an isothermal atmosphere with a scale height of H_e=7 km, a surface density, ρ_o = 1.2 kg/m^3, and a constant attenuation coefficient, k = 0.02 m^2/kg(i).

Then, for $\mu(z) = \mu_o \cdot \exp(-z/H_i)$, where μ_o at the surface = 0.01 and H_i = 4km:

a. Calculate the optical depth at 7 km and 2 km altitude (6%)

To calculate the optical depth, we just set in expressions for ρ and μ. Since it is isothermal, both the atmospheric and absorbing species fall off exponentially, but with different scale heights. So:

$$\tau = \int_z^\infty \rho\,\mu\,k\,dz \qquad \tau - k\,\rho 0\,\mu 0 \int_z^\infty e^{\left(-\frac{z}{He}\right)} e^{\left(-\frac{z}{Hi}\right)} dz$$

Which becomes:

$$\tau = k\,\rho 0\,\mu 0 \int_z^\infty e^{\left(-\frac{z(Hi+He)}{He\,Hi}\right)} dz$$

Which integrates to

$$\tau = \frac{k\,\rho 0\,\mu 0\,He\,Hi\,e^{\left(-\frac{z(Hi+He)}{He\,Hi}\right)}}{Hi+He}$$

Which we can also write as:

$$\tau = k \cdot \rho_o \cdot \mu_o \cdot \left(\frac{H_i \cdot H_e}{H_e + H_i}\right) \cdot e^{-\frac{z}{He}} \cdot e^{-\frac{z}{H_i}}$$

Setting in 7000 m and 2000 m altitude, and the appropriate scale heights and initial values, we get: **τ(7km) = 0.039, τ(2km) = 0.28**

b. If we look at the downward flux irradiance of solar radiation with a zenith angle (angle measured from the zenith) χ, give an expression for the flux irradiance at height z in terms of the flux irradiance at the top of the atmosphere, F_∞, the optical depth τ(z) derived in part a, and χ. (3%)

We take the flux irradiance (W/m^2) at the top of the atmosphere to be F_∞ in W/m^2, then the downward flux is given by $\mathbf{F^{\downarrow} = -F_\infty \cdot \cos(\theta) \cdot e^{\tau/\cos(\theta)}}$, where θ is the angle measured from the local vertical to the ray direction. Thus for a vertical downward path with the sun overhead it would be 180°. On the other hand, χ, is measured from the zenith to the sun, so that the sun overhead would have $\chi = 0°$. Thus, $\cos(\chi) = -\cos(\theta)$. So in terms of χ, **$F^{\downarrow} = F_{\infty} \cdot \cos(\chi) \cdot e^{-\tau/\cos(\chi)}$**. I will accept either one so long as the angles are defined properly. For the remainder of the problem, we defined heating rate in terms of net upward flux: $F = F^{\uparrow} - F^{\downarrow}$, and since $F^{\uparrow}=0$, the net upward flux:

$$F = -F^{\downarrow} = F_{\infty} \cdot \cos(\theta) \cdot e^{\tau/\cos(\theta)} = -F_{\infty} \cdot \cos(\chi) \cdot e^{-\tau/\cos(\chi)}$$

c. If we take $F_{\infty} = 400$ W/m^2 and $\chi = 45°$, what is the flux irradiance at 7 km and 2 km altitude? (3%)

For a zenith angle $\chi = 45°$ (or $\theta = 135°$), we end up with:

$F^{\downarrow}(7000\text{m}) = -267.6$ W/m^2, $F^{\downarrow}(2000\text{m}) = -190.8$ W/m^2,

Since the problem does not specify downward flux or net upward flux, there is no penalty for sign errors.

d. From the above results calculate the local warming in K/hour at 7 km and 2 km altitude due to the local absorption of solar ultraviolet light (note, this is an isobaric process with Cp = 1004 J/K/kg). (8%)

We had in class that the short wavelength diabatic heating rate in W/kg is given by:

$$\frac{dQ(z)}{dt} = -\frac{1}{\rho(z)} \cdot \frac{dF(z)}{dz}$$

Where F is the net upward flux. To make that a heating rate in K/s, we need to divide by Cp = 1004 W/K/kg.

$$\frac{dT(z)}{dt} = -\frac{1}{\rho(z) \cdot C_p} \cdot \frac{dF(z)}{dz} \frac{Kelvin}{s}$$

We have that $F = -F_{\infty} \cdot \cos(\chi) \cdot e^{-\tau/\cos(\chi)}$, so that $-dF/dz = -F_{\infty} \cdot e^{-\tau/\cos(\chi)} \cdot (d\tau/dz)$. We can write this in terms of the net upward flux as:

$-dF/dz = F/\cos(\chi) \cdot (d\tau/dz)$. (if you are working in θ, $-dF/dz = -F_{\infty} \cdot e^{\tau/\mu} \cdot d\tau/dz = -F/\mu \cdot d\tau/dz$).

So we have:

$$\frac{dT(z)}{dt} = -\frac{1}{\rho(z) \cdot C_p \cos(\chi)} \cdot F \cdot \frac{d\tau(z)}{dz}$$

And we have from differentiating τ from the first section:

$$\frac{d\tau(z)}{dz} = -k \cdot \rho_o \cdot \mu_o \cdot e^{-z\left(\frac{H_i + H_e}{H_e \cdot H_i}\right)} = -k \cdot \rho_o \cdot \mu_o \cdot e^{-\frac{z}{H_i}} \cdot e^{-\frac{z}{H_e}}$$

If we put them all together (we don't have to since we could calculate each element at the two altitudes. That is, calculate the density at the two altitudes, then the $d\tau/dz$ at the two altitudes and use F at the two altitudes from part c). If we do put them together, it can simplify a bit:

$$\frac{dT(z)}{dt} = \frac{1}{\rho(z) \cdot C_p \cos(\chi)} \cdot F \cdot k \cdot \rho_o \cdot \mu_o \cdot e^{-\frac{z}{H_i}} \cdot e^{-\frac{z}{H_e}}$$

And with $\rho = \rho_o \cdot e^{-z/He}$, this becomes:

$$\frac{dT(z)}{dt} = \frac{1}{C_p \cos(\chi)} \cdot F \cdot k \cdot \mu_o \cdot e^{-\frac{z}{H_i}}$$

Again, we have the flux at the two altitudes from part c, so substitute and go. The result is (remembering to convert from K/s to K/hr:

dT/dt @ 2 km = **0.117 K/hr** (or 3.26×10^{-5} K/s) and dT/dt @ 7 km = **0.047 K/hr** (1.31×10^{-5} K/s)