

Department of Physics

Examination paper for FY3201 / 8902 Atmospheric Physics and Climate Change

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Permitted examination support material: All support materials are allowed

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You may take:

Molar mass of dry air: ~ 29 kg/kmole

Molar mass of helium: ~ 4 kg/kmole

Molar mass of H₂O: ~ 18 kg/kmole

Molar mass of CO₂: ~ 44 kg/kmole

$N_A = 6.02 \times 10^{23}$ molecules/mole

Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K

$273 \text{ K} = 0 \text{ }^\circ\text{C}$ $1 \text{ hPa} = 10^2 \text{ Pa} = 10^2 \text{ N m}^{-2}$ $1 \text{ atm} = 1013 \text{ hPa}$ $g = 9.8 \text{ m s}^{-2}$ constant in z

Stefan–Boltzmann constant: $\sigma = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$

Solar photospheric temperature, $T_s = 5786 \text{ K}$

Radius of the Sun = 695800 km

Radius of the Earth = 6370 km

1 AU (Earth-Sun distance) = 150×10^6 km

Radius of Mercury = 6051 km

Mercury-Sun distance = 0.387 AU

Radius of Mars = 3396 km

Mars-Sun distance = 1.52 AU

Latent heat of vaporization water: $L_v = 2.5 \times 10^6 \text{ J} \cdot \text{kg}^{-1}$

Latent heat of sublimation ice: $L_i = 2.8 \times 10^6 \text{ J} \cdot \text{kg}^{-1}$

Gas constant for water vapour: $R_v = 461 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$

Values for dry air:

$C_p = 1004 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$

$C_v = 718 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$ $R_d = 287 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}$

$\gamma = C_p / C_v$

$\kappa = R_d / C_p$

$R_d = C_p - C_v$

$\Gamma_{da} = 9.8 \text{ K/km}$

Clausius–Clapeyron relation: $e_s = 6.112 \text{ hPa} \cdot \exp\left[\frac{L_v}{R_v}\left(\frac{1}{273 \text{ K}} - \frac{1}{T}\right)\right]$

Some integrals that may be of use:

$$\int x^m e^{(ax)} dx = \frac{x^m e^{(ax)}}{a} - \frac{m \int x^{(m-1)} e^{(ax)} dx}{a}$$

$$\int x e^{(ax)} dx = \frac{e^{(ax)} (ax - 1)}{a^2}$$

$$\text{For } a > 0 \int_0^\infty e^{(-ax)} dx = \frac{1}{a}$$

$$\int \frac{1}{a + bx} dx = \frac{\ln(a + bx)}{b}$$

Problem 1) Matching radiometric units:

Spectral radiance $\frac{\text{Watts}}{\text{m}^2 \text{ sr nm}}$

Spectral irradiance $\frac{\text{Watts}}{\text{m}^2 \text{ nm}}$

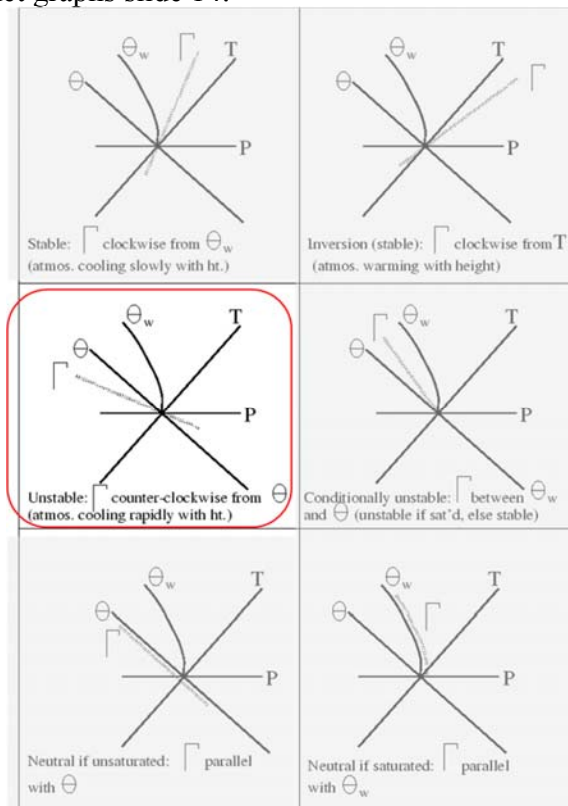
Radiance $\frac{\text{J}}{\text{s m}^2 \text{ sr}}$

Irradiance $\frac{\text{Watts}}{\text{m}^2}$

Problem 2) Atmospheric stability

Which of the following graphs depicts unstable conditions on a skew T-P diagram, where Γ is the atmospheric lapse rate?

From Lecture 18 met graphs slide 14:



As the air parcel at T and P rises and cools, it follows either θ (if it is dry) or θ_w if it condenses water. In either case, it will be warmer than the surrounding atmosphere and continue to rise. This is an unstable condition.

3) Meteorology Graphs

In the Skew-T log-P diagram, at a pressure of 1000 hPa an air parcel has a temperature $T=20\text{ }^\circ\text{C}$ and a dew-point temperature of $T_d=10\text{ }^\circ\text{C}$.

a) At what pressure is the Lifting Condensation Level (LCL)?

The LCL is at approximately 850 hPa:

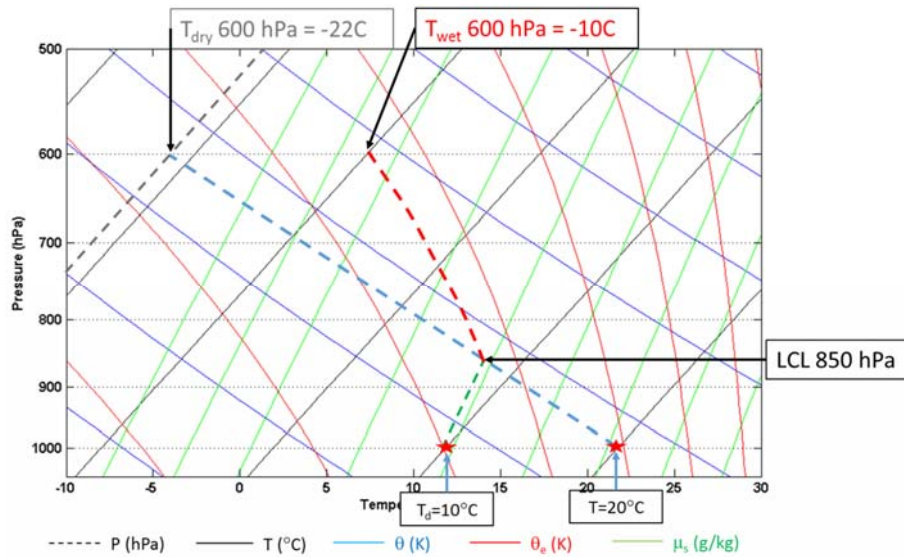
Since $T_d < T$, the water in the parcel is not saturated. Thus, it will follow the dry adiabat if lifted. The water amount in the parcel is the saturated mass mixing ratio, μ_s , at T_d . This mass mixing ratio will remain constant as the parcel is lifted until saturation occurs. To find the point at which the water in the parcel becomes saturated, follow the dry-adiabat line from the parcel temperature, T , upward (lower pressure), and follow the constant saturated mixing ratio line upward from T_d . Where these cross is the point where the water in the parcel (which is $\mu_s(T_d)$) is now equal to the saturated mixing ratio at P_{LCL} and T_{LCL} .

b) If the parcel contains no water, what is the air parcel's temperature when it is lifted to 600 hPa.

$T_{600\text{ hPa}}$ approximately $-20\text{ }^\circ\text{C}$:

If the parcel contained no water vapour, then no condensation would take place and it would continue to change its temperature with changing pressure so as to maintain the same potential temperature, θ . This means that it moves along the dry adiabat until it reaches 600 hPa. There, the chart shows the temperature to be $-22\text{ }^\circ\text{C}$.

If the parcel did contain water and have a $T_d=10\text{ }^\circ\text{C}$, it would have changed its temperature with changing pressure so as to conserve its equivalent potential temperature, θ_w , after reaching the LCL. Thus, it would have moved along the wet adiabat from 850 to 600 hPa where its temperature would be $-10\text{ }^\circ\text{C}$.



Problem 4) CO₂ radiation

At what altitude does the CO₂ molecule radiate to space with little chance of the radiation being absorbed?

Assume an isothermal atmosphere at 20 °C, with a constant mixing ratio of CO₂ = 400 ppmv and a surface pressure of 1000 hPa. Take the mass of dry air to be 29 g/mole, the mass of CO₂ to be 44 g/mole, and the absorption coefficient, $k = 0.4 \text{ m}^2/\text{kg}$.

The CO₂ radiation will escape to space with little chance of absorption from an altitude of 7784 m, or 8 km:

The chance of absorption is related to the optical depth, τ . When $\tau \leq 1$, there is virtually no chance of absorption. Since the optical depth will increase with decreasing altitude, solving for the altitude where $\tau = 1$ for CO₂ absorption will give the altitude from which CO₂ radiation can escape to space with little chance of absorption.

The optical depth is given by: $\tau = \int k \cdot \rho_{\text{CO}_2} \cdot dz$ where the integral goes from an altitude z to infinity. We are given an isothermal atmosphere and an initial pressure, so we can get the pressure profile of the atmosphere easily. One problem is that we are given a mixing ratio of CO₂ which is $P_{\text{CO}_2}/P_{\text{atm}}$, but we need the *density* profile of CO₂ to calculate τ .

Air pressure in an isothermal atmosphere, where H is the scale height of a dry atmosphere, is given by:

$$P = P_0 e^{\left(-\frac{z}{H}\right)}$$

Where H is the scale height given by $R_d \cdot T/g$. This comes from integrating the hydrostatic equation with ρ replaced by $P/(R_d \cdot T)$ using the ideal gas law. Now, P_{CO_2} is given by $P_{\text{CO}_2} = v \cdot P$, where v is the mixing ratio of CO₂ given as 400×10^{-6} (=400 ppmv).

Now we have that the partial pressure of CO₂, but to get the density of CO₂ we use the ideal gas law: $\rho_{CO_2} = P_{CO_2} / (R_{CO_2} \cdot T)$. Here R_{CO_2} is the specific gas constant for CO₂ where $R_{CO_2} = R_d \cdot M_d / M_{CO_2}$, with M_d and M_{CO_2} the molar masses of dry air and CO₂.

Putting this together, we have

$$\rho_{CO_2} = \frac{\nu \cdot P_o \cdot e^{-z/H}}{\frac{M_d}{M_{CO_2}} \cdot R_d \cdot T}$$

Where $H = R_d \cdot T / g$.

This lets us write the optical depth as:

$$\tau = \int_z^\infty k_\nu \cdot \rho_{CO_2} \cdot dz = \int_z^\infty k \cdot \frac{\nu \cdot P_o \cdot e^{-z'/H}}{\frac{M_d}{M_{CO_2}} \cdot R_d \cdot T} \cdot dz' = \frac{k \cdot \nu \cdot P_o}{\frac{M_d}{M_{CO_2}} \cdot R_d \cdot T} \int_z^\infty e^{-z'/H} dz'$$

From our equation sheet, we have that $\int_{x1}^{x2} e^{(-a \cdot x)} dx = -\frac{e^{(-a \cdot x2)} - e^{(-a \cdot x1)}}{a}$

Therefore our integral has been evaluated at the upper – lower limit. At the upper limit, $z = \infty$, the optical depth is zero (since the density goes to zero). Hence we have the integral evaluated at the lower limit, $(1/a) \cdot e^{-a \cdot x1}$.

In our case, the integral evaluates to:

$$\tau = k \cdot H \cdot \frac{\nu \cdot P_o \cdot e^{-z/H}}{\frac{M_d}{M_{CO_2}} \cdot R_d \cdot T} = k \cdot \frac{M_{CO_2}}{M_d \cdot g} \cdot \nu \cdot P_o \cdot e^{-\frac{z \cdot g}{R_d \cdot T}}$$

Setting $\tau = 1$ and solving for z , we get that

$$z_{\tau=1} = -\ln\left(\frac{M_d \cdot g}{k \cdot M_{CO_2} \cdot \nu \cdot P_o}\right) \frac{R_d \cdot T}{g}$$

Using $P_o = 100000$ Pa, $\nu_{CO_2} = 400 \cdot 10^{-6}$, $T = 20 + 273 = 293$ K, $k = 0.4$ m²/kg, $R = 287$ J/kg/K, and $g = 9.8$ m/s², we get that

$$z_{\tau=1} = 7783.57 \text{ m}$$

Problem 5) Water vapour

An air parcel at 300 K and a total pressure of 1000 hPa .

- a) What is the water vapour pressure, e , of the air parcel if it has a relative humidity of 40 %?

The partial pressure of water vapour, $e = 12.1$ hPa

The relative humidity is defined as $RH = 100 \cdot e / e_s(T)$, where $e_s(T)$ is the saturation or equilibrium vapour pressure of water at temperature T . That is, in steady state, $e_s(T)$ is

the pressure of water vapour that condenses into water at the same rate as the water evaporates into vapour at temperature T. The expression for $e_s(T)$ is given on the equation sheet as:

$$e_s(T) = 6.112 \text{ hPa} \cdot \exp \left[\left(\frac{L_v}{R_v} \right) \cdot \left(\frac{1}{273 \text{ K}} - \frac{1}{T} \right) \right]$$

Where L_v is the latent heat of vaporization, 2.5×10^6 J/kg and R_v is the specific gas constant for water vapour, 461 J/K/kg, both given on the equation sheet. Thus substituting values we get at $T=300$ K:

$$e_s(T) = 36.56 \text{ hPa}$$

And $e = RH/100 \cdot e_s(T) = 0.4 \cdot 36.59 \text{ hPa} = 14.6 \text{ hPa}$

- b) What is the dew point temperature of the air parcel if the partial pressure of water vapour is 6 hPa?

$$T_d = 4 \text{ }^\circ\text{C}$$

The dew-point temperature is the temperature at which the vapour pressure of water in the parcel, $e=6$ hPa is equal to the saturation pressure, $e_s(T)$. So we have to invert the expression for $T_d(e)$. That is:

$$T_d := 273. \frac{L_v}{L_v - 273. \ln(.1636125654 e) R_v}$$

Again, L_v is the latent heat of vaporization, 2.5×10^6 J/kg and R_v is the specific gas constant for water vapour, 461 J/K/kg. At $e = 6$ hPa, this give $T=272.7 \text{ K} \approx 0 \text{ }^\circ\text{C}$

- c) What is the mean molecular weight of the air with 3 hPa water?

$$M = 28.97 \text{ g/mole}$$

The mixing ratio of 3 hPa in the air is given by $v = e/P_0 = 3/1000 = 0.003$. To get the molecular weight of air where $1-v$ of the molecules have been replaced by water molecules, we use the formula:

$$M = (1 - v) M_d + v M_{wv}$$

$M_d =$ Mass of dry air = 29 g/mole, $M_{wv} =$ Mass of water vapour = 18 g/mole, gives:
 $M = 28.967 \text{ g/mole}$

- d) If the air parcel had a water vapour mass mixing ratio of 1 g/kg, what would its relative humidity be after lifting it adiabatically to 900 hPa?

$$RH = 6\%$$

We can approach this by first assuming that the parcel does not condense. Then we have two things conserved under adiabatic motion, the mass mixing ratio, $\mu(P, T)$, and the potential temperature, θ . We will calculate the temperature at 900 hPa, and the saturated mass mixing ratio, $\mu_s(900 \text{ hPa}, T_{900 \text{ hPa}})$. If this is less than 1 g/kg, the mass of water in the parcel, then we know we have not saturated and do not need to use the equivalent potential temperature.

So, first, what is the temperature at 900 hPa if the parcel is adiabatically lifted. Using the expression for potential temperature:

$$\theta = T \left(\frac{P_0}{P} \right)^\kappa$$

We can calculate θ at $P_0 = 1000$ hPa and $T_0 = 300$ K to be $\theta = 300$ K. Now, if the parcel is lifted adiabatically to $P_1 = 900$ hPa, we can find the temperature there via this same expression:

$$T_1 = \frac{\theta}{\left(\frac{P_0}{P_1} \right)^\kappa}$$

Substituting in $\theta = 300$ K, $P_0 = 1000$ hPa and $P_1 = 900$ hPa, we get $T_1 = 291$ K. Now, we need to calculate $\mu_s(P_1 = 900 \text{ hPa}, T_1 = 291 \text{ K})$ using the formula for the saturation vapour pressure:

$$e_s(T_1) = 6.112 \text{ hPa} \cdot \exp \left[\left(\frac{L_v}{R_v} \right) \cdot \left(\frac{1}{273} - \frac{1}{T_1} \right) \right]$$

Gives $e_s(T_1) = 21$ hPa. The saturated mass mixing ratio at P_1 and T_1 is just:

$$\mu_s(P_1, T_1) = \frac{M_w}{M_d} \cdot \frac{e_s(T_1)}{P_1}$$

$M_w = 18$ g/mole, and $M_d = 29$ g/mole, gives a saturated mass mixing ratio of 0.015, or 15 g/kg. We would need 15 g/kg of water to begin to condense, but the parcel only has 1 g/kg in it. Thus, we are not saturated and our motion was adiabatic after all!

Now, all that remains is to calculate the Relative Humidity. This is just $e/e_s(T_1)$ at P_1 , and we know e at P_1 from the mass mixing ratio that was given:

$$\mu(P_0, T_0) = \mu(P_1, T_1) = 0.001 = \frac{M_w}{M_d} \cdot \frac{e_{at\ p1}}{P_1}$$

This gives $e_{at\ P1}$ to be 1.45 hPa, and $RH = 100 \cdot e/e_s(T_1) = 100 \cdot 1.45/21 = 6.9 \%$

Problem 6) Atmospheric composition

At the surface of the Earth, there are 2.3×10^{25} molecules/m³. If the temperature is a constant 20 °C with altitude:

- a) How many molecules are there in the Earth's atmosphere?

There are 10^{44} molecules

There are two ways to approach this.

FIRST, The more complicated integral is to take spherical shells of atmosphere of volume $4\pi \cdot r^2 \cdot dr$ going from the Earth up to infinity. Each shell will contain $N(r)$ molecules. Since it is isothermal, we know how N varies with altitude.

$$N(z) = n_0 e^{\left(-\frac{z}{H}\right)}$$

Where $H=R_d \cdot T/g$ is constant, and, of course, $z = r - R_{earth}$. This gives the integral:

$$N_{tot} = \int_{R_{earth}}^{\infty} 4 n_0 e^{\left(\frac{-r + R_{earth}}{H}\right)} \pi r^2 dr$$

And we can pull some constant factors out of the integral

$$N_{tot} = 4 \pi n_0 e^{\left(\frac{R_{earth}}{H}\right)} \int_{R_{earth}}^{\infty} r^2 e^{\left(-\frac{r}{H}\right)} dr$$

This integral is of the form (from the help sheet):

$$\int x^m e^{(ax)} dx = \frac{x^m e^{(ax)}}{a} - \frac{m \int x^{(m-1)} e^{(ax)} dx}{a}$$

With $m=2$, we have:

$$\int x^2 e^{(ax)} dx = \frac{x^2 e^{(ax)}}{a} - \frac{2 \int x e^{(ax)} dx}{a}$$

And the second integral is of the form (again from the help sheet):

$$\int x e^{(ax)} dx = \frac{e^{(ax)} (ax - 1)}{a^2}$$

So that the total integral becomes:

$$\int x^2 e^{(ax)} dx = \frac{x^2 e^{(ax)}}{a} - \frac{2 e^{(ax)} (ax - 1)}{a^3}$$

We now have to substitute $a=-1/H$, and $x=r$. Then take the upper limit, ∞ , minus the lower limit, R_{earth} . Since the exponentials go to zero at ∞ , we just have -1 *this integral evaluated at $r=R_{earth}$. The result is:

$$N_{tot} = 4 \pi n_0 e^{\left(\frac{R_{earth}}{H}\right)} R_{earth}^2 e^{\left(-\frac{R_{earth}}{H}\right)} H - 2 H^3 e^{\left(-\frac{R_{earth}}{H}\right)} \left(-\frac{R_{earth}}{H} - 1\right)$$

Which simplifies to:

$$N_{tot} = 4 \pi n_0 R_{earth}^2 H + 2 H^2 e^{\left(-\frac{R_{earth}}{H}\right)} R_{earth} + 2 H^3 e^{\left(-\frac{R_{earth}}{H}\right)}$$

Just evaluate $H=R_d \cdot T/g = 297 \text{ J/kg/K} \cdot (273+20) \text{ K}/9.8 \text{ m/s}^2 = 8880 \text{ m}$, put in R_{earth} (in metres!) = $6470 \times 10^3 \text{ m}$, and $n_0=2.3 \times 10^{25} \text{ molecules/m}^3$, and you get $N_{tot} = 1.04 \times 10^{44}$ molecules.

SECOND: Making life easier, we can calculate the number of molecules in a column of area 1 m^2 , and then multiply by the surface area of the earth $= 4\pi \cdot R_{\text{earth}}^2$. We have:

$$N(z) = n_0 e^{\left(-\frac{z}{H}\right)}$$

And need to integrate from $z=0$ to $z=\infty$ to get the column density in molecules/ m^2 :

$$N_{\text{col}} = n_0 \int_0^{\infty} e^{\left(-\frac{z}{H}\right)} dz$$

Now this integral is of the form seen on the help sheet:

$$\int_0^{\infty} e^{(-ax)} dx = \frac{1}{a}$$

With $a=1/H$, this becomes $N_{\text{col}}=n_0 \cdot H = 2.3 \times 10^{25} \text{ molecules/m}^3 \cdot 8880 \text{ m}$ and we get $N_{\text{col}}=2 \times 10^{29} \text{ molecules/m}^2$.

When we multiply this by the surface area of the earth:

$A_e=4\pi \cdot R_{\text{earth}}^2 = 5.1 \times 10^{14} \text{ m}^2$, we get $N_{\text{tot}}=N_{\text{col}} \cdot A_e=1.04 \times 10^{44} \text{ molecules}$, as before, with much less maths!

- b) What is the surface pressure if the molar mass of dry air is taken as 27.9 g/mole ?

Po=930 hPa

Again, there are two ways to approach this. The **simplest** is to use the ideal gas law that relates the number density of molecules/ m^3 to the pressure, P, for the surface density given:

$$P_0 = n_0 \cdot k \cdot T$$

Where n_0 is given as $2.3 \times 10^{25} \text{ molecules/m}^3$, T is given as $(273+20)=290 \text{ K}$, and k is Boltzmann's constant given on the help sheet as $k=1.38 \times 10^{-23} \text{ J/K}$. This gives us $P_0=92988 \text{ Pa} = 929.88 \text{ hPa}$.

We could also take a more **masochistic route**, hoping our answer to part a) was correct, and take the column density, N_{col} , of molecules/ m^2 , convert to moles/ m^2 by dividing by Avogadro's number $= 6.02 \times 10^{23} \text{ molecules/mole}$, and then multiplying by the molar mass of dry air given $27.9 \times 10^{-3} \text{ kg/mole}$. This is the total mass per square metre, so $m \cdot g = \text{force per square metre}$, which is the pressure in Pa.

That is, take N_{tot}/A_e to get $N_{\text{col}} = 1.04 \times 10^{44} \text{ molecules}/5.1 \times 10^{14} \text{ m}^2 = 2 \times 10^{29} \text{ molecules/m}^2$. We can convert to moles by dividing by Avogadro's number $= 6.02 \times 10^{23} \text{ molecules/mole}$ to get $3.4 \times 10^5 \text{ moles/m}^2$, and the weight of each mole is $27.9 \times 10^{-3} \text{ kg/mole}$. This gives us 9465 kg/m^2 as the mass of the column, and $M_{\text{col}} \cdot g = F_{\text{col}} = P_0 = 927.6 \text{ hPa}$.

Problem 7) Ozone

In Antarctica, the winter ozone density is 250 DU (Dobson Units). In Spring, this drops to 100 DU in the ozone hole. What is the ozone column density loss (in molecules/m²) from Winter to Spring? Take 1 atm = 1013 hPa

4x10²² molecules/m² are lost in the ozone hole

This requires you to know/lookup the definition of the Dobson Unit. It is effectively a unit of length. If one took all the ozone molecules in the column, and condensed it to standard temperature and pressure (1 atmosphere = 1013 hPa and 0 °C = 273 K), the height in 10⁻⁵ m of this unit-area column is one Dobson Unit (DU). Makes you appreciate SI units, doesn't it. This unit was created by Gordon Dobson, a British meteorologist (of course, you know that Britain is going metric, inch-by-inch) to describe the column amount without all the exponents. The multiplier of 10⁻⁵ m was to give the precision he wanted for the column, where one unit was about the accuracy with which he could measure the column.

So, what is the density of pure ozone under Standard Temperature and Pressure (STP) conditions? Well, it is just the ideal gas law: $P = [n] \cdot k \cdot T$ where $[n]$ is molecules/m³, $P = 101300$ Pa and $T = 273$ K.

But, this density $[n]$ is also the column density in molecules/m² divided by the column height in m, or $N_{col} = [n] \cdot h$. By definition, the $DU \cdot 10^{-5}$ is the column height in m of a gas at STP. So, all we need to do is to calculate the ozone number density (molecules/m³) from the ideal gas law at STP, and then multiply by $DU \cdot 10^{-5}$ m, the column height in m.

So, $[n] = P / (k \cdot T) = 101300 \text{ Pa} / (1.38e^{-23} \text{ J/K} \cdot 273 \text{ K}) = 2.7 \times 10^{25}$ molecules/m³. This means that the column density in Winter is:

$$N_{col_W} = 2.7 \times 10^{25} \text{ molecules/m}^3 \cdot 250 \times 10^{-5} \text{ m} = 6.7 \times 10^{22} \text{ molecules/m}^2,$$

And in Spring:

$$N_{col_S} = 2.7 \times 10^{25} \text{ molecules/m}^3 \cdot 100 \times 10^{-5} \text{ m} = 2.7 \times 10^{22} \text{ molecules/m}^2.$$

So the ozone loss will be $N_{col_W} - N_{col_S} = 4.02 \times 10^{22}$ molecules/m².

The nice thing about the DU length is that since the number density of any gas is always the same at STP, you can just difference the $DU \times 10^{-5}$ m and then multiply by the STP density. Thus, we could say:

$$\text{Ozone loss} = (250 - 100) \times 10^{-5} \cdot 2.7 \times 10^{25} \text{ molecules/m}^3 = 4.02 \times 10^{22} \text{ molecules/m}^2.$$

Or 60% loss of column ozone during the spring ozone hole.

Problem 8) Radiative Equilibrium

- a) Given the black-body temperature of the Sun, what is the equilibrium temperature of Mercury. Mercury has an albedo of 0.068 and no atmosphere to absorb radiation.

$$T_M = 440 \text{ K}$$

For radiative equilibrium, the energy flux into the system is equal to the flux leaving. If there is an atmosphere with short-wavelength transmission TSW and long-wavelength transmission TLW that can interfere with those fluxes, then we have:

$$T_m^4 = \frac{1}{4} \frac{S_m (1 - \alpha) (1 + TSW)}{\sigma (1 + TLW)}$$

S_m is the irradiance of the Sun at the orbit of Mercury, α is the albedo (the energy reflected from the surface and the atmosphere without being absorbed by the system), and σ is the Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}$. This comes from the fact that the irradiance of any body with $T > 0 \text{ K} = \sigma T^4$.

Now, for Mercury we are told there is no atmosphere, so $TSW=TLW=1$. We are given the albedo $\alpha=0.068$, so all we need is S_m , the solar constant of Mercury.

To calculate the irradiance of the Sun at the orbit of Mercury, we can treat the sun as a blackbody at a temperature of 5786 K. Then $\sigma \cdot T^4$ will give you the energy per m^2 at the surface of the Sun. As one gets farther from the Sun, this energy is spread over a larger and larger area, and so the energy per m^2 decreases. To account for this, we take the total energy emitted by the Sun to be:

$$I \text{ (Watts)} = \sigma \cdot T^4 \text{ W/m}^2 \cdot 4\pi(R_s)^2 \text{ m}^2.$$

This energy will expand out spherically in all directions from a central point. When it reaches the radius of the Sun, the energy per m^2 will simply be $\sigma \cdot T^4$. When it reaches a distance further than the radius of the Sun, say the distance of Mercury's orbit, R_{mo} , it will be spread out over $4\pi(R_{mo})^2 \text{ m}^2$. Thus, the irradiance of the Sun at the orbit of Mercury is the $I \text{ (Watts)}$ divided by the surface area of the sphere or radius R_{mo} . So:
 $S_m \text{ (Watts/m}^2) = I \text{ (Watts)} / [4\pi(R_{mo})^2 \text{ m}^2] = \sigma \cdot T^4 \text{ W/m}^2 \cdot 4\pi(R_s)^2 \text{ m}^2 / [4\pi(R_{mo})^2 \text{ m}^2]$
 Is the irradiance of the Sun at the orbit of Mercury.

The solar blackbody temperature, the R_{mo} , and the radius of the Sun are given on the help sheet in terms of Astronomical Units (AU), where

$$1 \text{ AU (Earth-Sun distance)} = 150 \times 10^6 \text{ km.}$$

$$\text{Radius of the Sun} = 695800 \text{ km}$$

$$\text{Mercury-Sun distance} = 0.387 \text{ AU} \cdot 150 \times 10^6 \text{ km}$$

So, putting these together, and cancelling the 4π 's, we have that the solar constant at the orbit of Mercury is:

$$S_m = \sigma \cdot T^4 \cdot (R_s / R_{mo})^2 \text{ W/m}^2 = 9230 \text{ W/m}^2.$$

All that remains is to put $S_m = 9230 \text{ W/m}^2$, $TSW=TLW=1$, and $\alpha=0.068$ into our radiative equilibrium equation to get

$$T_m = 440.1 \text{ K}$$

- b) Given the solar flux at Mars is 586.2 W/m^2 , and the surface temperature is 242 K , what is the long wavelength transmission of the Martian atmosphere. The Martian atmosphere is transparent to short wavelengths, and the planetary albedo is 0.25 .

13% Transmission

Same problem, but we just need to invert

$$T_m^4 = \frac{1}{4} \frac{S_m (1 - \alpha) (1 + TSW)}{\sigma (1 + TLW)}$$

In order to solve for TLW in terms of the (where m=Mars now)

$$S_m = 586.2 \text{ W/m}^2$$

$$T_m = 242 \text{ K}$$

$$TSW = 1.0$$

$$\alpha = 0.25.$$

Now we have:

$$TLW = \frac{1}{4} \frac{S_m (1 - \alpha) (1 + TSW)}{\sigma T_m^4} - 1$$

And plugging in the above numbers we have $TLW = 0.13 = 13\%$

Problem 9) Vertical Structure

In a dry atmosphere with a surface pressure of 1000 hPa

- a) With a temperature profile given by: $T(z \text{ in metres}) = 263\text{K} + 30\text{K} \cdot \exp(-0.0004z)$, what is the buoyancy period in minutes at 8 km .

5.4 minutes

The Buoyancy or Brunt-Väisälä (BV) frequency is the natural period of oscillation that a displaced parcel will undergo when acted upon by the force of gravity and the buoyancy force. Thus, it will be proportional to the difference between a neutrally stable atmosphere (one with an environmental lapse rate $= \Gamma_d = g/C_p$) and the actual environmental lapse rate, $\Gamma_e = -dT/dz$. The expression is:

$$N \left(\frac{\text{rad}}{\text{s}} \right) = \sqrt{\frac{g}{T} \cdot (\Gamma_d - \Gamma_e)}$$

Since the period is $1/f$ (Hz), and $2\pi \cdot f = N$, the BV frequency in Hz is given by $N/(2\pi)$. The BV period is then $\tau(\text{s}) = 2\pi/N$. This then has to be converted to minutes by dividing by 60 s/minute .

So first, what is the atmospheric lapse rate, $\Gamma_e = -dT/dz$. Well we have to differentiate the expression for $T(z)$ with respect to z . This is of the form:

$$-\left(\frac{\partial}{\partial z} T \right) = -\left(\frac{\partial}{\partial z} (a + b e^{(-cz)}) \right)$$

Or

$$-\left(\frac{\partial}{\partial z} T \right) = b c e^{(-cz)}$$

With $a = 263 \text{ K}$, $b = 30 \text{ K}$, and $c = 0.004$. This becomes:

$$\Gamma_e = -\frac{dT}{dz} = 30 \cdot 0.0004 \cdot e^{-0.0004 \cdot z}$$

Or

$$\Gamma_e = 0.0120 \cdot e^{-0.0004 \cdot z}$$

At $z = 8000 \text{ m}$, this gives $\Gamma_e = 0.000490 \text{ K} \cdot \text{m}^{-1}$, and $T = 264 \text{ K}$

Likewise,

$$\Gamma_d = \frac{g}{C_p} = \frac{9.8 \text{ m} \cdot \text{s}^{-2}}{1004 \text{ J} \cdot \text{K}^{-1} \cdot \text{kg}^{-1}} = 0.00976 \text{ K} \cdot \text{m}^{-1}$$

This gives a BV frequency of 0.019 rad/s , or a BV period of $\tau_{\text{BV}} = 5.65 \text{ minutes}$.

- b) Approximating the temperature with $T(z) = 293 - 0.01215 \cdot z$, what is the pressure at 8 km ?

P=334 hPa

Now T is of the form $T(z) = T_0 - \Gamma \cdot z$, with $T_0 = 293 \text{ K}$ and $\Gamma = 0.01215$. Since T is no longer constant in altitude, when we integrate the hydrostatic equation we get:

$$P = P_0 e^{\left(-\frac{g}{R} \int_0^z \frac{1}{T} dz \right)}$$

Now, since T is of the form $a + b \cdot x$, with $a = 293 = T_0$, $b = -0.01215 = -\Gamma$, and $x = z$, the integral in the exponential is of the form (from the help sheet) at the upper and lower limits:

$$\int_0^x \frac{1}{a + b x} dx = \frac{\ln(a + b X) - \ln(a)}{b}$$

Replacing a , b and X with the variables T_0 , $-\Gamma$ and z , we have for P :

$$P = P_0 e^{\left(\frac{g (\ln(T_0 - \Gamma z) - \ln(T_0))}{R \Gamma} \right)}$$

With $P_0 = 100000 \text{ Pa}$, $T_0 = 293 \text{ K}$, $\Gamma = 0.01215 \text{ K/m}$, $g = 9.8 \text{ m/s}^2$, and $R = 297 \text{ J/k/kg}$, we find that at $z = 8000 \text{ m}$ that $P = 334.65 \text{ hPa}$.

- c) Using the approximation of an average temperature of 220 K , what is the height in km above the surface of the 200 hPa level?

The 200 hPa level is 11 km above the surface.

This is a straight application of the hypsometric equation:

$$z_2 - z_o = \frac{R_d \cdot \bar{T}}{g} \cdot \ln\left(\frac{P_1}{P_o}\right)$$

Where $z_o = 0$, $P_o = 100000$ Pa, $P_1 = 20000$ Pa, $\bar{T} = 220$ K, $R_d = 297$ J/k/kg, $g = 9.8$ m/s², we find that $z_2 = 10731$ m = 11 km.

Problem 10) General Stratosphere

The most abundant gas in the stratosphere is:

- a. oxygen (O₂).
- >b. nitrogen (N₂).**
- c. carbon dioxide (CO₂).
- d. ozone (O₃).
- e. chlorofluorocarbons (CFCs).

Don't be fooled, the maximum mixing ratio of ozone is about 5 ppmv at its maximum in the stratosphere. The atmosphere is well mixed, meaning that about 80% of the gas is still nitrogen (N₂).

Problem 11) General stratosphere

In the stratosphere, the air temperature normally:

- a. decreases with increasing height.
- >b. increases with increasing height.**
- c. both increases and decreases depending on the season.
- d. cannot be measured.

Due to solar heating of the ozone, the temperature rises with altitude in the stratosphere up to about 55 km. While the heating per m³ ($\rho \cdot \dot{Q}$) maximizes near the peak of the ozone at 30 km, the heating per kg (\dot{Q}) maximizes near 50 km. In the winter pole, where it is dark, the general meridional circulation creates converging and downward motion that creates a maximum of adiabatic heating in the 40-50 km region. Thus, the temperature rises from the tropopause up to about 55 km, the stratopause.

Problem 12) General Thermodynamics

The processes of condensation and freezing:

- >a. both release sensible heat into the environment.**
- b. both absorb sensible heat from the environment.
- c. do not affect the temperature of their surroundings.
- d. do not involve energy transport.

A bound system is typically at a lower energy level than its unbound constituents. The condensation or freezing of a gas represents forming stronger intermolecular bonds. In the gas phase, there is no energy tied up in intermolecular bonds and all the energy is in the kinetic energy of the molecules. You can think of the condensation or freezing process as an inelastic collision. Before the binding, the two molecules are moving toward each other. The momentum might be zero, but the center of mass energy is

positive. When they collide and bind together, the centre of mass energy is now zero. In a car crash, we know this energy is dissipated by deforming metal, breaking glass, and goes into heating the two bodies with the subsequent heat given off to the surroundings. In molecular collisions, this energy is dissipated by giving off heat to the surroundings. The opposite occurs when evaporating or melting, where energy from the surroundings must be put into the system to separate the bound state. Just like the “jaws of life” have to put energy into the system to separate the two cars fused together in the car crash!

Problem 13) General radiation

If the Earth's average surface temperature were to increase, the amount of radiation emitted from the Earth's surface would ____, and the wavelength of peak emission would shift toward ____ wavelengths.

- >a. **increase; shorter**
- b. increase; longer
- c. decrease; shorter
- d. decrease; longer

From Planck's radiation law, we know that the amount of radiation that a body gives off is proportional to its temperature. If we integrate the Planck equation over all wavelengths and solid angles, we obtain the Stefan-Boltzmann law that the total irradiance is equal to σT^4 Watts/m². Thus, the radiation from the Earth's surface would increase. To find the peak radiance, we differentiate Planck's Radiation law with respect to wavelength and solve for the wavelength that makes the differential zero. This gives rise to Wein's displacement law, that the peak wavelength in μm (10^{-6} m) is $\lambda_{\text{peak}} = (2898 \mu\text{m}\cdot\text{K}) / T$. Thus, higher temperatures will shift the peak of the emission toward shorter wavelengths. Think of a light on a dimmer switch; the less current you send to the light bulb the cooler it is, and the redder or warmer the colour. Increase the current to the light bulb and it heats up, resulting in the light becoming much “whiter”.

Problem 14) General structure

Which is not the case in baroclinic stratification?

- >a. **is the lowest energy state of the atmosphere**
- b. isentropes are not parallel to isobars
- c. potential energy can be converted to kinetic energy
- d. can be caused by horizontal temperature gradients

Barotropic stratification is when the height of a pressure surface does not vary horizontally. As a result, there are no horizontal pressure gradients and, thus, no pressure force to initiate motions. From the hypsometric equation, we know that to keep a uniform vertical spacing between pressure surfaces, the temperature cannot vary horizontally either. Under these conditions, the lines of constant potential temperature (isentropes) are parallel to the lines of constant pressure (isobars), and there is no further conversion of potential energy to kinetic energy.

Baroclinic stratification occurs when there are horizontal gradients in the height of a pressure surface. Again, the hypsometric equation tells us that there must also be

horizontal temperature gradients. Now, there is a pressure gradient force at each altitude that can initiate the conversion of potential energy into kinetic energy, and the temperature gradients cause a displacement of the isentropes from the isobars. Since potential energy can be converted to kinetic energy, this is not the lowest energy state of the atmosphere.

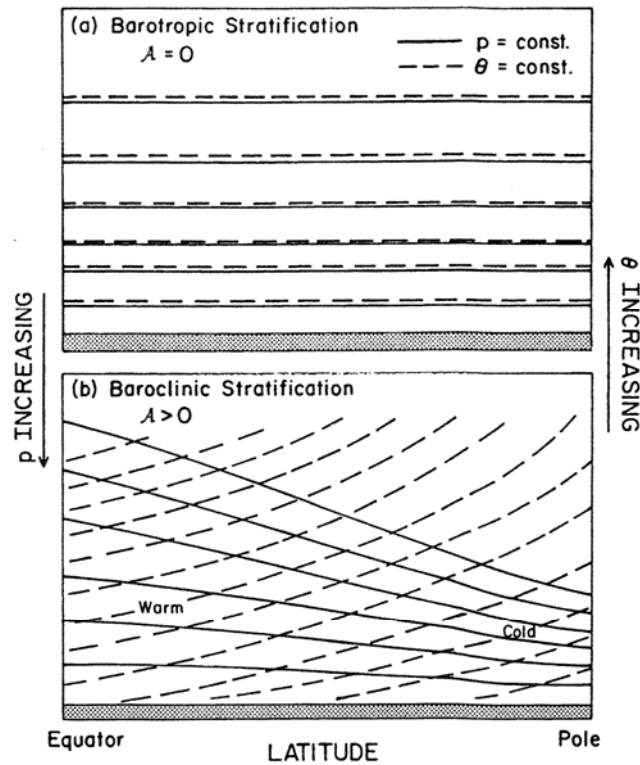


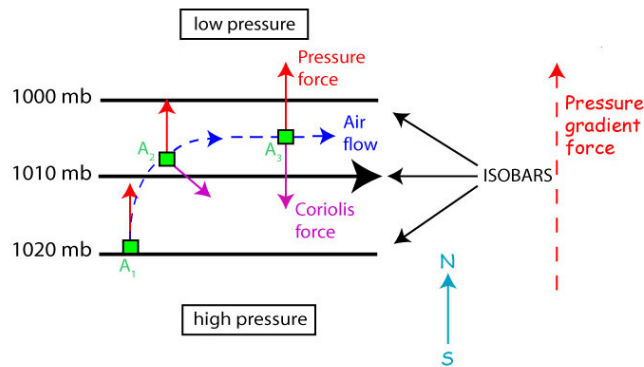
Figure 12.5 Thermal structure corresponding to (a) barotropic stratification, wherein isentropic surfaces coincide with isobaric surfaces and available potential energy \mathcal{A} is zero (Sec. 15.1.3), and (b) baroclinic stratification, wherein isentropic surfaces do not coincide with isobaric surfaces and \mathcal{A} is positive. The rotation of isobaric and isentropic surfaces from their positions under barotropic stratification is symbolic of atmospheric heating at low latitude and cooling at middle and high latitudes (refer to Fig. 1.29c).

14) General dynamics

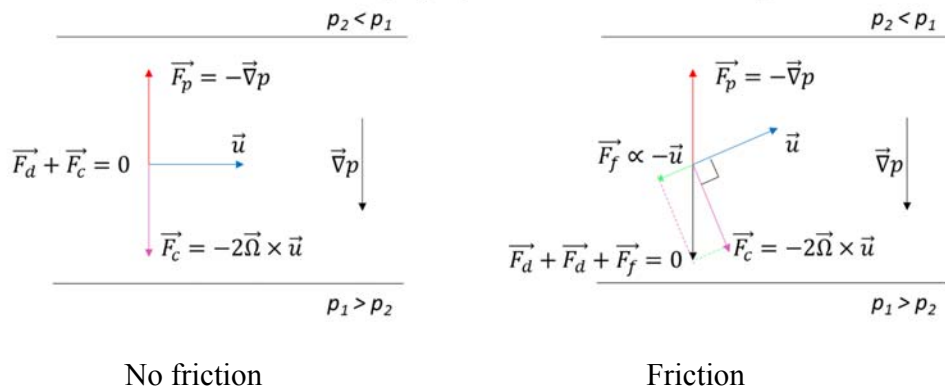
The geostrophic wind results from a balance between:

- a) Coriolis force and centripetal force.
- b) centripetal force, pressure gradient force, and Coriolis force.
- >c) pressure gradient force and Coriolis force.**
- d) pressure gradient force, Coriolis force, and friction.

Geostrophic conditions occur when the wind flow is parallel to the lines of constant pressure (isobars) in steady state. This occurs when the non-dissipative Coriolis force, which always acts perpendicular to the velocity, balances the pressure gradient force.

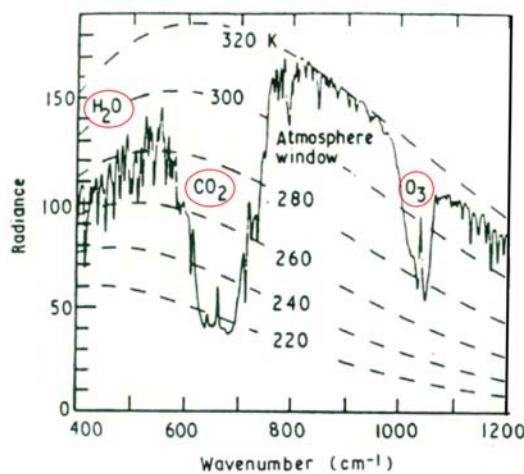


Some might be tempted to say friction must also be a part of the equation. However, friction will be in a direction anti-parallel to the wind. As a result, the pressure gradient force, Coriolis force, and the drag or friction force must balance in steady state, and the Coriolis force is always perpendicular to the velocity. Thus:



Problem 16) General radiation

A downward looking satellite takes the following spectrum of the outgoing long-wavelength radiation from the Earth.



Why does the CO₂ radiate at a blackbody temperature of 220 K while water vapour and ozone radiate at a temperature of 280 K?

>a) CO₂ radiate in a region with a negative lapse rate above H₂O while O₃ radiates above them in a region of positive lapse rate.

b) H₂O and O₃ radiate from the same region of the atmosphere that is warmer than where the CO₂ radiates.

c) CO₂ absorbs more radiation than H₂O and O₃ and so has a deeper minimum.

d) The O₃ and H₂O absorptions are from smog pollution near the surface while the CO₂ absorbs higher in the troposphere.

The CO₂, O₃ and H₂O in the atmosphere all have densities high enough to absorb all the radiation emitted by the Earth within their absorption bands. While they re-radiate that energy in all directions, it is quickly reabsorbed. However, the energy going upward encounters less absorbers, and travels farther before being absorbed than the downward energy. In addition, if the temperature decreases with altitude, there are fewer absorptions and also fewer re-emissions (Kirchoff; emission = absorption).

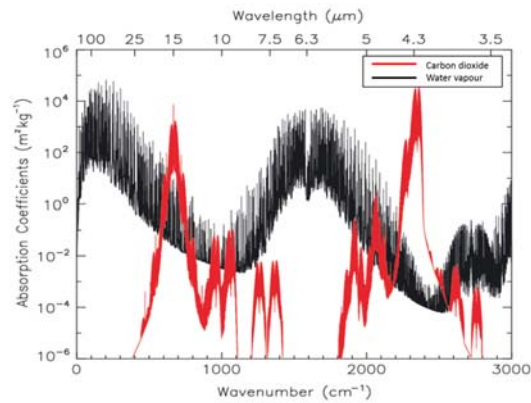
The combination of these reduced absorber amounts (density fall off) and reduced absorber efficiency (temperature fall off) means that eventually there are no absorptions and the energy escapes to space. However, it is emitted to space at a rate proportional to the local temperature of the gas emitting it.

For water vapour (H₂O), the density falls exponentially with the total density for the first 3 km above ground. Then the water vapour density drops sharply due to condensation. Thus, it emits to space from an altitude of about 3 km, where the gas temperature, and hence the emission temperature (σT^4) is about 280 K.

The carbon dioxide (CO₂), however, does not condense and does not react strongly with other gases. Thus, it is well mixed and continues to follow the total density fall-off with altitude up to 75-80 km. That means that it is still optically thick at 3 km (complete absorption and re-emission), and it is not until about 7 km that the density and temperature have fallen enough to minimize the chance of upward radiation being absorbed. Thus it escapes to space from an altitude of around 7 km, where the gas and emission temperature is about 220 K.

Ozone (O₃), on the other hand, is a very minor constituent in the troposphere, but is produced in situ in the stratosphere through the UV dissociation of O₂ into O and subsequent recombination into O₃. It reaches its peak density around 35 km, after which it exponentially falls slightly faster than the total density. However, the UV absorption heats the atmosphere, with a peak heating per kg at 55-60 km. Thus, while the ozone density falls with altitude, its temperature increases. Thus, the upward radiation is no longer absorbed above about 40 km where the gas and emission temperature are 280 K.

This explains the “bite-outs” in the Earth’s blackbody spectrum. For those that think that it is because the CO₂ absorption coefficient is larger than that of water, they are, in fact, nearly the same:



Thus, the correct answer is that CO₂ radiates in a region with a negative lapse rate above H₂O while O₃ radiates above them in a region of positive lapse rate.

Problem 17) General structure

The rate at which temperature decreases with increasing altitude is known as the:

- a. temperature slope.
- >b. lapse rate.**
- c. sounding.
- d. thermocline.

This is nothing more than the definition of lapse rate:

$$\Gamma = \text{lapse rate} = -\frac{dT}{dz}$$

So if the temperature decreases with altitude, it would have a negative slope, but the lapse rate would be positive. Thus, it describes the rate of “decrease” with altitude.