Department of Physics

Examination paper for FY3201 / 8902 Atmospheric Physics and Climate Change

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Permitted examination support material: All support materials are allowed

Academic contact during examination: Patrick Espy
Phone: +47 73 55 10 95

Technical support during examination: Orakel support services:
Link: innsida.ntnu.no/wiki/-/wiki/English/Orakel+Support+Services

Phone: +47 73 59 16 00

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Each incorrect or blank answer is 0 points. Answers have been randomized and are not exact. You must choose the closest answer!

You may take:

Molar mass of dry air: ~29 kg/kmole
Molar mass of helium: ~4 kg/kmole
Molar mass of H_2O: ~18 kg/kmole
Molar mass of CO_2: ~44 kg/kmole

$N_A = 6.02\text{x}10^{23}$ molecules/mole   Boltzmann's constant $k = 1.38\text{x}10^{-23}$ J/K

273 K = 0 °C   1 hPa = 10^2 Pa = 10^2 N m^{-2} 1 atm = 1013 hPa   g=9.8 m s^{-2} constant in z

Stefan–Boltzmann constant: $\sigma = 5.67\times10^{-8}$ $W\cdot m^{-2}\cdot K^{-4}$

Solar photospheric temperature, T_s = 5786 K   Radius of the Sun = 695800 km

Radius of the Earth = 6370 km   1 AU (Earth-Sun distance) =$150\text{x}10^6$ km

Radius of Mercury = 6051 km   Mercury-Sun distance = 0.387 AU

Radius of Mars = 3396 km   Mars-Sun distance = 1.52 AU

Latent heat of vaporization water: $L_v=2.5\text{x}10^6$ J· kg^{-1} Density of liquid water = 1000 $kg\cdot m^{-3}$

Latent heat of sublimation ice: $L_i=2.8\text{x}10^6$ J· kg^{-1} Density of water vapour = $5\text{x}10^{-3}$ $kg\cdot m^{-3}$

Gas constant for water vapour: R_v=461 $J\cdot K^{-1}\cdot kg^{-1}$ Surface tension of water droplet $75\text{x}10^{-3}$ $N\cdot m^{-1}$

Values for dry air: C_p=1004 $J\cdot K^{-1}\cdot kg^{-1}$ C_v=718 $J\cdot K^{-1}\cdot kg^{-1}$ R_d=287 $J\cdot K^{-1}\cdot kg^{-1}$

$\gamma = C_p / C_v$ $\kappa = R_d / C_p$ $R_d=C_p - C_v$ Γ_{da}=9.8 K/km

Clausius–Clapeyron relation: $e_s = 6.112\,hPa \cdot \exp\left[\frac{L_v}{R_v}\left(\frac{1}{273K} - \frac{1}{T}\right)\right]$

Some integrals that may be of use:

$$\int x^m e^{(a x)}\, dx = \frac{x^m e^{(a x)}}{a} - \frac{m \int x^{(m-1)} e^{(a x)}\, dx}{a}$$

$$\int x\, e^{(a x)}\, dx = \frac{e^{(a x)} (a x - 1)}{a^2}$$

$$\text{For a > 0} \quad \int_0^{\infty} e^{(-a x)}\, dx = \frac{1}{a}$$

$$\int \frac{1}{a + b x}\, dx = \frac{\ln(a + b x)}{b}$$

1. Geostrophic flow occurs when the wind blows

(1) parallel to the isobars or contours or pressure.
(2) in the direction of the Coriolis force.
(3) in the direction of the pressure gradient force.
(4) at an angle between 10 and 30 to the contours and towards low pressure.
(5) at an angle between 10 and 30 to the contours and towards high pressure.

In geostrophic flow, the pressure gradient force is balanced by the Coriolis force. When this happens, the wind ceases to change, and results in the wind blowing parallel to the isobars.

2. Of the gases listed below, which is NOT believed to be responsible for enhancing the earth's greenhouse effect?
a. molecular oxygen (O2)
b. chlorofluorocarbons (CFCs)
c. nitrous oxide (N2O)
d. carbon dioxide (CO2)
e. methane (CH4)

A greenhouse gas has high transmission in the visible, but absorbs appreciably in the infrared. Since infrared wavelengths interact with molecular vibrations and rotations, the molecule must possess a permanent dipole moment, or one that is induced by vibration or rotation. O2 is a symmetric (homonuclear) molecule that does not possess a permanent dipole moment. Since the only vibrations are along the internuclear axis, one cannot be induced. Hence it cannot interact with infrared light.

3. The most abundant gas in the stratosphere is:

a. nitrogen (N2).
b. oxygen (O2).
c. carbon dioxide (CO2).
d. ozone (O3).
e. chlorofluorocarbons (CFCs).

Ozone is never the major species, but always a minor one. In the well-mixed atmosphere is N_2 always the major species.

4. How do you find the Lifting Condensation Level (LCL) on a Skew-T diagram?

a. Find the intersection between a parcel’s dry adiabat and constant $\mu_s=\mu$ of the parcel
b. Find a parcel’s water vapour mixing ratio using its dew point temperature.
c. Find the region where the temperature starts to rise with altitude.
d. Find where the atmospheric lapse rate is equal to the dry adiabatic lapse rate.

The LCL is where the water vapour contained in the parcel, μ, may begin to condense. The parcel will therefore follow the dry adiabat until it reaches the point where $\mu_s=\mu$. This point can then be found by tracing the isopleth of μ_s (= μ) until it intersects the parcels dry adiabt.

5. If the earth's average surface temperature were to increase, the amount of radiation emitted from the Earth's surface would ____, and the wavelength of peak emission would shift toward ____ wavelengths.

a. increase; shorter
b. increase; longer
c. decrease; shorter
d. decrease; longer

The total energy emitted is found from Stefan-Boltzmann relation F(w/m2) = $\sigma_{SB}*T^4$, where σ_{SB} is a constant. The peak wavelength of an emission is given by Wein displacement law λ_{peak}=Constant/T. Therefore, an increase in temperature will result in a shorter wavelength of peak emission.

6. a) A spherical drop of water of radius 20 nm is stable (neither growing nor shrinking) at a temperature of 15 C. what is the relative humidity around the droplet?

For a drop, one must use Kelvin's formula where ρ_L= density of liquid water, 1000 kg/m^3, R_v is the specific gas constant for water vapour, 461 J/K/kg, and r is the radius of a stable drop (the turning point is the free energy equation)

$$e = e_s \, \mathrm{e}^{\left(2\frac{\gamma}{\rho_L R_v T r}\right)}$$

Here, γ is the surface tension = 75x10^{-3} kg/m. The equilibrium or saturated partial pressure of water vapour over a flat surface of water, e_s, is given as

$$e_s = 6.112 \, \mathrm{e}^{\left(\frac{L_v\left(\frac{1}{273}-\frac{1}{T}\right)}{R_v}\right)}$$

Where the latent heat of vaporization, L_v =2.5x10^6 J/kg. Substituting in these values for T=(15+273)K and a droplet radius of 20x10^{-9} m gives: e_s = 17.63 hPa, and e = 18.66 hPa.
The relative humidity is therefore e/e_s = 106%

b) An airplane economy cabin is pressurized to 818 hPa with a temperature 20 C and a relative humidity of 15% when flying at 11 km. What is the dew point temperature in the cabin?

The dew point temperature is the temperature where the partial pressure of water vapour in the air, e, is equal to the saturation vapour pressure, $e_s(T_d)$. The saturation vapour pressure is as above, and if we substitute $e=e_s$, and $T=T_d$, we can solve for T_d

$$T_d = 273. \frac{L_v}{L_v - 273. \ln(.1636125654\, e)\, R_v}$$

Where L_v ad R_v are given as above. We still need e, but this is given by RH=e/e_s(T), where e_s(T) is, as before:

$$e_s = 6.112\,\mathrm{e}^{\left(\frac{L_v\left(\frac{1}{273}-\frac{1}{T}\right)}{R_v}\right)}$$

At a temperature of T=(20+273)K, the saturated partial pressure of water vapour is e_s=24.50 hPa. With a relative humidity of 15%, the partial pressure of water vapour, e = 0.15·24.5 hPa = 3.67 hPa. Given this, we can calculate the dew point temperature to be: T_d = 266.3 K = -6.66 C.

c) A person perspires. How much liquid water (as a percentage of the person's mass) must evaporate to lower the temperature of the person by 5.0 C. Take the specific heat of the human body to be that of water, C_{pw} = 4200 J/kg/K.

Evaporating a mass of water, M_w, will release an amount of heat, dQ = $L_v \cdot M_w$ into the environment
Losing an amount of heat, dQ, from a body of mass M_p with a heat capacity C_{pw}= 4200 J/kg/K will drop its temperature by dQ= $C_{pw} \cdot M_p \cdot dT$
Equating the dQ's and solving for M_w/M_p= $dT \cdot C_{pw}/L_v$
For dT=5 K (remember a change in temperature is the same in K as in C) and the values for C_{pw} and L_v we get that M_w/M_p= 0.0075=0.75%

7. An exoplanet orbits its star at a distance R_{orbit}= 0.41 AU, and has a radius R_p=1.34*R_{earth}. The planet has an albedo of 0.4 and emissivity of 1.

The star it orbits has a radius R_{st}= 0.6*R_{sun} and a photosphere blackbody temperature of T_{st}=4400 K

a. What is the planet's equilibrium temperature assuming it has no atmosphere?

We need to know the "stellar" constant for this planet. that is the W/m^2 it puts on the planet. The irradiance W/m^2 at the photosphere of the star is given (with σ_{SB} = Stefan Boltzmann constant):

$$F_{st} = \sigma_{SB} \cdot T_{st}^4$$

The star's total power in watts is this times the area of the photosphere. This is the luminosity of the point source radiating in all directions (4π sr).

$$L_{st} = 4\pi \cdot R_{st}^2 \cdot \sigma_{SB} \cdot T_{st}^4$$

At the planet, this luminosity is distributed over a sphere 4π*$(R_{orbit})^2$ giving the planet's stellar constant as:*

$$F_p = \frac{R_{st}^2}{R_{orbit}^2} \cdot \sigma_{SB} \cdot T_{st}^4$$

We can calculate this intermediate step and find that the stellar constant for this planet is 979.3 W/m^2
Now the planet absorbs (1-albedo) of this stellar flux over its cross-sectional area of π $(R_p)^2$, and radiates over its surface area = 4*π*R_p^2 as $\varepsilon \cdot \sigma_{SB}$*$(T_p)^4$*
At equilibrium, energy in = energy out and the radius of the planet cancels:

$$\pi \cdot R_p^2 \cdot F_p(1-a) = 4\pi \cdot R_p^2 \cdot \epsilon \cdot \sigma_{SB} \cdot T_{st}^4$$

We can solve for T_{st} and find:

$$T_p = \frac{1}{2}\,\frac{\left(\sqrt{2}\,(-F_p\,(-1+a)\,\varepsilon^3\,\sigma_{SB}^3)\right)^{\left(\frac{1}{4}\right)}}{\varepsilon\,\sigma_{SB}}$$

Using the values given, Tp=225.6 K = -47.4 C

b. At what wavelength does the radiance of this star with a blackbody temperature of 4400 K peak?

For this we can use Wiens displacement law for a stellar temperature of 4400 K, the peak wavelength in microns is:

$$\lambda = 2898\,\frac{1}{Tst}$$

λ=0.6586 μ = 658.6 nm

c. Another exoplanet in the same system has a stellar flux constant F_p of 800 W/m^2 at its orbit, The planet has an albedo of 0.4, and emissivity of 1, but there is a compound in the atmosphere that passes all infrared light but absorbs in the visible.
A probe has told us that the temperature of the surface of this planet is -85 C. What is the short wavelength transmission of the atmosphere of this planet?

We could re-derive the radiative equilibrium equation with an atmosphere, but if we take the short and long wavelength transmissions to be TS and TL, we know from the class notes that a planet at temperature Tp, with albedo a and stellar flux Fp, has a radiative balance equation given by:

$$\varepsilon\,\sigma\,Tp^4 = \frac{1}{4}\,\frac{(1-a)\,Fp\,(1+TS)}{1+TL}$$

We can solve for TS,

$$TS = \frac{4(1+TL)\cdot\epsilon\cdot\sigma_{SB}\cdot T_p^4}{(1-a)\cdot F_p} - 1$$

And find that TS=0.18 = 18%

8. Sunlight coming into the Earth's atmosphere can be absorbed and heat the atmosphere.
The Sun is at a solar ZENITH angle (angle of the Sun from overhead) of χ= 45 degrees. You may assume no scattering, and take the optical depth of incoming solar radiation (averaged over all wavelengths), the density of air, and the mass mixing ratio of the absorber to be defined as:

$$\tau(Z) = \int_Z^{\infty} \rho(z)\,\mu(z)\,k\,dz$$

$$\rho(z) = \rho 0\,\mathbf{e}^{\left(-\frac{z}{Hm}\right)}$$

$$\mu(z) = \mu 0\,\mathbf{e}^{\left(-\frac{z}{Hv}\right)}$$

Where $\rho(z)$ is the mass density of air, $\mu(z)$ is the mass mixing ratio of the component in the air that attenuates the radiation, and k is the mass absorption coefficient.

The air density at the surface, ρ_0= 1.2 kg/m^3 and the scale height of air is H_m=7 km. In addition, $\mu 0$=0.01 (kg of absorbing component)/(kg air), and its scale height H_v=4 km. Finally, the mass absorption coefficient, k is constant and k=0.11 m^2/(kg of absorbing compound).

a) Calculate the optical depth τ and its derivative with height $d\tau/dz$ at a height of 5 km in the atmosphere

So, the first thing is to substitute in for ρ and μ into the integral

$$\tau = \int_0^\infty \rho 0\, e^{\left(-\frac{z}{Hm}\right)} \mu 0\, e^{\left(-\frac{z}{Hv}\right)} k\, dz$$

This is not too hard to integrate as the exponents of the exponentials add, and can be factored out. With k a constant, we get:

$$\tau = \frac{\rho 0\, \mu 0\, k\, Hm\, Hv\, e^{\left(-\frac{Z\,(Hv+Hm)}{Hm\,Hv}\right)}}{Hv+Hm}$$

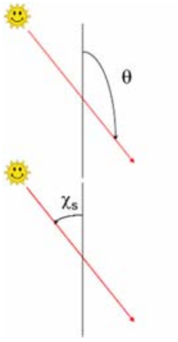
and of course, its derivative

$$\frac{d\tau}{dZ} = -\rho 0\, \mu 0\, k\, e^{\left(-\frac{Z\,(Hv+Hm)}{Hm\,Hv}\right)}$$

Substituting in the values given yields:

τ *= 0.47, and* $\frac{d\tau}{dZ} = -0.00019\ m^{-1} = -0.19\ km^{-1}$

b) If the incoming solar flux at the top of the atmosphere is 400 W/m^2, and the solar ZENITH angle, χ, is 45 degrees, calculate the NET UPWARD flux of solar radiation at 7 km if the optical depth is 0.2 at this altitude



First things first. The ZENITH angle vs μ=cos(θ) is shown here:
As you can see cos(χ) = -μ. Therefore, we can use the equation for downward flux from the short wavelength heating lecture, but the one for χ instead of μ. This is given by:

$$F_\nu^{\downarrow}(z) = -F_{\nu\infty}^{\downarrow}\cdot\mu\cdot e^{\frac{\tau(z)}{\mu_s}} \quad = \quad F_{\nu\infty}^{\downarrow}\cdot\cos(\chi_s)\cdot e^{-\frac{\tau(z)}{\cos(\chi_s)}}$$

Where $F_{\nu\infty}^{\downarrow} = 400\ W\cdot m^{-2}$
All we have to do now is pay attention to the fact that we want the net UPWARD flux, which is:

$$F_\nu(z) = \left[F_\nu^{\uparrow}(z) - F_\nu^{\downarrow}(z)\right] = F_{\nu\infty}^{\downarrow}\cdot\mu\cdot e^{\frac{\tau(z)}{\mu_s}} = -F_{\nu\infty}^{\downarrow}\cdot\cos(\chi_s)\cdot e^{-\frac{\tau(z)}{\cos(\chi_s)}}$$

Putting in the numbers, we get the net upward flux F_ν(Z=7km) = -213 W/m^2.

c) Calculate the heating rate in Kelvin/day due to short wavelength radiation at 3 km if the downward solar flux at the top of the atmosphere is 400 W/m^2.
The solar ZENITH angle is χ=45 degrees, and at 3 km, the downward solar flux is 70 W/m^2, the density is ρ=0.78 kg/m^3, the optical depth is τ=1 and its derivative, $d\tau/dz$, is -0.4 km^{-1}.

Well, the heating rate in W/kg, when divided by the specific heat Cp in J/kg/K, will give the heating rate in K/sec. That heating rate is given in terms of the net upward flux as:

$$\frac{\partial}{\partial t}Q = -\frac{\frac{\partial}{\partial z}F_\nu(z)}{\rho(z)\,Cp}$$

And in the previous part (b.) we have an expression for $F_\nu(z)$*:*

$$F_\nu(z) = -F^{\downarrow}_{\nu\infty}\cdot\cos(\chi_s)\cdot e^{-\frac{\tau(z)}{\cos(\chi_s)}}$$

When we differentiate this with respect to z, we get:

$$\frac{\partial}{\partial t}Q = -\frac{-F^{\downarrow}_{\nu\infty}\left(\frac{\partial}{\partial z}\tau(z)\right)e^{\left(-\frac{\tau(z)}{\cos(\chi)}\right)}}{\rho(z)\,Cp}$$

Now, kindly old professor Espy actually gave you everything you need to calculate this without differentiating τ*. That's because you needed to do that in part a. and if you didn't, then you shouldn't lose points twice! Plugging in the numbers, you get:*

$$\frac{dQ}{dt} = 0.00005\frac{K}{s} = 0.003\frac{K}{min} = 0.17\frac{K}{hr} = 4\frac{K}{day}$$

d) In an isothermal atmosphere of temperature 220 K, at what pressure level will the Doppler-line width be the same as the collision-line width at a wavelength of 15 microns.

We can use the formula for line shape as a function of pressure and temperature given in lecture

$$\gamma_L = 0.01\,cm^{-1}\cdot\left(\frac{p}{1000\,hPa}\right)\left(\frac{273\,K}{T}\right)^{1/2}$$

$$\gamma_D = 0.003\,cm^{-1}\cdot\left(\frac{\nu}{1000\,cm^{-1}}\right)\left(\frac{T}{300\,K}\right)^{1/2}$$

Equate them and solve for P. We of course need to turn 15 μ *into wavenumbers in* cm^{-1}*. That comes out to be* $\nu = 1/(15\times10^{-6}\,m\cdot100\,cm/m) = 666.7\,cm^{-1}$*.*
If we equate γ_L *and* γ_D*, and solve for P, we get:*

$$P = 0.3\frac{\nu\cdot T}{\sqrt{273\cdot300}}$$

At 666.7 cm-1 and 220 K, we get a pressure P=154 hPa

9. On Venus g=8.87 m/s^2, the specific gas constant for dry air is R_{dv} = 195.5 J/K/kg, and the specific heat at constant pressure, Cp = 846 J/K/kg.
The temperature profile is given by T=T$_o$*exp(-a*Z), with T$_o$=783K and a=1.68e-5 m^{-1}
The surface pressure is P$_o$ = 93320.33 hPa, and the atmosphere behaves like a dry ideal gas.

a. What is the buoyancy period at 59 km

The Brunt frequency, or buoyancy frequency, in radians/sec is given by:

$$N = \sqrt{\frac{g\left(\frac{g}{Cp} + \frac{dT}{dz}\right)}{T}}$$

Of course, we have a different g, Cp, and we need to calculate T(z=59 km) and dT/dz there. We are asked for a period, and the brunt period is given by:

$$T_{Brunt} = \frac{2\pi}{N} \text{ in seconds}$$

Putting in the values for Venus, and calculating the temperature and temperature gradient:

$$T = To\, e^{(-a\, z)} \text{ and } \frac{dT}{dz} = -To \cdot a \cdot e^{(-a \cdot z)}$$

At 59 km, we get values of T=291 K, $\Gamma_d = g/Cp = 0.01\,K/m$, $\Gamma_{env} = -\frac{dT}{dz} = 0.005\ K/m$, *giving N=0.013 rad/sec and* $T_{Brunt} = 480.5\ s = 8$ *minutes*

Just as a note, at about 59 km, Venus has a temperature and pressure about the same as at the surface of the Earth!

b. Is the atmosphere stable with respect to vertical motions at 59 km?

It is stable, because $N^2 > 0$*, so that a displaced parcel will return to its original position, oscillating under the influence of gravity and buoyancy. Some answered conditionally stable or conditionally unstable. No information is given from which one could deduce this. However, if the 2 points from this answer changed the grade then it was given.*

c. Is the atmosphere stable with respect to vertical motions at the surface?

Here we have to calculate N^2 *for the surface conditions. Now we find that at the surface, we get values of T=783 K,* $\Gamma_d = g/Cp = 0.01\,K/m$, $\Gamma_{env} = -\frac{dT}{dz} = 0.013\ K/m$, *giving* $N^2 = -0.00003$ *rad/s. So it is unstable since N would be imaginary, and a displaced parcel would continue in the direction of the displacement*

d. What is the Scale Height, H, at 59 km and at the surface?

Hopefully, you did not have the same brain fart that I did during the spørretime and remembered that:

$$H = \frac{R\,T}{g} = \frac{R\,To\, e^{(-a\,Z)}}{g}$$

Where R is now R_{dV} and g is appropriate for Venus, at 59 km, this gives H=6.4 km, and at the surface H=17.3 km.

e. What is the pressure at 59 km?

The hydrostatic equation is $\frac{dP}{dz} = -g \cdot \rho$. *Substituting in for* ρ *using the ideal gas law,* $P=\rho RT$*, we get that:*

$$\frac{dP}{P} = -\frac{g}{R \cdot T(z)} \cdot dz$$

Or

$$P = Po\,\mathrm{e}^{\left(\int_0^Z \frac{g}{R\,\mathrm{T}(z)}\,dz\right)}$$

But we know that T(z) is, $\mathrm{T}(z) = To\,\mathrm{e}^{(-a\,z)}$ *, so we need to integrate:*

$$\int_0^Z -\frac{g\,\mathrm{e}^{(a\,z)}}{R\,To}\,dz = -\frac{g\,(\mathrm{e}^{(a\,Z)}-1)}{R\,To\,a}$$

So, now P is:

$$P = Po\,\mathrm{e}^{\left(-\frac{g\,(\mathrm{e}^{(a\,Z)}-1)}{R\,To\,a}\right)}$$

Putting in the appropriate values (including $R=R_{dV}$*), we get P=27072 Pa = 270 hPa = 27 kPa*

f. If a parcel of air at the surface, where the pressure is P_o=9332033 Pa, is moved adiabatically to an altitude of 700 m where the pressure is 0.96*P_o, what is the parcel's final temperature?

This is our old friend the potential temperature, and the temperature T at pressure P is:

$$Tp = \frac{\theta}{\left(\frac{Po}{P}\right)^{\kappa}}$$

Where θ *is the temperature at our reference pressure, Po. We do need to calculate the new value for* κ *on Venus, however. That would be* $\kappa=R_{dV}/C_{pV}$ *= 0.23.*
If we choose our reference pressure at the surface, where we know the temperature is To = θ*, we can calculate the temperature of the parcel at P = 0.96 Po. We get that Tp=776 K.*
Note, the atmospheric temperature at 700 m is 773 K, so cooler than the parcel. Thus the atmosphere is unstable with regard to vertical motion.

10. Sulfur dioxide, (SO_2) is a pollutant released in combustion of coal and other high-sulfur fuels that can cause acid rain. Although Norway produces very little, it is transported by the prevailing winds to Norway from large industrial regions in Europe as well as from volcanic sources in Iceland. Thus, its concentration in Norway is highly variable between 1 and 25 ppbv (parts per billion by volume).

a. At 1000 hPa and 22 C, the mixing ratio of SO_2 measured one day in Bergen was 5 ppbv. What was the mass per unit volume of SO_2?

We will need the number of SO_2 *molecules and multiply by the weight of each molecule. To get the number of molecules, we can go back to the definition of the mixing ratio:* $\nu = \frac{N_{SO_2}}{N_{total}}$*, and we can obtain* N_{total} *from the ideal gas law:* $N_{total} = \frac{P}{kT}$*, where k is Boltzmann's constant. Thus,* $N_{SO_2} = \frac{\nu P}{kT}$*. We have everything we need with P=100000 Pa, T=22+273=295K, and* ν*=5e-9 giving:*

$$N_{SO_2} = 1.23x10^{17} molecules \cdot m^{-3}.$$

We can verify this yields the correct mixing ratio by calculating $N_{total}=2.46x10^{25}$ *giving*

$$\nu = \frac{1.23\times10^{17}}{2.46\times10^{24}} = 5\,ppb_v$$

Next, we need the mass of an SO_2 *molecule. From the periodic table, this is* $32+2\cdot16=64\ g\cdot mole^{-1}$*, and there are* $6.02x10^{23}$ *molecules*$\cdot$*mole*$^{-1}$*. This means the mass per unit volume, or density,* ρ*, is:*

$$\rho_{SO_2} = \frac{1.23x10^{17} molecules \cdot m^{-3}}{6.02\times10^{23}\ molecules \cdot mole^{-1}} \cdot 64\ g \cdot mole^{-1} \cdot 10^{-3}\ kg \cdot mole^{-1}$$

This gives a density of $\rho_{so_2} = 1.31\times10^{-8}\ kg\cdot m^{-3} = 13.1\ \mu g\cdot m^{-3}$

b. If a mass of 50 μg/m^3 of SO_2 is entirely converted into sulfuric acid, H_2SO_4, what is the resulting mass per volume of sulfuric acid at 1000 hPa and 22 C?

We could start over, or recognize that:
The number of sulphur atoms S = number of SO_2 *molecules = number of* H_2SO_4 *molecules.*
Instead of each S atom being associated with a 64 g/mole SO_2 *molecule, it is associated with a molecule of* $H_2SO_4 = 1 + 32 + 4*16 = 98$ *g/mole. We can take the 50 micrograms/m*^{3} *of* SO_2*, multiply by the ratio of molecular weights: 98/64 to get mass* H_2SO_4*:*

$$\rho_{H_2SO_4} = 50\times10^{-9}\ kg\cdot m^{-3}\cdot\frac{98}{64} = 7.66\times10^{-8}\ kg\cdot m^{-3} = 76.6\ \mu g\cdot m^{-3}$$

c. Originally, the Martian atmosphere was thought to consist only of 95% CO_2, 3% N_2 and 2% Ar. However, recently, it was discovered that it also contains 0.2% O_2. By how much did the initial estimate of the mean molecular weight of the Martin atmosphere change when this additional species, O_2 was taken into account.

The molecular weight of a mixture of gases is the sum of the products of the mixing ratios of each component times its molar mass $M_m = \Sigma\nu_i\cdot M_{mi}$*. For Mars, we can get the molar masses from the periodic table where we have that* M_m *of* $CO_2=44$*,* $N_2 = 28$*, Ar, = 40, and for later,* $O_2 = 32$ *all in g/mole.*
The molecular weight of the Martian atmosphere without O2 is therefore:

$$M_{m_{no\,O_2}} = 0.95\cdot44 + 0.03\cdot28 + 0.02\cdot40 = 43.44\ g\cdot mole^{-1}$$

Now, if we replace 0.2% of this mixture with O_2*, we calculate the new molecular weight by:*

$$M_{m_{O_2}} = \left(1-\nu_{O_2}\right)\cdot M_{m_{no_{O_2}}} + \ \nu_{O_2}\cdot M_{M_{O_2}} = (1-0.002)\cdot43.44 + 0.002\cdot32 = 43.41\ g\cdot mole^{-1}$$

Thus, the molecular mass has gone down, decreasing by 0.03 kg/kmole