NTNU
Department of Physics

Exam FY3490 Observational Astrophysics

Lecturer: Prof. Manuel Linares

Examination date: June 2nd 2022.

Examination time (from-to): 9am – 12pm

Permitted examination support material: Approved calculator + formula sheet

Other information: This exam accounts for 50% of the final grade. Total score: 10 points.

Official formula sheet provided. Read carefully. Good luck!

Problem 1 [2 points]. An "interloper" in optical astronomy is a star whose angular position in the sky is very close to our science target (say < 0.5"). The interloper (star 1) and science target (star 2) appear very close in an optical image, even if they can be far from each other.

a. In bad observing conditions (e.g. with a seeing > 1.5"), we cannot resolve the two stars and we measure an apparent magnitude $m_3 = 18.64$ for the resulting blend of the two sources. Calculate the true apparent magnitude of our science target, m_2, knowing that the interloper has an apparent magnitude $m_1 = 20$. [1 p]

b. Give the distance to the interloper (in kpc), if it has an absolute magnitude M = 5 and the extinction in this band is A = 0.5 mag. [0.5 p]

c. If we observe from space with a 1m telescope (avoiding atmospheric turbulence), what sets the minimum/best possible angular resolution? In which wavelength range can we observe to resolve an interloper which is 0.3" from our target? [0.5 p]

Problem 2 [2 points]. Consider the declination (δ), maximum altitude (h) and zenith angle (z) of a star, as seen from a geographical latitude L.

a. Derive the relation between h, δ and L. What is the minimum declination observable from Barcelona (L = 41° N) above the horizon? And at an altitude of at least 30°? [0.5 p]

b. In the common plane-parallel atmosphere approximation, calculate the dimensionless airmass (column density relative to its minimum value towards the zenith) for a zenith angle z = 35°. [0.5 p]

c. Calculate the fraction of the sky which is visible throughout a full year from Barcelona (L = 41° N) above the horizon. [1 p]

Problem 3 [2 points]. On a particular night, the planet Mars has an angular diameter of 15 arcsec and an energy flux of 1.0×10^{-7} W m^{-2}. Two astronomers observe the planet, using identical CCD cameras whose pixels are 25 μm apart. Albert uses a telescope of 0.3m aperture whose focal ratio is f/8. Bertha uses a telescope of 30 m aperture whose focal ratio is f/4.

a. Calculate the surface brightness (energy flux per unit area in the sky) of Mars in units of W m^{-2} arcsec^{-2}. [0.5 p]
b. How much energy accumulates in a single pixel of Albert's CCD image of Mars in a 100 s exposure? Give your answer in Joules. [0.75 p]
c. How much energy (in J) accumulates in the same time in a single pixel of Bertha's image of Mars? [0.75 p]

Question 4 [1 point]. The bolometric peak luminosity of thermonuclear bursts is $[3.79 +/- 0.15] \times 10^{38}$ erg/s. Observing a newly discovered neutron star, we measure during a thermonuclear burst a bolometric peak flux of $[6.7 +/- 0.7] \times 10^{-8}$ erg/s/cm^2.

a. Assuming that this is a standard candle, calculate the distance and its uncertainty, both in kiloparsecs (kpc). [0.5 p]
b. Assuming that the measurements are normally distributed and that the quoted uncertainties represent 1-sigma confidence regions: estimate the probability that the newly discovered source is more than 7.64 kpc away. [0.5 p]

Question 5 [1 point]. In a long-slit optical spectrum:

a. If the dimensionless spectral resolution is R=1000, what is the resolution in Å and in km/s at the Hydrogen alpha line (6563 Å)? [0.25 p]
b. If the spectrum covers the range 3650-7110 Å with 2035 pixels, what is the average dispersion? [0.25 p]
c. From your answers above, how many pixels sample one full resolution element? [0.25 p]
d. State the Nyquist criterion and, based on it, explain whether this spectrum is over, under or critically sampled. [0.25 p]

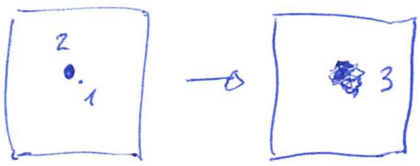
Question 6 [1 point]. A CCD pixel has 95000 e-. If the full-well depth of the pixel is 100000 e-, the gain is 1.4 e-/ADU and a 16-bit ADC is used, will the pixel be saturated? Explain why.

Question 7 [1 point]. Optical and X-ray telescopes.

a. Draw schematically a Cassegrain reflector telescope, showing the primary and secondary mirrors and the Cassegrain focus. [0.5 p]
b. Explain briefly the difficulties in focusing X-rays and the physical principle used to overcome them. [0.5 p]

OBS - ASTRO - EXAM

P1



a)

(I) $m_1 - m_2 = 2{,}5 \log \frac{F_2}{F_1}$

(II) $m_1 - m_3 = 2{,}5 \log \frac{F_3}{F_1}$

(III) $F_3 = F_1 + F_2$ (0.25p)

$m_3 = 18{,}64$ mag

$m_1 = 20$ mag

$m_2 = ?$

II & III :

$$m_1 - m_3 = 2{,}5 \log\left(\frac{F_1 + F_2}{F_1}\right) = 2{,}5 \log\left(1 + \frac{F_2}{F_1}\right)$$

$$\rightarrow \frac{F_2}{F_1} = 10^{\frac{m_1 - m_3}{2{,}5}} - 1 = 2{,}5$$

I :

$$\rightarrow \boxed{m_2 = m_1 - 2{,}5 \log \frac{F_2}{F_1} = 19{,}01 \simeq 19 \text{ mag}}$$

b)

$$m - M = 5 \log d_{pc} - 5 + A$$

$$\boxed{d_{pc} = 10^{\left(\frac{m - M + 5 - A}{5}\right)} = 7943 \text{ pc} \simeq 7{,}9 \text{ kpc}}$$

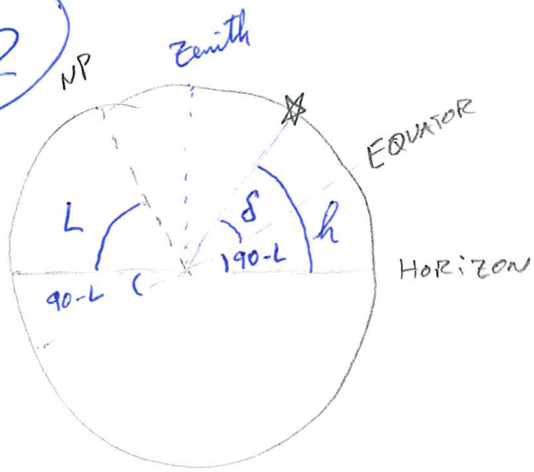
c) DIFFRACTION LIMIT. AIRY DISC RADIUS:

$$\theta_{MIN} = 1{,}22\,\frac{\lambda}{D} \Longrightarrow \lambda = \frac{D \cdot \theta_{MIN}}{1{,}22} = 1{,}2 \times 10^{-6}\ m$$

$$\theta_{MIN} = 0{,}3'' = 1{,}45 \times 10^{-6}\ rad$$

$$\boxed{\lambda \lesssim 1{,}2\ \mu m}$$

P2



DECLINATION : δ

ALTITUDE : h

ZENITH ANGLE : z

LATITUDE : L

a) $\Rightarrow$ $\boxed{h = 90 - L + \delta}$

AT HIGHEST POINT:

- $h = 90 - L + \delta > 0°$

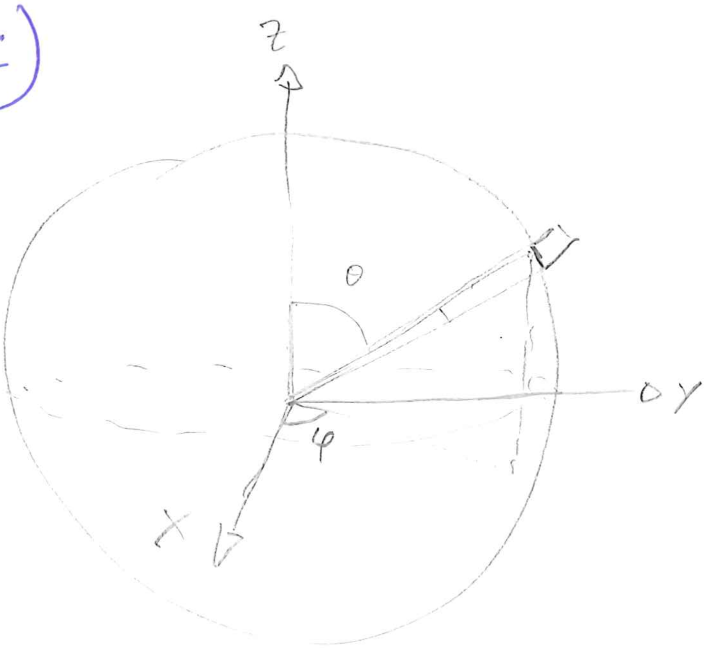
  $\Rightarrow \delta > L - 90° = -49°$

- $h = 90 - L + \delta > 30°$

  $\Rightarrow \delta > L - 60° = -19°$

b) $X \simeq \dfrac{1}{\cos z} = 1{,}22$

c)



BCL ($L = 41°$)

above horizon:

$\delta > -49°$

$\Rightarrow \theta = 90 + 49 = 139$

$\theta < 139°$

if only $\theta < 90 + 41 = 141°$

$\Rightarrow 0{,}5\,\checkmark$

$d\Omega = \sin\theta \, d\theta \, d\varphi$

VISIBLE SKY: $\Omega_{vis} = \int_{vis} d\Omega = \underbrace{\int_0^{2\pi} d\varphi}_{2\pi} \underbrace{\int_{\theta=0°}^{\theta=\theta_{max}} \sin\theta \, d\theta}_{} = 2\pi(1 - \cos\theta_{max})$

$-\cos\theta \Big|_0^{\theta_{max}} = \underbrace{\cos 0}_{1} - \cos\theta_{max}$

FULL SKY: $\Omega_{SKY} = 4\pi$

$\Rightarrow$ FRACTION: $\dfrac{\Omega_{vis}}{\Omega_{SKY}} = \dfrac{1 - \cos\theta_{max}}{2} = 0{,}877$

87,7 %

P3

d $\oslash$ $\pi\left(\frac{d}{2}\right)^2$

25 μm

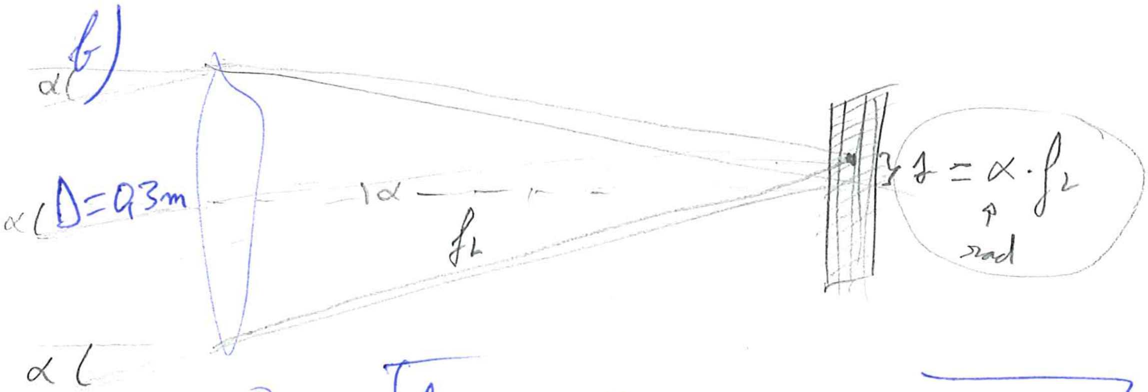
25 μm

$$\begin{cases} d = 15'' \\ F = 10^{-7}\ W\,m^{-2} \end{cases}$$

$$R = \frac{f_L}{D}$$

a) $\boxed{B = \frac{F}{A} = \frac{4F}{\pi d^2} = 5{,}66 \times 10^{-10}\ W\,m^{-2}\,arcsec^{-2}}$

b)



$R = 8$

$(f/8)$

$\boxed{f_L = D \cdot R = 0{,}3\,m \cdot 8 = 2{,}4\,m}$ 0,25

AREA PIXEL : 25 μm × 25 μm.

ANGULAR " " : $\Delta\theta \times \Delta\theta$ where $\Delta\theta = \frac{\Delta x}{f_L} = 1{,}0416 \times 10^{-5}$ rad

$\times \frac{180°}{\pi\,rad} \times \frac{3600''}{1°} \rightarrow \boxed{\Delta\theta = 2{,}15''}$

$$E = B \times A_{TEL} \times A_{Pix} \times T = B \times \pi\left(\frac{D}{2}\right)^2 \times \Delta\theta^2 \times T = \boxed{1{,}85 \times 10^{-8}\ J}$$

c) NEW $\left\{ \begin{array}{l} D = 30 \text{ m} \\ R = 4 \end{array} \right\} \rightarrow f_L = D \cdot R = 120 \text{ m}$ 0,25

$$\Delta\theta = \frac{\Delta x}{f_L} = 2{,}08 \times 10^{-7} \text{ rad} = 0{,}0429''$$

$$E = B \times A_{TEL} \times A_{PIX} \times T = 7{,}36 \times 10^{-8} \text{ J}$$

Q4

$$L_{peak} = (3,79 \pm 0,15) \times 10^{38}\ erg/s$$

$$F_{peak} = (6,7 \pm 0,7) \times 10^{-8}\ erg/s\,cm^2$$

$$F = \frac{L}{4\pi d^2} \Rightarrow d = \sqrt{\frac{L}{4\pi F}} = 2,12 \times 10^{22}\ cm = 6875\ pc$$

$$\underline{6,88\ kpc}$$

$$\delta d = \sqrt{\delta d_F^2 + \delta d_L^2}$$

$$\delta d_F = \frac{\partial d}{\partial F}\delta F = \frac{-1}{2} \cdot \underbrace{\sqrt{\frac{L}{4\pi F}}}_{d} \cdot \frac{\delta F}{F}$$

$$\delta d_L = \frac{\partial d}{\partial L}\delta L = \frac{1}{2}\underbrace{\sqrt{\frac{L}{4\pi F}}}_{d}\ \frac{\delta L}{L}$$

$$\Rightarrow \frac{\delta d}{d} = \sqrt{\frac{1}{4}\left(\frac{\delta F}{F}\right)^2 + \frac{1}{4}\left(\frac{\delta L}{L}\right)^2} = \frac{1}{2}\sqrt{\left(\frac{\delta F}{F}\right)^2 + \left(\frac{\delta L}{L}\right)^2} = 0,0556$$

$$\frac{1}{2}\sqrt{(0,104)^2 + (0,0396)^2} \qquad (5,6\%)$$

$$\Rightarrow \delta d = 383\ pc = 0,38\ kpc$$

$$\Rightarrow \underline{d = 6,88 \pm 0,38\ kpc = 6,9 \pm 0,4\ kpc}$$

0,25 p. 0,25 p.

b)

$$\frac{x-\bar{x}}{\sigma} : \frac{7,64-6,88}{0,38} = 2$$

"2 sigma":

Probab (> +2σ deviation):

From Table 6.2 : $P = \frac{0,0455}{2} = 0,0228$

2,2%

2,3%

factor 2 off: 0,25 p.

Correct reasoning but wrong σ : 0,5 p.

Q5

a) $R = 1000 = \frac{\lambda}{\Delta\lambda} \rightarrow \underline{\Delta\lambda} = \frac{\lambda}{R} = \underline{6{,}6\ \text{Å}}$

$\frac{\Delta v}{c} = \frac{\Delta\lambda}{\lambda} \rightarrow \underline{\Delta v} = \frac{c}{R} = 299{,}8\ \text{km/s}$

$\underline{\simeq 300\ \text{km/s}}$

b) $\boxed{\langle disp \rangle = \frac{7110 - 3650}{2035} = 1{,}7\ \text{Å/pix}}$

c)

$\frac{\Delta\lambda = 6{,}6\ \text{Å}}{disp = 1{,}7\ \text{Å/pix}} = \underline{3{,}86 \simeq 3{,}9\ \text{pixels}}$

d) Nyquist: $V_{sampl} \geq V_{Nyq}$ (> 2 pixels per resolution element)

(slightly) <u>OVER SAMPLED</u>

Q6

95.000 e^- < 100.000 e^- FULL-WELL DEPTH ✓

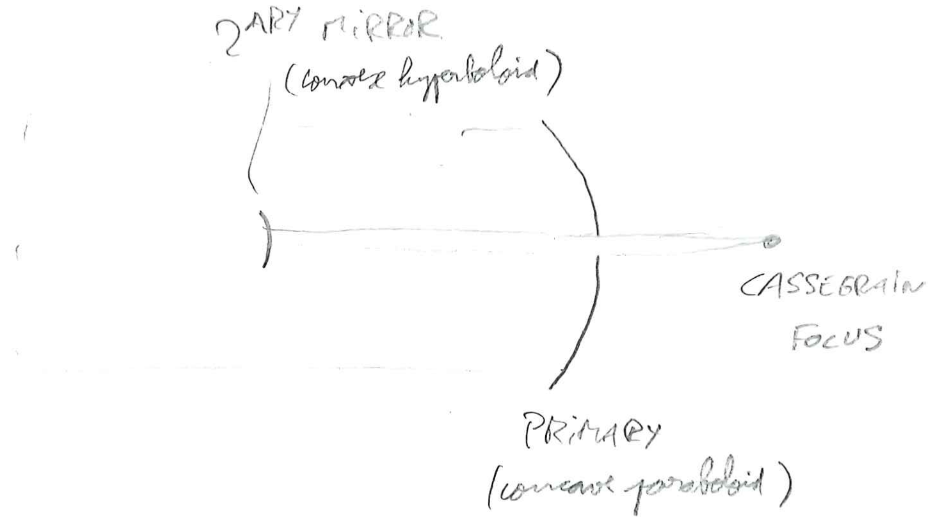
~~...~~ PIXEL "COUNTS" $= \frac{95000 e^-}{1,4 e^-/ADU} = \underline{\underline{67857 \text{ ADU}}}$

16-bit ADC: $2^{16} - 1 = \underline{65535 \text{ MAX}}$

COUNTS > MAX ⇒ <u>SATURATED</u>

Q7

a)



b) X-rays penetrate matter / metals, reflected fraction drops drastically with increasing photon energy.

GRAZING INCIDENCE MIRRORS are used to focus X-rays, since reflection is still efficient at very small incident angles ($\ll 1°$).

This results in long focal lengths...