(I)

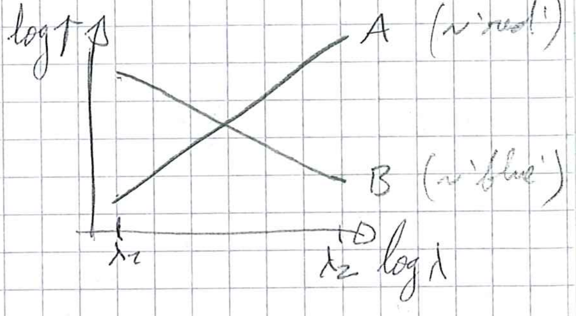
EXAM - OBSERVATIONAL ASTROPHYSICS - MAY 15, 2023

P1

2 sources, same band: 400-600 nm $\left\{ \begin{array}{l} \lambda_1 = 400\,nm = 0{,}4\,\mu m \\ \lambda_2 = 600\,nm = 0{,}6\,\mu m \end{array} \right\}$

SPECTRAL PHOTON FLUX DENSITY: $\left[\frac{phot}{cm^2 \cdot s \cdot \mu m}\right]$ (*)

$$\left\{ \begin{array}{l} f_{\lambda A} = f_A(\lambda) = A\,\lambda^3 \\ f_{\lambda B} = f_B(\lambda) = B\,\lambda^{-2} \end{array} \right\}$$



- Assume flat response: $\varepsilon(\lambda) = constant\ (\equiv 1)$
- Area of telescope: A_{TEL} (but not needed).

Note: λ in microns, μm (*)

a) The two sources will "generate photoelectrons" at a rate ("count rate") R:

$$\frac{R_A}{A_{TEL}} = \int_{\lambda_1}^{\lambda_2} f_A(\lambda)\,d\lambda = A \int_{\lambda_1}^{\lambda_2} \lambda^3\,d\lambda = A\left[\frac{\lambda^4}{4}\right]_{0,4}^{0,6} = \underbrace{\frac{0{,}6^4 - 0{,}4^4}{4} \cdot A}_{0{,}026 \cdot A} \quad \left[\frac{counts}{s\,cm^2}\right]$$

(0,15)

$$\frac{R_B}{A_{TEL}} = \int_{\lambda_1}^{\lambda_2} f_B(\lambda)\,d\lambda = B \int_{\lambda_1}^{\lambda_2} \lambda^{-2}\,d\lambda = B\left[-\lambda^{-1}\right]_{0,4}^{0,6} = \underbrace{\left(\frac{1}{0{,}4} - \frac{1}{0{,}6}\right)B}_{0{,}833 \cdot B} \quad \left[\frac{counts}{s \cdot cm^2}\right]$$

But they "generate photoelectrons at exactly the same rate":

"normalized" $-0{,}1_p$

(0,15) $$R_A = R_B \Rightarrow \boxed{\frac{A}{B} = \frac{0{,}833}{0{,}026} = 32{,}05} \quad \left[\frac{1}{\mu m^5}\right] (*)$$

b) Brightness ratio F_B/F_A? Now it's energy flux: $\times\, h\nu = \frac{hc}{\lambda}$

$$F_A = \int_{\lambda_1}^{\lambda_2} f_A(\lambda) \cdot h\nu \cdot d\lambda = Ahc \int_{\lambda_1}^{\lambda_2} \lambda^2\,d\lambda = Ahc\left[\frac{\lambda^3}{3}\right]_{0,4}^{0,6} = 0{,}05067 \cdot Ahc$$

(0,15)

$$F_B = \int_{\lambda_1}^{\lambda_2} f_B(\lambda) \cdot h\nu \cdot d\lambda = Bhc \int_{\lambda_1}^{\lambda_2} \lambda^{-3}\,d\lambda = Bhc\left[-\frac{\lambda^{-2}}{2}\right]_{0,4}^{0,6} = 1{,}736 \cdot Bhc$$

$$\Rightarrow \boxed{\frac{F_B}{F_A} = \frac{1{,}736\,B}{0{,}05067\,A} = 1{,}069}$$

(7% difference in F leads to the same R)

(wrong units but OK otherwise: $0{,}05_p$)

$\boxed{\text{NOTE:}}$ (SPECTRAL) PHOTON FLUX DENSITY $\longrightarrow$ (SPECTRAL) ENERGY FLUX DENSITY

$$p(\lambda) \quad \times \frac{hc}{\lambda} \rightarrow f_\lambda = f(\lambda) \text{ "f-lambda"}$$

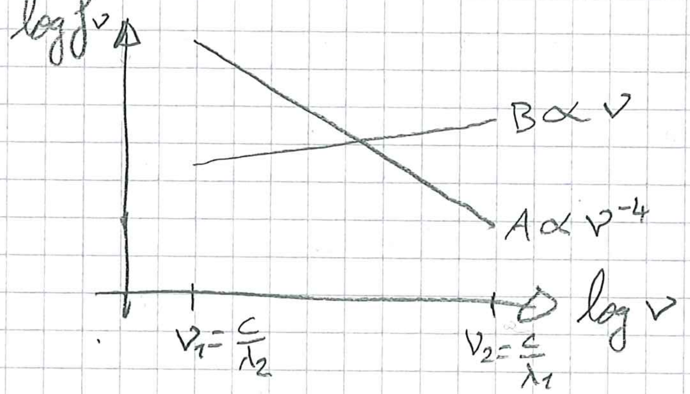
$$f_\nu = \frac{\lambda^2}{c} f_\lambda$$

(0,15)

$$p(\nu) \quad \times h\nu \rightarrow f_\nu = f(\nu) \text{ "f-nu"}$$

IF you want to complicate things, this can be also done in ν space:

$$p_A(\lambda)\cdot h\nu \cdot \frac{\lambda^2}{c} = f_A(\nu) = \frac{Ahc^4}{\nu^4} \quad ; \quad f_B(\nu) = \frac{Bh}{c}\nu = p_B(\lambda)\cdot h\nu\cdot\frac{\lambda^2}{c}$$



$$F_A = \int_{\nu_1}^{\nu_2} f_A(\nu)\,d\nu = \dots = \frac{Ahc}{3}\cdot 9{,}152$$

$$F_B = \int_{\nu_1}^{\nu_2} f_B(\nu)\,d\nu = \dots = \frac{Bhc}{2}\cdot 3{,}472$$

$\rightarrow$ And reach the same $\frac{F_B}{F_A} = 1{,}069$

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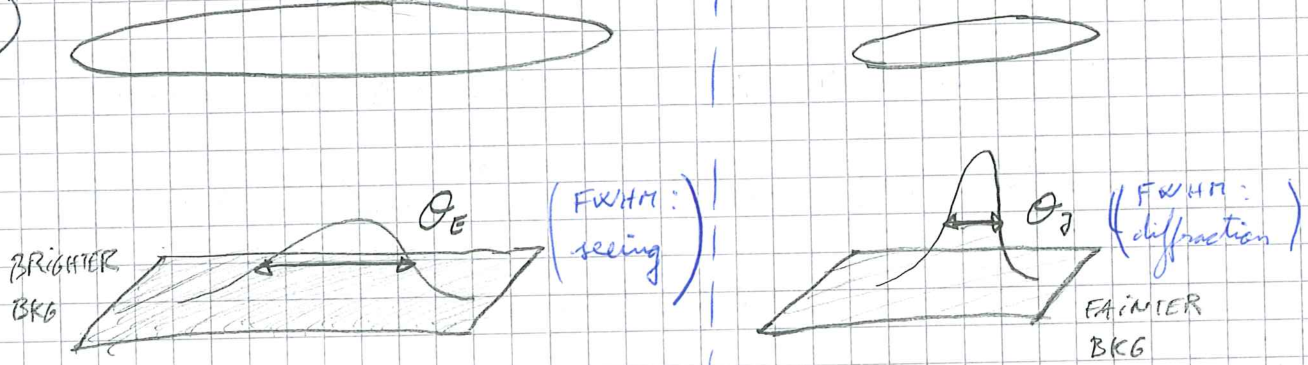
P2

ATMOSPHERE: TURBULENCE | SPACE

ELT: $D_E = 24\,m$ | JWST: $D_J = 6{,}5\,m$



TO DETECT FAINT SOURCES: PROS & CONS!

• APERTURE:

⊕ PRO (larger $A_E \propto D_E^2$) | ⊖ CON (smaller $A_J \propto D_J^2$)

• BACKGROUND:

⊖ CON (brighter background lowers S/N) | ⊕ PRO (fainter background boosts S/N)

brighter due to atmosphere scattering (e.g. moon light)

"The background for the ELT is 2 magnitudes per square arcsec brighter"

Background flux per square arcsec: B

$$\Rightarrow \underbrace{b_J - b_E}_{\text{"SURFACE BRIGHTNESS"}} = 2\,\frac{mag}{arcsec^2} = 2{,}5 \log \frac{B_E}{B_J} \Rightarrow \frac{B_E}{B_J} = 10^{\frac{2}{2{,}5}} \simeq 6{,}31$$

(0,25 p)

• POINT-SPREAD FUNCTION (PSF):

With a better (smaller) PSF, the source counts are distributed over a smaller area, so the S/N increases (less background counts in that area).

⊖ CON (seeing spreads counts over larger area) | ⊕ PRO (diffraction-limited PSF is narrower)

"Magnitude threshold" implies FLUX LIMIT : what is the faintest source flux we need to do something? ("certain application": e.g. detect a source).

- SAME APPLICATION : SAME SIGNAL TO NOISE RATIO (SNR).

Recall: we measure TOTAL (N_T) and BACKGROUND (N_B) counts and then:

$$N_S = N_T - N_B \Rightarrow \sigma_S^2 = \underbrace{\sigma_T^2}_{N_S+N_B} + \underbrace{\sigma_B^2}_{N_B} = N_S + 2N_B$$

Poisson : $N_S + N_B$ $\quad N_B$

AT THE FAINT SOURCE LIMIT, BACKGROUND DOMINATES : $N_B \gg N_S$:

- So the NOISE : UNCERTAINTY ON N_S : $\sigma_S \simeq \sqrt{2N_B} \propto \sqrt{B \cdot A \cdot \theta^2}$

"is proportional to $\sqrt{\text{collected background counts in source region}}$"

B: bkg flux per arcsec^2; A: collecting area; θ: angular area source (PSF)

- While our SIGNAL are the net source counts :

$$N_S = K\,F \cdot A \propto F \cdot D^2$$

(0,25 p)

K: constant: bandpass, detector; F: flux density; A: collecting area

$$\Rightarrow \boxed{SNR \simeq \frac{N_S}{\sigma_S} \propto \frac{F \cdot D^2}{\sqrt{B \cdot A \cdot \theta^2}} = \frac{F}{\sqrt{B}} \cdot \frac{D}{\theta}}$$

$$SNR_J = SNR_E \Leftrightarrow \frac{F_E}{\sqrt{B_E}} \cdot \frac{D_E}{\theta_E} = \frac{F_J}{\sqrt{B_J}} \cdot \frac{D_J}{\theta_J}$$

Ratio of flux limits : $\frac{F_E}{F_J} = \left(\frac{B_E}{B_J}\right)^{1/2} \cdot \frac{D_J}{D_E} \cdot \frac{\theta_E}{\theta_J} = 10^{1/2,5} \cdot \frac{6,5}{24} \cdot 4 = 2,72$

Magnitude difference : $m_J - m_E = 2,5 \log \frac{F_E}{F_J} = 1,087$ (0,25 p)

$$\Rightarrow \boxed{m_E = 26,9 \text{ mag}}$$

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II[1]

... P2

NOTE: INCOMPLETE ALTERNATIVES:

A1 Consider only aperture: $N_s \propto D^2 F$ (P 1.6 class)

$E \propto D^2 F$

"same N_s": $F_E \pi \left(\frac{D_E}{2}\right)^2 \Delta t = F_J \pi \left(\frac{D_J}{2}\right)^2 \Delta t$

$$\frac{F_E}{F_J} = \left(\frac{D_J}{D_E}\right)^2 \Rightarrow \boxed{m_E = m_J - 2{,}5 \log \left(\frac{D_J}{D_E}\right)^2 = 30{,}8 \text{ mag}}$$

× (+0,5 p)

A2: Consider aperture & PSF: $\frac{N_S}{\text{arcsec}^2} \propto \frac{D^2 F}{\theta^2}$

"same $\frac{N_S}{\text{arcsec}^2}$" ... $\rightarrow \frac{F_E}{F_J} = \left(\frac{D_J}{D_E}\right)^2 \left(\frac{\theta_E}{\theta_J}\right)^2 \Rightarrow \boxed{m_E = m_J - 5 \log\left(\frac{D_J}{D_E}\right) - 5 \log\left(\frac{\theta_E}{\theta_J}\right) = 27{,}8}$

× (+1 p)

A3: Consider aperture, PSF & background: $\frac{N_S}{N_B} \propto \frac{D^2 F}{B \theta^2}$

"same $\frac{N_S}{N_B}$" ... $\rightarrow \frac{F_E}{F_J} = \frac{B_E}{B_J} \left(\frac{D_J}{D_E}\right)^2 \cdot \left(\frac{\theta_E}{\theta_J}\right)^2$

$$\Rightarrow \boxed{m_E = m_J - 5 \log \frac{D_J}{D_E} - 5 \log \frac{\theta_E}{\theta_J} - 2{,}5 \log \frac{B_E}{B_J} = 25{,}8}$$

× (+1,5 p.)

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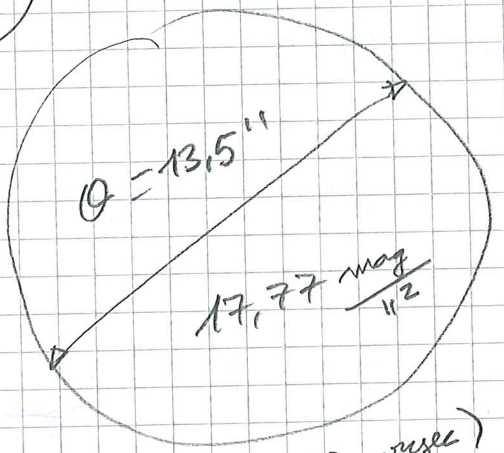
P3



Uniform nebula with a surface brightness

$$m_S = 17,77 \ \frac{mag}{arcsec^2}$$

Recall: flux is additive (energy/photon), magnitudes are not!

$$F_{nebula} = \pi \left(\frac{\theta}{2}\right)^2 \cdot F_{1as}$$

where F_{1as} is the flux in 1 arcsec^2.

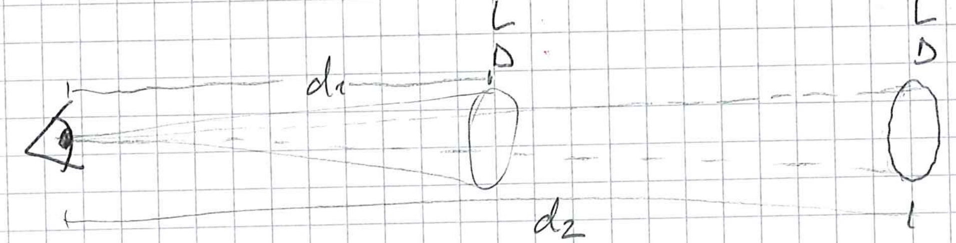
a)

$$m_S - m = 2,5 \log \frac{F_{nebula}}{F_{1as}} = 2,5 \log \left(\pi \left(\frac{\theta}{2}\right)^2\right) = 5,389$$

$$\Longrightarrow \boxed{m = m_S - 5,39 = 12,38 \ mag}$$

wrong order: 0,25 p.

b)



$$d_2 = 2 d_1$$

$$\left(\begin{array}{l} A_1 = 143,1''^2 \\ A_2 = 35,8''^2 \end{array}\right)$$

$$D = d_1 \theta_1 = d_2 \theta_2 \Longrightarrow \frac{\theta_2}{\theta_1} = \frac{d_1}{d_2} = \frac{1}{2}$$

$$A = \pi \left(\frac{\theta}{2}\right)^2 \propto \theta^2 \Longrightarrow \boxed{\frac{A_2}{A_1} = \left(\frac{\theta_2}{\theta_1}\right)^2 = \left(\frac{d_1}{d_2}\right)^2 = \frac{1}{4}}$$

angle but no area: 0,25

The angular area decreases by a factor $\frac{1}{4}$.

c) Flux $F \propto \frac{L}{d^2}$; $m_2 - m_1 = 2,5 \log \frac{F_1}{F_2}$

$$\frac{F_1}{F_2} = \left(\frac{d_2}{d_1}\right)^2 = 4 \longrightarrow m_2 - m_1 = 1,505 \text{ mag}$$

$$\Rightarrow \boxed{m_2 = 12,38 + 1,505 = 13,89 \text{ mag}}$$

The apparent magnitude increases by $\simeq 1,5$ mag (since it becomes fainter).

d)

TOTAL FLUX DECREASES $\times \frac{1}{4}$

ANGULAR AREA " $\times \frac{1}{4}$

$\frac{\text{FLUX}}{\text{AREA}} = \text{constant}$

Surface brightness stays constant!

- correct "increase" or "decrease" without quantifying: 0,25 p.

IV

EXAM - OBSERVATIONAL ASTROPHYSICS - MAY 15, 2023

Q4

The number of pixels to read would be simply 8192 × 8192.

At a pixel readout speed of 25 kHz this would take

$$\boxed{t = \frac{(8192)^2}{25 \times 10^3\ \mathrm{Hz}} = 2684\ \mathrm{s} \simeq 45\ \mathrm{min}}$$

factor 3 (charge transfer): −0,25

Q5

a) (see L3.2 class notes)

MADE:

- A metal box with inert gas (Ar, Xe), crossed by a:
- wire/anode at high voltage (attracting e^-), open from one side with a:
- thin transparent ($E \gtrsim 2$ keV) window.

OPERATE:

- Photoelectric effect ionizes/ejects inner-shell e^- from gas
- ionization track is formed (outer shell e^- from other atoms): AMPLIFY.
- CHARGE COLLECTED @ ANODE is PROPORTIONAL to E (incoming X-ray photon)

b) $$\nu_{\mathrm{Nyquist}} \equiv \frac{1}{2 \cdot t} = 500\ \mathrm{Hz} \quad (t = 1\ \mathrm{ms})$$

(see P5.3)

Interpretation: signals with $\nu > \nu_{\mathrm{Nyquist}}$ can't be reliably/fully detected.

c) For $T = 1024$ s:
Frequency resolution, first term in Fourier power spectrum @: $\nu = \frac{1}{T} = 9.8 \times$ $\simeq 1$ mHz

Q6

$$\begin{cases} \Delta = 15\ cm = 150\ mm \\ \lambda = 650\ nm \end{cases} ; \quad R = \frac{f_L}{\Delta} = 10$$

"f/10"

⟹ DIFFRACTION-LIMITED RESOLUTION:

Radius Airy disk: $\boxed{\theta_{min} = 1,22 \frac{\lambda}{\Delta}}$

$$\boxed{\theta_{min} = 1,22 \frac{650 \times 10^{-9}\ m}{15 \times 10^{-2}\ m} = 5,29 \times 10^{-6}\ rad \simeq 1,1''}$$

0,5 p.

⟹ AT THE FOCAL PLANE:

Focal length: $f_L = R\Delta = 1500\ mm$

0,5 p.

$$\to \boxed{s_{min} = \theta_{min}(rad) \cdot f_L = 7,94\ \mu m} = \boxed{7,9 \times 10^{-3}\ mm}$$

Q7

In space: cosmic rays can hit the CCD at almost any angle.

On Earth: from its surface the atmosphere prevents ("shields from") nearly all glancing angle cosmic-ray impacts.

(CCDs are typically "facing up" to the sky)