NTNU Trondheim, Institutt for fysikk

Examination for FY3403 Particle Physics
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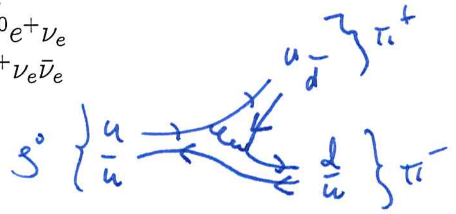
Allowed tools: mathematical tables, pocket calculator
Some formulas and data can be found on p. 2ff.

1. Interactions.

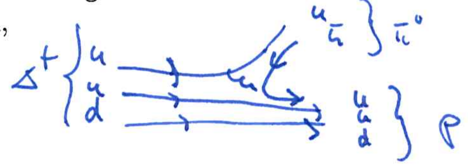
Draw the lowest order Feynman diagram(s) for the following decays or scatterings, if they exist. Name the interaction(s) which are responsible or the reason why the process is forbidden:

a.) $\rho^0 \to \pi^+\pi^-$ (2 pts)
b.) $e^+e^+ \to \mu^+\mu^+$ (2 pts)
c.) $\Delta^+ \to p\pi^0$ (2 pts)
d.) $K^+ \to \pi^0 e^+\nu_e$ (2 pts)
e.) $K^+ \to \pi^+\nu_e\bar{\nu}_e$ (2 pts)

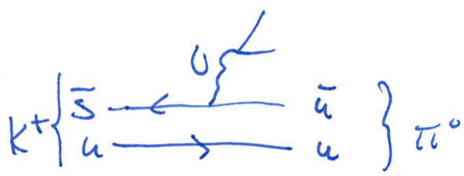
a.) strong,



b) forbidden, since elecromagnetic and neutral current conserve lepton number L_e and L_μ
c.) strong interaction,



d.) weak interaction,



e.) FCNC, forbidden in the SM

2. Scattering in QED.

a.) Consider elastic electron-muon scattering $e^- + \mu^+ \to e^- + \mu^+$. Draw the lowest order Feynman diagram(s) and write down the Feynman amplitude $\mathcal{M}$ in momentum space. (4 pts)
b.) Calculate $|\bar{\mathcal{M}}|^2$ and show that it can be written in the form

$$|\bar{\mathcal{M}}|^2 = \frac{e^4}{q^4} L_{\mu\nu} H^{\mu\nu}$$

where $L_{\mu\nu}$ and $H^{\mu\nu}$ are determined by the electron and the muon current, respectively. (20 pts)
c.) Consider now the case of elastic electron-proton scattering $e^- + p \to e^- + p$ with $H^{\mu\nu}$ now determined by the proton current. Use instead the tensor method, i.e. express $M^{\mu\nu}$

and

$$\psi_R = \frac{1}{2}(1+\gamma^5)\psi = \frac{1}{2}\begin{pmatrix} \chi+\phi \\ \chi+\phi \end{pmatrix}.$$

b.) Left-chiral quark fields live in the fundamental representation of $SU(2)_L$, i.e. $Q = (u,d)_L^T$, while right-chiral fields are singlets, i.e. u_R and d_R. Singlets are invariant under $SU(2)_L$ transformation, $u'_R = u_R$ and $d'_R = d_R$, while the doublets tranform as

$$Q \to Q' = \exp\left\{\frac{\mathrm{i}\boldsymbol{\alpha}\cdot\boldsymbol{\tau}}{2}\right\} Q$$

where $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \alpha_3\}$ parameterise the transformation, and the generators $\boldsymbol{T} = \boldsymbol{\tau}/2$ are given by the Pauli matrices. Specialising to the case $\boldsymbol{\alpha} = \{0, 0, 2\pi\}$, it is

$$U = \exp\{\pi \mathrm{i}\tau_3\} = \begin{pmatrix} \mathrm{e}^{\mathrm{i}\pi} & 0 \\ 0 & \mathrm{e}^{-\mathrm{i}\pi} \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Thus

$$u' = u'_L + u'_R = -u_L + u_R = \frac{1}{2}\begin{pmatrix} -\chi+\phi+\chi+\phi \\ \chi-\phi+\chi+\phi \end{pmatrix} = \begin{pmatrix} \phi \\ \chi \end{pmatrix}.$$

4. Abelian Higgs model.

Consider the free complex scalar field ϕ with the Lagrangian

$$\mathscr{L} = (\partial_\mu\phi)^\dagger(\partial^\mu\phi) + \mu^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2. \tag{1}$$

a) Show that the Lagrangian is invariant under global U(1) transformations. (4 pts)

b.) Show that converting normal into covariant derivatives makes this Lagrangian invariant under local U(1) transformations. (6 pts)

c.) Write out the part of the Lagrangian describing interactions between the scalar and the U(1) gauge boson. Draw the resulting interaction vertices between the scalar and the U(1) gauge boson. [Just drawings, the mathematical expressions for the Feynman rules are not needed.] (5 pts)

d.) Consider now the complete Abelian Higgs model, i.e. including also the free Lagrangian $-F_{\mu\nu}F^{\mu\nu}/4$ of the gauge field. Explain the differences between the two cases $\mu^2 > 0$ and $\mu^2 < 0$. (8 pts)

a.) The scalar ϕ transforms as

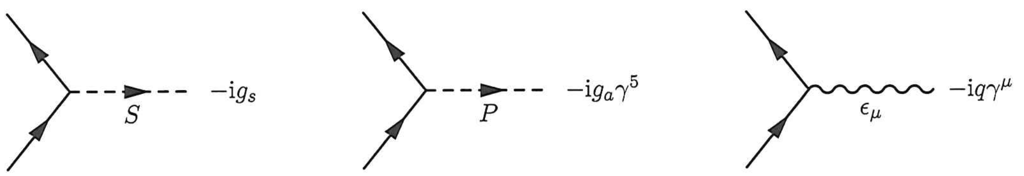
$$\phi \to \phi' = U\phi = \exp\{\mathrm{i}\vartheta\}\phi,$$

while $\phi^\dagger$ transform as

$$\phi^\dagger \to \phi^\dagger \exp\{-\mathrm{i}\vartheta\}.$$

Since for global transformations ϑ is constant and $U^\dagger U = 1$, the transformation cancels in $\mathscr{L}$.

Feynman rules and useful formulas



The Pauli matrices are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

They satisfy $\sigma^i\sigma^j = \delta^{ij} + \mathrm{i}\varepsilon^{ijk}\sigma^k$.
The gamma matrices form a Clifford algebra,

$$\{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}. \tag{3}$$

The matrix

$$\gamma^5 \equiv \gamma_5 \equiv \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3. \tag{4}$$

satisfies $(\gamma^5)^\dagger = \gamma^5$, $(\gamma^5)^2 = 1$, and $\{\gamma^\mu, \gamma^5\} = 0$. In the Dirac representation

$$\gamma^0 = 1 \otimes \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \gamma^i = \sigma^i \otimes \mathrm{i}\tau^2 = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \tag{5}$$

$$\gamma^5 = 1 \otimes \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad C = \mathrm{i}\gamma^2\gamma^0 = -\mathrm{i}\sigma^1 \otimes \tau^2 = \begin{pmatrix} 0 & -\mathrm{i}\sigma^2 \\ -\mathrm{i}\sigma^2 & 0 \end{pmatrix}. \tag{6}$$

$$P_L = \frac{1}{2}(1-\gamma^5)\psi \quad \text{and} \quad P_R = \frac{1}{2}(1+\gamma^5).$$

$$\overline{\Gamma} \equiv \gamma^0\Gamma^\dagger\gamma^0. \tag{7}$$

The trace of an odd number of γ^μ matrices vanishes, as well as

$$\mathrm{tr}[\gamma^5] = \mathrm{tr}[\gamma^\mu\gamma^5] = \mathrm{tr}[\gamma^\mu\gamma^\nu\gamma^5] = 0. \tag{8}$$

Non-zero traces are

$$\mathrm{tr}[\gamma^\mu\gamma^\nu] = 4\eta^{\mu\nu} \quad \text{and} \quad \mathrm{tr}[\not{a}\not{b}] = 4a\cdot b \tag{9}$$

$$\mathrm{tr}[\not{a}\not{b}\not{c}\not{d}] = 4[(a\cdot b)(c\cdot d) - (a\cdot c)(b\cdot d) + (a\cdot d)(b\cdot c)] \tag{10}$$

$$\mathrm{tr}\left[\gamma^5\gamma_\mu\gamma_\nu\gamma_\alpha\gamma_\beta\right] = 4\mathrm{i}\varepsilon_{\mu\nu\alpha\beta} \tag{11}$$

as a linear combination of all relevant tensors. On which variables do the scalar functions depend on? [Note: You do not have to take into account that current conservation implies $q_\mu H^{\mu\nu} = 0$.] (8 pts)

a+b.) See e.g. Griffiths 7.7.

c.) Since the proton is a composite particle, we cannot use the Feynman rule for a charged point-like fermion. using instead the tensor method, we know that $H^{\mu\nu}$ is a symmetric second rank tensor which can depend possibly on q, p_2 and p_4. Since $q = p_4 - p_2$, only two of them are independent and we choose them as q and $p \equiv p_2$. Another symmetric second rank tensor is the metric tensor $\eta^{\mu\nu}$. Thus

$$H^{\mu\nu} = A\eta^{\mu\nu} + Bp^\mu p^\nu + Cq^\mu q^\nu + D(p^\mu q^\nu + q^\mu p^\nu),$$

where A, B, C and D are scalar functions. Lorentz invariance implies that they can depend only on scalar variables: Since $p^2 = m^2$ and $p_4^2 = m_p^2$ are constants, and $p_4^2 = m_p^2 = (q+p)^2 = q^2+2q\cdot p+m_p^2$ implies $q^2 = -2q \cdot p$, there is only one independent scalar variable which one chooses usually as q^2. Hence

$$H^{\mu\nu}(q^2) = A(q^2)\eta^{\mu\nu} + B(q^2)p^\mu p^\nu + C(q^2)q^\mu q^\nu + D(q^2)(p^\mu q^\nu + q^\mu p^\nu).$$

3. Chiral spinors and SU(2)$_L$ transformations.

a.) Assume that the Dirac spinor of a fermion is given by

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = \begin{pmatrix} \chi \\ \phi \end{pmatrix}$$

in the Dirac representation. Find the left- and right-chiral components of the field ψ. (6 pts)

b.) Consider now one generation of quarks, e.g. up and down quarks. How do they transform under a global SU(2)$_L$ transformation

$$U = \exp(\mathrm{i}\boldsymbol{\alpha\tau}/2)$$

with $\boldsymbol{\alpha} = (0, 0, 2\pi)$? (6 pts)

a.) The left- and right-chiral fields are defined by

$$\psi_L = P_L\psi = \frac{1}{2}(1-\gamma^5)\psi \quad \textit{and} \quad \psi_R = P_R\psi = \frac{1}{2}(1+\gamma^5)\psi\,.$$

Using γ^5 in the Dirac representation gives

$$\psi_L = \frac{1}{2}(1-\gamma^5)\psi = \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right]\begin{pmatrix} \chi \\ \phi \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \chi - \phi \\ -\chi + \phi \end{pmatrix}$$

b.) Substituting normal with covariant derivatives means

$$\partial^\mu \phi \to D^\mu \phi = (\partial^\mu + \mathrm{i}gB^\mu)\,\phi\,,$$

where B^μ is the gauge field of the U(1) group. Thus it transforms as

$$B_\mu \to B'_\mu = B_\mu - \partial_\mu \Lambda$$

under local gauge transformations. We choose now $\vartheta(x) = g\Lambda(x)$. Then the covariant derivative transforms homogenously as

$$D_\mu \phi \to [D_\mu \phi]' = (\partial_\mu + \mathrm{i}gB'_\mu)\,\phi' = U\,[\partial_\mu + \mathrm{i}g(\partial_\mu \Lambda) + \mathrm{i}g(B_\mu - \partial_\mu \Lambda]\,\phi = UD_\mu \phi.$$

Thus the transformation cancels in $(D_\mu \phi)^\dagger (D^\mu \phi)$.

c.) Expanding the $(D_\mu \phi)^\dagger (D^\mu \phi)$ term, one obtains

$$\mathscr{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi \underbrace{-\mathrm{i}qB_\mu \phi^\dagger \partial^\mu \phi + \mathrm{i}qB^\mu (\partial_\mu \phi^\dagger)\phi + q^2 B_\mu B^\mu \phi^\dagger \phi}_{\mathscr{L}_I}.$$

Thus the Lagrangian contain a vertex connecting two scalar and a gauge boson, and a vertex connecting two scalars and two gauge bosons,

d.) Sketch of V:

$\mu^2 < 0$: normal mass term for the scalar, the $\phi \to -\phi$ symmetry is respected by the groundstate. The theory contains two scalar degrees of freedom and two degrees of freedom in the massless spin-1 field. Perturbation theory around the minumum $v = 0$ in the scalar field.
$\mu^2 > 0$: the $\phi \to -\phi$ symmetry is broken, one of the minima has to be chosen as the groundstate. Perturbation theory has to be performed around the minumum $v \neq 0$, and the field shifted, $\phi(x) = v + \xi(x)$. Using the new $\xi(x)$, a mass term for one of the scalar degrees of freedom and the gauge boson appears. The other, massless one, becomes the longitudinal degree of freedom of the massive gauge boson.

Useful are also $\not{a}\not{b} + \not{b}\not{a} = 2a \cdot b$, $\not{a}\not{a} = a^2$ and the following contractions,

$$\gamma^\mu\gamma_\mu = 4\,, \qquad \gamma^\mu\not{a}\gamma_\mu = -2\not{a}\,, \qquad \gamma^\mu\not{a}\not{b}\gamma_\mu = 4a\cdot b\,, \qquad \gamma^\mu\not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a}\,. \tag{12}$$

Completeness relations

$$\sum_s u_a(p,s)\bar{u}_b(p,s) = (\not{p}+m)_{ab}\,, \tag{13}$$

$$\sum_s v_a(p,s)\bar{v}_b(p,s) = (\not{p}-m)_{ab}\,. \tag{14}$$

Breit-Wigner formula

$$\sigma(12 \to R \to 34) = \frac{4\pi s}{p_{\rm cms}^2}\frac{(2s_r+1)}{(2s_1+1)(2s_2+1)}\frac{\Gamma(R\to 12)\Gamma(R\to 34)}{(s-M_R^2)^2+(M_R\Gamma_R)^2} \tag{15}$$

where Γ_R is the total decay width of the resonance with mass m_R and spin s_R, while s_1 and s_2 are the spins of the particles in the initial state.

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m_\phi^2\phi^2 + \bar{\psi}(\mathrm{i}\not{\partial} - m_\psi)\psi$$

$$D_\mu = \partial_\mu + \mathrm{i}gA_\mu$$

pseudoscalar mesons: $\pi^\pm$, $m = 139.6\,\text{MeV}$, $(u\bar{d}), (d\bar{u})$.
π^0, $m = 135.0\,\text{MeV}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$.
$K^\pm$, $m = 493.68\,\text{MeV}$, $(u\bar{s}), (s\bar{u})$.
vector mesons: ρ^0, $m = 775.5\,\text{MeV}$, $(u\bar{u} - d\bar{d})/\sqrt{2}$.
baryons: Δ^+, $m = 1232\,\text{MeV}$, (uud).

	mass	energy	1/length	1/time	temperature
GeV	1.78×10^{-24} g	1.60×10^{-3} erg	5.06×10^{13} cm^{-1}	1.52×10^{24} s^{-1}	1.16×10^{13}K