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Exam FY3403: Particle Physics
Saturday December 14th 2013
09.00-13.00
English

The exam consists of 2 problems. Each problem counts for a specific percentage of the total weight of the exam. This percentage is indicated for each problem on the following pages. Note that the sub-exercises **(a)**, **(b)**, etc. do not necessarily count equally.

In all the problems below you may use words, equations, and figures in your response. Your response is expected to be sufficiently detailed - short and inaccurate answers will be given a lower score. Read each problem carefully in order to avoid unnecessary mistakes.

Allowed material to use at exam: C.

- Approved, simple calculator.
- K. Rottmann: Matematisk formelsamling.
- K. Rottmann: Mathematische Formelsammlung. Barnett & Cronin: Mathematical Formulae.

Also consider the Supplementary Material on the last page of this exam.

PROBLEM 1 (50%)

(a) Describe the reason for why the following particles do not decay: neutrino, electron, photon, proton. Consider now the following scattering process: $e + p \to e + X$ where e is an electron, p is a proton, and X is a particle with positive charge. The 4-momentum of the electron before and after the collision is q and q', respectively. Working in the CM-frame, find a simple expression for the 4-momentum transfer $K^2 = -(q - q')^2$ in terms of the energies E and E' of the incoming and outgoing electrons and the scattering angle θ. You may neglect the electron mass in this problem.

(b) Which of the following expressions are Lorentz covariant and why:

- $p^\mu = m$
- $\varepsilon^\mu p_\mu p^\alpha = p_\sigma p^\sigma \varepsilon^\alpha$
- $p_\nu q^\nu = q_\nu p^\nu$

Consider now a process $\zeta(p_1) + \bar{\zeta}(p_2) \to \phi(q_1) + \phi(q_2)$ in the center of mass frame. The particles have masses m_ζ and m_ϕ respectively. What is the energy of the final particles and the magnitude of the momentum of the initial and final particles? Explain your answers (writing the answer without explanation gives zero points).

(c) A massless neutrino with energy E collides with a massive lepton of the same species which is at rest. How high must E be in order to produce a W boson (mass m_W) via the reaction $\bar{\nu}_l + l \to W$? Express your answer only in terms of m_e, m_W, and c and then insert numerical values to obtain E. Is the required energy realistic to obtain?

(d) Why are the weak interactions usually much slower than electromagnetic processes? Are there exceptions to this? What is the gauge group of the Standard Model, excluding the quarks? How many colors are there for each of the following particles: electron, neutrino, down quark, top quark, gluon, photon, Z?

(e) Describe in detail what is meant by the following concepts: renormalization, asymptotic freedom, gauge theories.

(f) The following ratio of cross-sections:

$$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

is usually quite flat as a function of energy E. Ocassionally, however, it undergoes strong steps up where the constant value increases to a new, higher constant. What is the physical reason for this?

PROBLEM 2 (50%)

(a) Consider top quark production via collision of neutrinos with positrons:

$$e^+ + \nu_e \to t + \bar{f}$$

where f is a fermion. Name some possible fermions f that would be allowed in the above process.

(b) Draw the lowest-order Feynman diagram for this process and write down the Feynman amplitude without any approximations made for the propagator. You may represent the vertex involving t simply as $ig_c\gamma^\mu(1-\gamma_5)$ where g_c is a constant. How does the amplitude simplify if you assume that the positron and neutrino may be treated as massless?

Consider now a different problem. Consider the following expression for the Feynman amplitude squared:

$$|\mathcal{M}|^2 = \frac{g_e^4}{(p_1-p_3)^4}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)][\bar{u}(3)\gamma^\nu u(1)]^*[\bar{u}(4)\gamma_\nu u(2)]^*.$$

Find by direct calculation (using Casimir's trick) an expression for $\langle|\mathcal{M}|^2\rangle$ expressed in terms of one or more traces and a prefactor.

(c) Explain in detail what is meant by spontaneous symmetry breaking and how this may generate interactions between gauge fields and matter fields. After providing a general description, use the Dirac Lagrangian $\mathcal{L}$ as a concrete example and introduce gauge field(s), transformation rules, and a coupling between the gauge field(s) and Dirac spinor in a gauge-invariant way. After doing this, explain qualitatively how spontaneous symmetry breaking of a global vs. local symmetry may result in different physics.

(d) Interpret what each term in the following Lagrangian represents physically:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + 1 - \alpha^2\phi^2 + \frac{1}{2}\alpha^4\phi^4.$$

Imagine that the two last terms in $\mathcal{L}$ had the opposite sign. What would this mean and how would you proceed to treat the Lagrangian?

Supplementary Information

The regime of validity and the meaning of the symbols below are assumed to be known by the reader.

Mass of W: $m_W \simeq 82$ GeV. Mass of K^-: 493 MeV$/c^2$. Mass of electron and muon: $m_e = 0.51$ MeV$/c^2$ and $m_\mu = 105$ MeV$/c^2$.

$\hbar = 6.58 \times 10^{-16}$ eV·s. Weak coupling constant: $g_W = 0.66$.

Photon propagator: $-\mathrm{i}g_{\mu\nu}/q^2$. QED vertex factor: $\mathrm{i}g_e\gamma^\mu$.

W propagator: $\frac{-\mathrm{i}(g_{\mu\nu}-q_\mu q_\nu/M_W^2c^2)}{q^2-M_W^2c^2}$. Weak vertex factor: $-\frac{\mathrm{i}g_W}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$.

Completeness relations ($\bar{u} = u^\dagger\gamma^0$):

$$\sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = (\gamma^\mu p_\mu + mc), \ \sum_{s=1,2} v^{(s)}\bar{v}^{(s)} = (\gamma^\mu p_\mu - mc).$$

We also have that $\gamma^0(\gamma^\nu)^\dagger\gamma^0 = \gamma^\nu$ whereas $\gamma^5 = (\gamma^5)^\dagger$ and γ^5 anticommutes with γ^μ.

Dirac Lagrangian: $\mathcal{L} = \mathrm{i}\hbar c\bar{\psi}\gamma^\mu\partial_\mu\psi - mc^2\bar{\psi}\psi$.

Charge of the top quark t: +2e/3.

Trace theorems: (below I use the notation $a' \equiv a^\mu\gamma_\mu$)

$$Tr(A+B) = Tr(A)+Tr(B),\ Tr(\alpha A) = \alpha Tr(A),\ Tr(ABC) = Tr(CAB) = Tr(BCA).$$

$$g_{\mu\nu}g^{\mu\nu} = 4,\ \{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu},\ a'b'+b'a' = 2ab.$$

$$\gamma_\mu\gamma^\mu = 4,\ \gamma_\mu\gamma^\nu\gamma^\mu = -2\gamma^\nu,\ \gamma_\mu\gamma^\nu\gamma^\lambda\gamma^\mu = 4g^{\nu\lambda}.$$

$$\gamma_\mu\gamma^\nu\gamma^\lambda\gamma^\sigma\gamma^\mu = -2\gamma^\sigma\gamma^\lambda\gamma^\nu,\ \gamma_\mu a'\gamma^\mu = -2a'.$$

$$\gamma_\mu a'b'\gamma^\mu = 4ab,\ \gamma_\mu a'b'c'\gamma^\mu = 2-c'b'a'.$$

$$Tr(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu},\ Tr(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma) = 4(g^{\mu\nu}g^{\lambda\sigma} - g^{\mu\lambda}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\lambda}).$$

$$Tr(a'b') = 4ab,\ Tr(a'b'c'd') = 4[(ab)(cd)-(ac)(bd)+(ad)(bc)], \gamma^5 = \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3.$$

$$Tr(\gamma^5) = 0,\ Tr(\gamma^5\gamma^\mu\gamma^\nu) = 0,\ Tr(\gamma^5\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma) = 4\mathrm{i}\varepsilon^{\mu\nu\lambda\sigma}.$$

$$Tr(\gamma^5 a'b') = 0,\ Tr(\gamma^5 a'b'c'd') = 4\mathrm{i}\varepsilon^{\mu\nu\lambda\sigma}a_\mu b_\nu c_\lambda d_\sigma,$$

where $\varepsilon^{\mu\nu\lambda\sigma}$ is -1 if $\mu\nu\lambda\sigma$ is an even permutation of 0123, +1 if it is an odd permutation, 0 if any two indices are the same. Finally, the trace over an odd number of γ matrices is zero.