NTNU Trondheim, Institutt for fysikk

Examination for FY3403 Particle Physics
Contact: Jan Myrheim, tel. 900 75 172
Allowed tools: mathematical tables, pocket calculator
Some formulas and data can be found on p. 2ff.

1. Chirality vs. helicity.
a.) Helicity for a free, massive particle is (3 pts)
☐ frame dependent
☐ conserved
b.) Chirality for a free, massive particle is (3 pts)
☐ frame dependent
☐ conserved

2. SU(2)$_L$.
Consider one generation of quarks before spontanous symmetry breaking, i.e. when they are massless.
a.) How do the quarks transform under SU(2)$_L$ transformations? (5 pts)
b.) Show that the Lagrangian describing these fields (without interactions) is invariant under global, but not under local SU(2)$_L$ transformations. (6 pts)

3. Polarization sum for a massive spin-1 particle.
a.) Write down a possible set of polarisation vectors $\varepsilon_\mu^{(r)}$, $r = 1, \ldots, n$ for a massive spin-1 particle. (4 pts)
b.) Derive the completeness relation for a massive spin-1 particle, (6 pts)

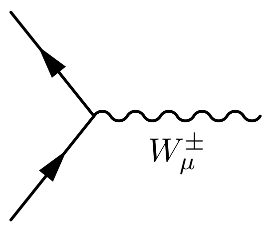
$$\sum_{r=1}^{n} \varepsilon_\mu^{(r)} \varepsilon_\nu^{(r)*} = -\eta_{\mu\nu} + k_\mu k_\nu / m^2 \,.$$

4. W decays.
Calculate the squared matrix element $|\bar{\mathcal{M}}|^2$ for the decay rate of a W boson into a massless fermion pair, summed over final state spins and averaged over intial ones. Express $|\bar{\mathcal{M}}|^2$ as function of invariant scalar products, $f(p_1 \cdot p_2, p_1 \cdot k, \ldots)$, where k, p_1 and p_2 are the four-momenta of the three particles. (16 pts)

5. Neutrino scattering.
Let us assume that at the Galactic Center a source of high-energy neutrons exist. These

neutrons escape from the source and produce neutrinos via beta-decay which we aim to measure.
a.) Neglect neutrio masses and oscillations. Draw the lowest order Feynman diagrams for the interaction of these neutrinos with matter (assumed to consist of e^-, u- and d-quarks). (6 pts)
b.) In which of the diagrams can the virtual particle be on mass-shell? Find the required neutrino energy. (6 pts)
c.) Estimate the maximal cross section (in SI or cgs units) using the Breit-Wigner formula. (6 pts)
d.) Describe qualitatively, how neutrino mixing and oscillations would change this picture (maximal 100 words). (6 pts)

Feynman rules and useful formulas



$$-\frac{\mathrm{i}g}{2\sqrt{2}}\gamma^\mu(1-\gamma^5)$$

The gamma matrices form a Clifford algebra,

$$\{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}\,. \tag{1}$$

The matrix

$$\gamma^5 \equiv \gamma_5 \equiv \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3\,. \tag{2}$$

satisfies $(\gamma^5)^\dagger = \gamma^5$, $(\gamma^5)^2 = 1$, and $\{\gamma^\mu,\gamma^5\} = 0$.

$$\overline{\Gamma} \equiv \gamma^0\Gamma^\dagger\gamma^0\,. \tag{3}$$

The trace of an odd number of γ^μ matrices vanishes, as well as

$$\mathrm{tr}[\gamma^5] = \mathrm{tr}[\gamma^\mu\gamma^5] = \mathrm{tr}[\gamma^\mu\gamma^\nu\gamma^5] = 0\,. \tag{4}$$

Non-zero traces are

$$\mathrm{tr}[\gamma^\mu\gamma^\nu\ ] = 4\eta^{\mu\nu} \quad \text{and} \quad \mathrm{tr}[\not{a}\not{b}] = 4a\cdot b \tag{5}$$

$$\mathrm{tr}[\not{a}\not{b}\not{c}\not{d}] = 4[(a \cdot b)\,(c \cdot d) - 4(a \cdot c)\,(b \cdot d) + 4(a \cdot d)\,(b \cdot c)] \tag{6}$$

$$\mathrm{tr}\big[\gamma^5 \gamma_\mu \gamma_\nu \gamma_\alpha \gamma_\beta\big] = 4\mathrm{i}\varepsilon_{\mu\nu\alpha\beta} \tag{7}$$

Useful are also $\not{a}\not{b} + \not{b}\not{a} = 2a \cdot b$, $\not{a}\not{a} = a^2$ and the following contractions,

$$\gamma^\mu \gamma_\mu = 4\,, \qquad \gamma^\mu \not{a} \gamma_\mu = -2\not{a}\,, \qquad \gamma^\mu \not{a}\not{b} \gamma_\mu = 4a \cdot b\,, \qquad \gamma^\mu \not{a}\not{b}\not{c} \gamma_\mu = -2\not{c}\not{b}\not{a}\,. \tag{8}$$

Completeness relations

$$\sum_s u_a(p,s)\bar{u}_b(p,s) = (\not{p} + m)_{ab}\ , \tag{9}$$

$$\sum_s v_a(p,s)\bar{v}_b(p,s) = (\not{p} - m)_{ab}\ . \tag{10}$$

Decay rate

$$\mathrm{d}\Gamma_{fi} = \frac{1}{2E_i}\,|\mathcal{M}_{fi}|^2\,\mathrm{d}\Phi^{(n)}\,. \tag{11}$$

The two particle phase space $\mathrm{d}\Phi^{(2)}$ in the rest frame of the decaying particle is

$$\mathrm{d}\Phi^{(2)} = \frac{1}{16\pi^2}\frac{|\boldsymbol{p}'_{\mathrm{cms}}|}{M}\,\mathrm{d}\Omega\,, \tag{12}$$

Breit-Wigner formula

$$\sigma(12 \to R \to 34) = \frac{4\pi s}{p^2_{\mathrm{cms}}}\,\frac{(2s_r+1)}{(2s_1+1)(2s_2+1)}\,\frac{\Gamma(R \to 12)\Gamma(R \to 34)}{(s - M_R^2)^2 + (M_R\Gamma_R)^2} \tag{13}$$

where Γ_R is the total decay width of the resonance with mass m_R and spin s_R, while s_1 and s_2 are the spins of the particles in the initial state.

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \frac{1}{2}m_\phi^2\phi^2 + \bar{\psi}(\mathrm{i}\not{\partial} - m_\psi)\psi$$

	mass	energy	1/length	1/time	temperature
GeV	1.78×10^{-24} g	1.60×10^{-3} erg	5.06×10^{13} cm^{-1}	1.52×10^{24} s^{-1}	1.16×10^{13}K