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Department of physics

Examination paper for FY3403 Particle physics

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Examination date: December 6, 2017

Examination time: 9–13

Permitted support material:
All calculators, mathematical and physical tables, Particle physics booklet

Language: English

Number of pages (front page excluded): 6

Number of pages enclosed: 0

Checked by:

Date Signature

Informasjon om trykking av eksamensoppgave
Originalen er:

1-sidig	☐	2-sidig	☒
sort/hvit	☒	farger	☐
skal ha flervalgskjema			☐

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Examination, course FY3403 Particle physics
Wednesday December 6, 2017
Time: 09:00–13:00

Grades made public: Saturday January 6, 2018

Allowed to use: All calculators, mathematical and physical tables, including the Particle physics booklet.
Some constants are given at the end of this problem set.
All subproblems are given the same weight in the grading.

Problem 1:

Neutral K mesons decay mostly to two π mesons, either $\pi^0\pi^0$ or $\pi^+\pi^-$. Even though isospin is not conserved in the decay, it makes sense to ask what is the total isospin of the two pions. We consider the decay in the rest frame of the K meson.

a) We know that the orbital angular momentum of the two pions must be $\ell = 0$. Why?

We also know that the total isospin must be either $I = 0$, $I = 1$, or $I = 2$. Why?

b) For simplicity we write, for example, $|+0\rangle$ for a state of two pions where particle 1 is a π^+ and particle 2 is a π^0. States like $|+0\rangle \pm |0+\rangle$ are symmetric or antisymmetric under interchange of particles 1 and 2.

How many states can we make that are symmetric, and how many that are antisymmetric?

How many states have isospin 0, 1, or 2?

Which isospin states are symmetric, and which are antisymmetric?

Use the table of Clebsch–Gordan coefficients (page 6 below) to answer this question, if you do not find the answer simply by counting.

c) The two pions from the decay of a neutral K meson can not have total isospin $I = 1$. Why not?

d) An approximate selection rule for hadronic decays by the weak interaction says that if the strangeness changes, then the total isospin changes by $\Delta I = \pm 1/2$. Explain how this selection rule, together with other information, determines uniquely the total isospin of the two pions from the decay of a neutral K meson.

What prediction does this give for the branching ratio between the two charge states $\pi^0\pi^0$ and $\pi^+\pi^-$?

The experimental result is 30.69 % for $\pi^0\pi^0$ and 69.20 % for $\pi^+\pi^-$, which in total over many decays gives nearly equal numbers of π^0, π^+, and π^-. Comment?

Problem 2:

Assume that a K^0 or $\overline{K}^0$ is at rest. The following analysis is much simplified if we assume that the CP symmetry is exact. In this approximation, which is less than one percent wrong, the short lived and long lived neutral K mesons, K_S and K_L, are eigenstates of CP,

$$|K_S\rangle = |K_1\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K}^0\rangle\right) ,$$
$$|K_L\rangle = |K_2\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K}^0\rangle\right) .$$

a) We define the charge conjugation operator C such that

$$C|K^0\rangle = |\overline{K}^0\rangle , \qquad C|\overline{K}^0\rangle = |K^0\rangle .$$

What are the eigenvalues of CP in the two states $|K_1\rangle$ and $|K_2\rangle$?

b) Assuming CP conservation we would expect the states $K_S = K_1$ and $K_L = K_2$ to have very different lifetimes. Explain why.

A K^0 or $\overline{K}^0$ is usually produced in a strong interaction process with a definite strangeness S, that is, either as a K^0 or a $\overline{K}^0$. Assume that a K^0 (with $S = +1$) is produced at $t = 0$, that is, the state of the particle at time $t = 0$ is

$$|\psi(0)\rangle = |K^0\rangle = \frac{1}{\sqrt{2}}\left(|K_1\rangle + |K_2\rangle\right) .$$

Still assuming CP conservation, the state at a later time t is

$$|\psi(t)\rangle = C(t)\left(\mathrm{e}^{-\left(\mathrm{i}m_S+\frac{\Gamma_S}{2}\right)t}\,|K_1\rangle + \mathrm{e}^{-\left(\mathrm{i}m_L+\frac{\Gamma_L}{2}\right)t}\,|K_2\rangle\right) ,$$

where $C(t)$ is a time dependent normalization factor. Here m_S and m_L are the masses of K_S and K_L, whereas $\Gamma_S = 1/\tau_S$ and $\Gamma_L = 1/\tau_L$ are the inverse lifetimes. We use natural units with $\hbar = 1$ and $c = 1$.

c) Write the state $|\psi(t)\rangle$ as a linear combination of $|K^0\rangle$ and $|\overline{K}^0\rangle$.

We see that the amplitudes of $|K^0\rangle$ and $|\overline{K}^0\rangle$ oscillate, this means that the strangeness of the particle oscillates.

Assuming that we observe the particle at time t as either a K^0 or a $\overline{K}^0$, what are the relative probabilities of the two alternatives K^0 or $\overline{K}^0$?

These relative probabilities go to constant limits when $t \to \infty$.

What are the limits, always assuming CP conservation?

d) One way to observe the particle as either a K^0 or a $\overline{K}^0$ is to observe a semileptonic decay and assume that the selection rule $\Delta S = \Delta Q$ holds. This is because the two processes $K^0 \to \pi^- e^+ \nu_e$ and $\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$ have $\Delta S = \Delta Q$ and are allowed, whereas $K^0 \to \pi^+ e^- \overline{\nu}_e$ and $\overline{K}^0 \to \pi^- e^+ \nu_e$ have $\Delta S = -\Delta Q$ and are forbidden. Hence we conclude that the decaying particle was a K^0 if we observe $\pi^- e^+$, or else a $\overline{K}^0$ if we observe $\pi^+ e^-$.

The selection rule $\Delta S = \Delta Q$ for strangeness changing semileptonic decays says that the change in strangeness equals the change in electric charge of the hadrons. Explain this selection rule by drawing Feynman diagrams for the allowed decays $K^0 \to \pi^- e^+ \nu_e$ and $\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$.

e) How are the decay rates for $K^0 \to \pi^- e^+ \nu_e$ and $\overline{K}^0 \to \pi^+ e^- \overline{\nu}_e$ related, assuming that the CP invariance is exact? Explain your reasoning.

This means that the probabilities for observing $\pi^- e^+$ or $\pi^+ e^-$ are directly related to the probabilities that the decaying particle is a K^0 or a $\overline{K}^0$.

f) What we observe in our experiment, where we produce a K^0 at $t = 0$, are time dependent decay rates

$$\Gamma_+(t) = \Gamma(\psi(t) \to \pi^- e^+ \nu_e) \qquad \text{and} \qquad \Gamma_-(t) = \Gamma(\psi(t) \to \pi^+ e^- \overline{\nu}_e)\ .$$

We write Γ_+ for the rate to the final state with a positron, and Γ_- for the rate to the final state with an electron.

From these rates we may define a time dependent asymmetry parameter

$$\delta(t) = \frac{\Gamma_+(t) - \Gamma_-(t)}{\Gamma_+(t) + \Gamma_-(t)}\ .$$

What is $\delta(0)$ when we start with a K^0 at $t = 0$?

Show that the time dependence is, in the approximations we have used,

$$\delta(t) = \frac{\cos(\Delta m\, t)}{\cosh(\gamma\, t)}\ ,$$

where $\Delta m = m_L - m_S$ and $\gamma = (\Gamma_S - \Gamma_L)/2$. Remember that we have assumed CP invariance.

g) The figure below (next page) shows experimental results for the asymmetry $\delta(t)$, from an eksperiment at CERN (Gjesdal et al., Physics Letters B52, 113 (1974)). The time axis goes from 0 to 2.5 ns.

Use the figure to estimate approximate values for the mass difference Δm and the rates γ and Γ_S.

Remember that we have used natural units with $\hbar = 1$ and $c = 1$. Hence we need to insert appropriate factors of $\hbar$ and c in order to convert Δm to units of MeV/c^2.

Does this experiment give any information about the sign of Δm?

What is the ratio $|\Delta m|/m_K$, where m_K is the mass of a neutral K meson? Comment?

h) In what way does the figure show that CP invariance is broken in the semileptonic decay of neutral K mesons?

From the figure, what is the approximate size of the CP breaking?

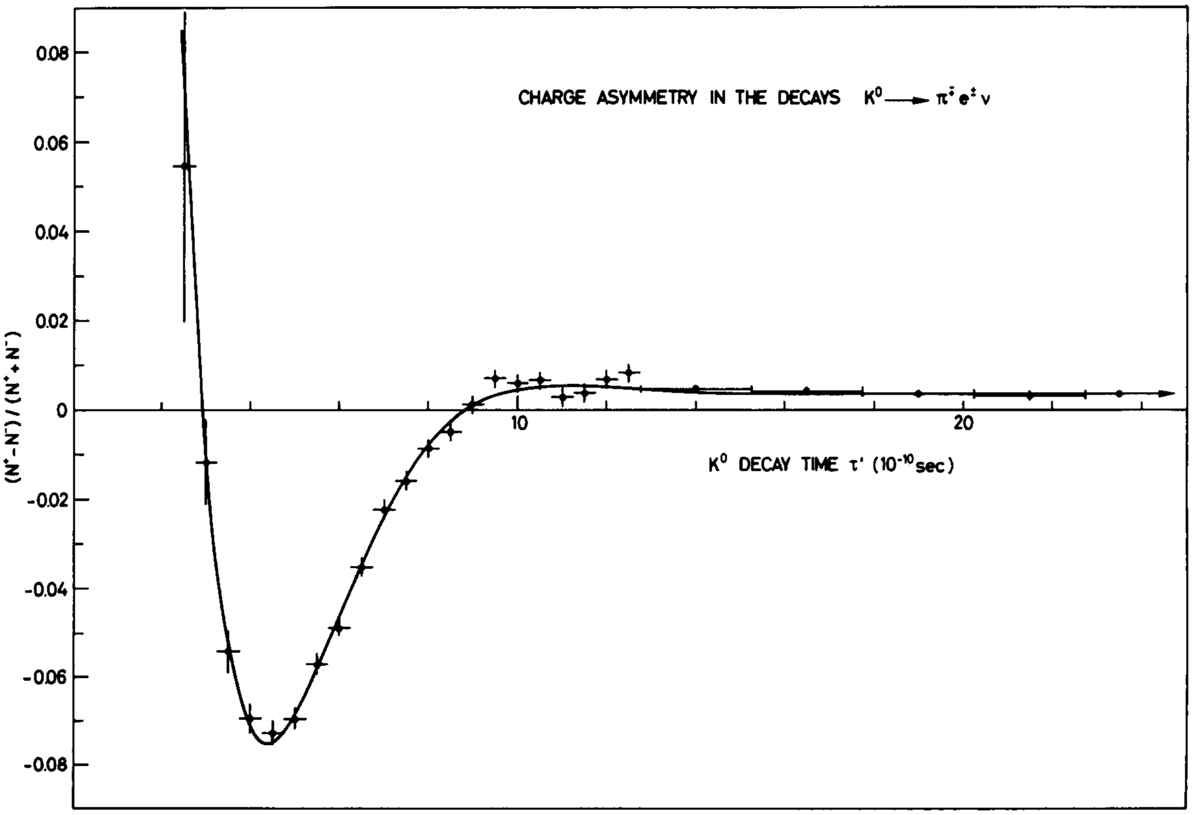


Fig. 1. The charge asymmetry as a function of the reconstructed decay time τ' for the K_{e3} decays. The experimental data are compared to the best fit as indicated by the solid line.

Some (potentially) useful constants:

The speed of light in vacuum	$c = 299\,792\,458\,\mathrm{m/s}$
The permeability of vacuum	$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{N/A^2}$
The permittivity of vacuum	$\epsilon_0 = 1/(\mu_0 c^2) = 8.854\,817\,187 \times 10^{-12}\,\mathrm{F/m}$
The reduced Planck's constant	$\hbar = 1.054\,571\,6 \times 10^{-34}\,\mathrm{J\,s}$
	$= 6.582\,119\,0 \times 10^{-22}\,\mathrm{MeV\,s}$
	$\hbar c = 197.732\,70\,\mathrm{MeV\,fm}$
The elementary charge	$e = 1.602\,176\,5 \times 10^{-19}\,\mathrm{C}$
The electron mass	$m_\mathrm{e} = 9.109\,382 \times 10^{-31}\,\mathrm{kg} = 0.510\,998\,9\,\mathrm{MeV}/c^2$
The proton mass	$m_\mathrm{p} = 1.672\,622 \times 10^{-27}\,\mathrm{kg} = 938.272\,0\,\mathrm{MeV}/c^2$
The neutron mass	$m_\mathrm{n} = 1.674\,927 \times 10^{-27}\,\mathrm{kg} = 939.565\,4\,\mathrm{MeV}/c^2$
The deuteron mass	$m_\mathrm{d} = 3.343\,583 \times 10^{-27}\,\mathrm{kg} = 1\,875.612\,8\,\mathrm{MeV}/c^2$

Particle data (showing quark content):
m = mass in MeV/c^2, S = strangeness,
I = isospin, G = G-parity, J = spin, P = parity,
C = charge conjugation symmetry for a neutral particle.

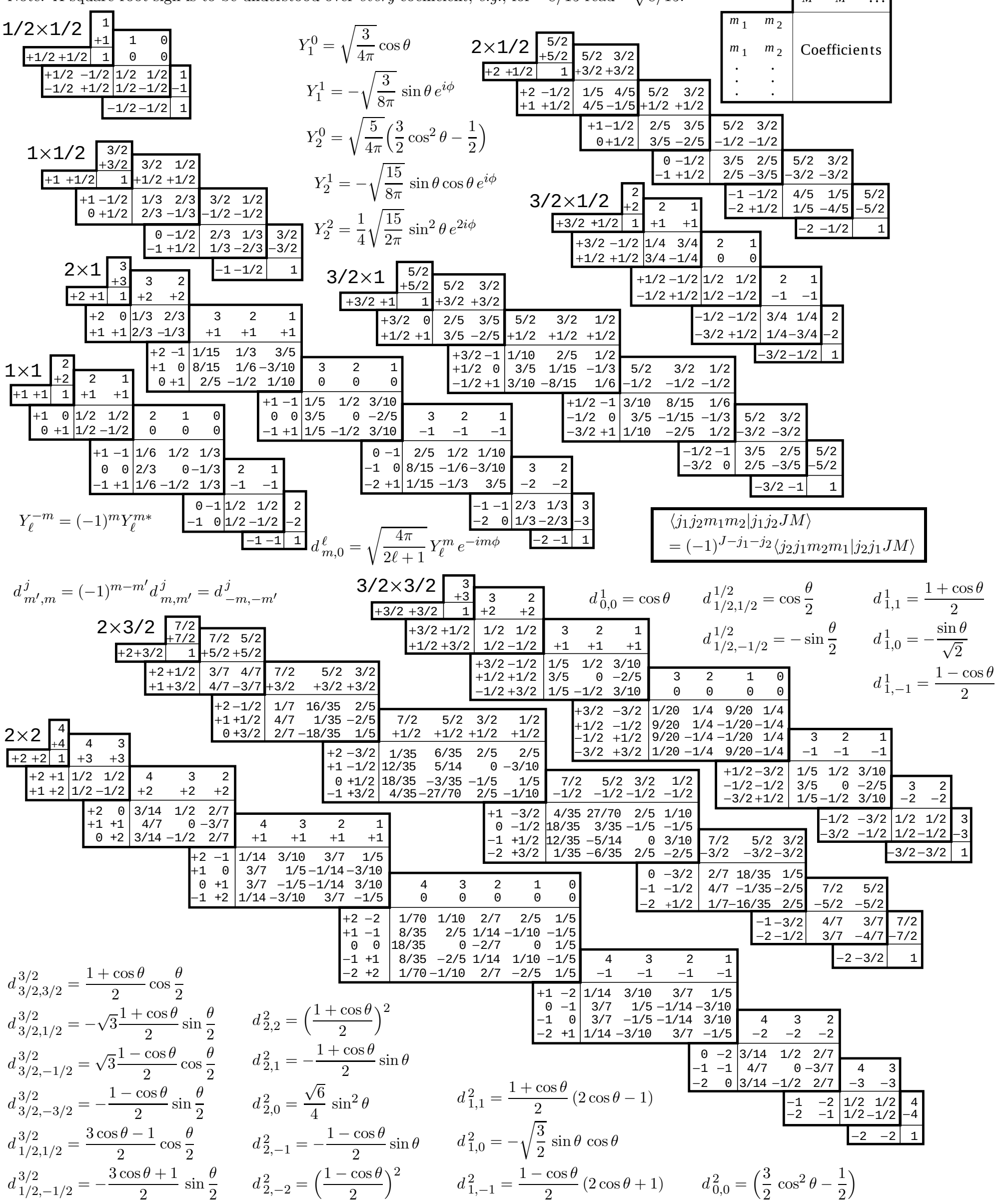
Mesons	m	S	$I^G(J^{PC})$ or $I(J^P)$	Mesons	m	S	$I^G(J^{PC})$ or $I(J^P)$
$\pi^0\,(u\overline{u} - d\overline{d})$	135.0		$1^-(0^{-+})$	$\pi^\pm\,(u\overline{d}, d\overline{u})$	139.6		$1^-(0^-)$
$K^0\,(d\overline{s})$	497.6	1	$\frac{1}{2}(0^-)$	$K^+\,(u\overline{s})$	493.7	1	$\frac{1}{2}(0^-)$
$\overline{K}^0\,(s\overline{d})$	497.6	-1	$\frac{1}{2}(0^-)$	$K^-\,(s\overline{u})$	493.7	-1	$\frac{1}{2}(0^-)$

Baryons	m	S	$I(J^P)$	Baryons	m	S	$I(J^P)$
p (proton,uud)	938.3		$\frac{1}{2}(\frac{1}{2}^+)$	n (neutron,udd)	939.6		$\frac{1}{2}(\frac{1}{2}^+)$
d (deuteron, pn)	1875.6		$0(1^+)$	$\Delta\,(uuu, uud, udd, ddd)$	1232		$\frac{3}{2}(\frac{3}{2}^+)$
$\Lambda^0\,(uds)$	1115.7	-1	$0(\frac{1}{2}^+)$	$\Sigma^0\,(uds)$	1192.6	-1	$1(\frac{1}{2}^+)$
$\Sigma^+\,(uus)$	1189.4	-1	$1(\frac{1}{2}^+)$	$\Sigma^-\,(dds)$	1197.4	-1	$1(\frac{1}{2}^+)$

34. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for $-8/15$ read $-\sqrt{8/15}$.

Notation: J J … / M M … ; m_1 m_2 / m_1 m_2 … Coefficients



$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta - 1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta - 1}{2}\cos\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta + 1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2 \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta + 1) \qquad d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 34.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.