NTNU Trondheim, Institutt for fysikk

Examination for FY3403, Particle Physics

This examination paper consists of 5 pages (including this title page)

Units, masses and other values: The convention: $\hbar = c = 1$ will be used throughout this paper. Constants and useful identities are listed on page 5 of this paper.

Please write your candidate number on every page and number the pages.

Please write only on ONE side of each sheet you hand in.

Answer ALL questions

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Question 1 (18 marks)

[Part Marks]

(a) The highest energy cosmic rays have energy $E \sim 10^{20}$ eV. Assuming the cosmic ray collides with a proton in the atmosphere, determine the ratio of the centre-of-mass energy in such a collision to the centre-of-mass energy for proton-proton collisions at the Large Hadron Collider, where each of two colliding proton beams has energy 7 TeV. **[8]**

(b) Particle C of mass m_C decays into two particles, A and B, with masses m_A and m_B. Determine the energies of the decay products, E_A, and E_B, in the rest frame of the parent particle in terms of the masses of A, B and C. **[5]**

(c) Also for $C \to A + B$, show that in the rest frame of the decaying particle, C, the outgoing momentum can be written in terms of the masses of A, B and C as

$$|\mathbf{p}| = \left[m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_B^2 m_C^2 - 2m_A^2 m_C^2\right]^{1/2} / 2m_C \,,$$

where $\mathbf{p}$ is the outgoing momentum of either A or B. **[5]**

Question 2 (10 marks)

The muon decay rate, Γ, and the muon lifetime, τ, are given by:

$$\Gamma \equiv \frac{1}{\tau} = \frac{G_F^2 m_\mu^5}{192\pi^3} \qquad (1)$$

where m_μ is the mass of the muon, and $G_F = \sqrt{2}g^2/(8m_{\mathrm{W}}^2)$ is Fermi's coupling constant, with g the coupling strength of the weak interaction and m_{W} the mass of the W boson.

(a) Draw the Feynman diagram for the dominant muon decay channel. Referring to the diagram, explain why $\Gamma \propto G_F^2$. What is the relative coupling of this type of interaction to different lepton flavours? **[4]**

(b) Calculate the lifetime of the muon. **[2]**

(c) Draw the Feynman diagram for the decay of the τ^- lepton to an electron and a pair of neutrinos (noticing that one or both of them may be anti-neutrinos). Given that the branching fraction for the process is 17.85%, use the above formula to predict the lifetime of the τ^- lepton. **[3]**

(d) State whether your prediction agrees with the measured value, $(290.6 \pm 1.0) \times 10^{-15}$ s. **[1]**

Question 3 (16 marks)

The K^0 and $\bar{K^0}$ mesons have quark contents d$\bar{s}$ and s$\bar{d}$ respectively, and belong to a meson nonet with quantum numbers $J^{PC} = 0^{-+}$.

(a) Draw a Feynman diagram of the interconversion between the K^0 and $\bar{K^0}$ mesons. **[2]**

(b) Explain how CP eigenstates K_1 and K_2 can be constructed in the $K^0 - \bar{K^0}$ system, and explain how, neglecting CP violation, these can be related to the states K_S and K_L by writing down the decay channels of the latter two states into pions, and considering the CP state of the final state system of pions. **[7]**

(c) A beam of neutral kaons is produced in an initial state which is known to consist purely of K_0 mesons. In the absence of CP violation, and considering only the hadronic decays of neutral kaons into pions, what would you expect to detect at different distances (near, and far) from the production point of the kaons? **[4]**

(d) If a small degree of CP violation occurs in the neutral kaon sector, how do the CP eigenstates K_1 and K_2 relate to the states K_L and K_S? Considering again the experimental setup of 3c, and neglecting all kaon decays except the dominant hadronic decays to pions, how does such CP violation manifest experimentally? **[3]**

Question 4 (23 marks)

Consider the decay $\pi^- \to \mu^- \bar{\nu}_\mu$ where the π^- has quark content $d\bar{u}$.

(a) Draw a Feynman diagram for this decay. **[2]**

(b) Draw a diagram showing the momentum and spin directions of the outgoing particles in the centre-of-mass frame, explaining clearly the reasons for your choice of spin state. **[3]**

(c) The lepton four-vector current for the final state can be written as

$$j_l^\nu = \bar{u}(p_3)\gamma^\nu \frac{1}{2}(1-\gamma^5)\upsilon(p_4)$$

where p_3 is the four-momentum of the muon and p_4 is the four-momentum of the $\bar{\nu}_\mu$. Forms for the γ-matrices and spinors can be found on page 5. Show that the time-like (zeroth) component of the lepton current is

$$j_l^0 = \frac{(E+m-|\mathbf{p}|)\sqrt{|\mathbf{p}|}}{\sqrt{E+m}} \tag{2}$$

where E, m, and $|\mathbf{p}|$ are the energy, the mass, and the magnitude of the three-momentum of the μ^- respectively. [You may assume that the muon direction aligns with the z-axis, so $\theta = 0$ and $\phi = 0$, and for the antineutrino, $\theta = \pi$, and $\phi = \pi$.] **[8]**

(d) The two-body decay rate is

$$\Gamma = \frac{|\mathbf{p}|}{8\pi m_\pi^2}\langle|\mathcal{M}|^2\rangle,$$

where m_π is the mass of the pion, and $\mathcal{M}$ is the matrix element for the decay. Given that only the time-like component of the lepton current contributes to the final amplitude, estimate the ratio

$$\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)}.$$

You may assume that the momentum of the muon emitted in the pion decay is 30 MeV while that of the electron is 70 MeV. Their respective masses are given on page 5. **[6]**

(e) Explain how measurements of the decay

$$^{60}\text{Co} \to ^{60}\text{Ni}^* + e^- + \bar{\nu}_e$$

can be used to show that the laws of nature are not symmetric under parity. **[4]**

[Total Marks = 67]

END OF PAPER

Constants

Meaning	Value
Masses of u,d,s,c,b,t quarks	1 MeV, 2 MeV, 0.2 GeV, 1.5 GeV, 4.5 GeV, 172 GeV
Masses of e, μ, τ leptons	0.511 MeV, 105.7 MeV, 1.777 GeV
Mass of all neutrinos	0
Mass of the $\pi^{+/-}, \pi^0, K^{+/-}, K^0$	139.6 MeV, 135 MeV, 493.7 MeV, 497.6 MeV
Mass of the W boson	80.39 GeV
Mass of the Z boson	91.19 GeV
Fine structure constant	1/137.036
Fermi coupling constant, G_F	$1.16637 \times 10^{-5}\,\text{GeV}^{-2}$
Reduced Planck constant, $\hbar$	6.58×10^{-25} GeV $\cdot$ sec
Speed of light, c	3×10^8 m/s
$\hbar c$	0.197 GeV $\cdot$ fm
$(\hbar c)^2$	0.0389 GeV^2 $\cdot$ fm^2

γ-matrices (Dirac-Pauli representation)

$$\gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \tag{3}$$

$$\gamma^2 = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & i & 0 & 0 \\ -i & 0 & 0 & 0 \end{pmatrix}, \quad \gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{4}$$

$$\gamma^5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}.$$

Helicity spinors

Particles:

$$u_\uparrow = \sqrt{E+m} \begin{pmatrix} c \\ se^{i\phi} \\ \alpha c \\ \alpha se^{i\phi} \end{pmatrix} \quad u_\downarrow = \sqrt{E+m} \begin{pmatrix} -s \\ ce^{i\phi} \\ \alpha s \\ -\alpha ce^{i\phi} \end{pmatrix}, \tag{5}$$

where $s = \sin\left(\frac{\theta}{2}\right)$, $c = \cos\left(\frac{\theta}{2}\right)$, $\alpha = \frac{|\mathbf{p}|}{E+m}$, and where θ and ϕ are the usual spherical angles (polar and azimuthal respectively) and where E, p, and m are the energy, momentum and mass of the particle (or antiparticle).

Antiparticles:

$$v_\uparrow = \sqrt{E+m} \begin{pmatrix} \alpha s \\ -\alpha ce^{i\phi} \\ -s \\ ce^{i\phi} \end{pmatrix} \quad v_\downarrow = \sqrt{E+m} \begin{pmatrix} \alpha c \\ \alpha se^{i\phi} \\ c \\ se^{i\phi} \end{pmatrix}. \tag{6}$$