Løsningen, Eksamen Partikkelfysikk
Høst 2003

Oppgave 1.

a) En relativistisk invariant har samme verdi i alle Lorentzsystemer.
For prosess $A + B \to C_1 + C_2 + \cdots$ er
for lab. systemet $\vec{p}_B = 0$ og for cm. $\vec{p}_A + \vec{p}_B = 0$.
For Comptoneffekt bruker vi ($\hbar = 1, c = 1$)

$$p_1 k_1 = \text{invariant} = (E_1 \omega_1 - \vec{p}_1 \vec{k}_1)_{lab} = (E_1 \omega_1 - \vec{p}_1 \vec{k}_1)_{cm} \text{ eller}$$

$$m\omega_1^L = (E_1 \omega_1 + p_1 \omega_1)_{cm} = \omega_1^c (E_1^c + \omega_1^c) \text{ og med } E_1 = \sqrt{\omega_1^2 + m^2}, |\vec{p}_1| = \omega_1,$$

$$m\omega_1^L = \omega_1^c \left(\sqrt{\omega_1^{c2} + m^2} + \omega_1^c\right).$$

b) Energi-impuls balanse $p_1 + k_1 = p_2 + k_2$ eller

$p_2 = p_1 + k_1 - k_2$, kvadrering gir

$$p_2^2 = p_1^2 + k_1^2 + k_2^2 + 2p_1 k_1 - 2p_1 k_2 - 2k_1 k_2$$

$$m^2 = m^2 + 0 + 0 + 2(E_1\omega_1 - \vec{p}_1 \vec{k}_1) - 2(E_1 \omega_2 - \vec{p}_1 \vec{k}_2) - 2\omega_1 \omega_2 (1 - \cos\theta).$$

Med $\vec{p}_1 = 0$, $E_1 = m$ er da

$$0 = m\omega_1 - m\omega_2 - \omega_1 \omega_2 (1 - \cos\theta) \quad \text{og}$$

$$\omega_2 = \frac{m\omega_1}{m + \omega_1(1-\cos\theta)} = \frac{\omega_1}{1 + \frac{\omega_1}{m}(1 - \cos\theta)}.$$

c) I cm. er $\vec{p}_1 + \vec{k}_1 = 0$ og $\vec{p}_2 + \vec{k}_2 = 0$ så

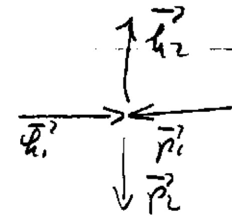
$|\vec{p}_1| = |\vec{k}_1|$, $|\vec{p}_2| = |\vec{k}_2|$.

Energiligningen $E_1 + \omega_1 = E_2 + \omega_2$

$$\sqrt{\vec{p}_1^{\,2} + m^2} + \omega_1 = \sqrt{\vec{p}_2^{\,2} + m^2} + \omega_2$$

$$\sqrt{\omega_1^2 + m^2} + \omega_1 = \sqrt{\omega_2^2 + m^2} + \omega_2$$

som betyr at $\omega_1 = \omega_2$ og derfor $E_1 = E_2$.



I cm er $\theta^c = \pi/2$.

Bruk at $p_1 k_2$ er invariant.

$$(E_1\omega_2 - \vec p_1 \vec k_2)_{cm} = (E_1\omega_2 - \vec p_1 \vec k_2)_L$$

$$E_1^c\omega_2^c - |\vec p_1||\vec k_2|\cos\theta^c = (E_1\omega_2 - \vec p_1 \vec k_2)_L$$

(Kan bruke $k_1 k_2$ = inv.)

$$E_1^c\omega_2^c = m\,\omega_2^L$$

$$E_1^c\omega_1^c = m\,\frac{\omega_1^L}{1+\frac{\omega_1^L}{m}(1-\cos\theta^L)}$$

$$E_1^c\omega_1^c = \omega_1^c(E_1^c+\omega_1^c)\Big/\left(1+\frac{\omega_1^L}{m}(1-\cos\theta^L)\right)$$

$$1+\frac{\omega_1^L}{m}(1-\cos\theta) = (1+\omega_1^c/E_1^c)$$

$$1-\cos\theta = \frac{m\,\omega_1^c}{\omega_1^L E_1^c} = \frac{m\,\omega_1^c}{\frac{\omega_1^c}{m}(E_1^c+\omega_1^c)E_1^c} = \underline{\frac{m^2}{E_1^c(E_1^c+\omega_1^c)}}$$

$\omega_1^c = m$, $E_1^c = \sqrt{m^2+m^2} = \sqrt{2}\,m$

$$1-\cos\theta = \frac{1}{\sqrt2(1+\sqrt2)}$$

$$\cos\theta = 1-\frac{1}{2+\sqrt2} = \frac{1+\sqrt2}{2+\sqrt2} = \frac{(1+\sqrt2)(2-\sqrt2)}{4-2} = \frac{\sqrt2}{2} = \frac{1}{\sqrt2}.$$

$\underline{\theta = 45°}$

d)

I cm, all energi går til dannelse av de tre massene, $3m$.

$$\omega_1^c + E_1^c = 3m$$

$$\omega_1^c + \sqrt{\omega_1^{c2}+m^2} = 3m$$

$$(\omega_1^c - 3m)^2 = \omega_1^{c2}+m^2$$

$$-6m\omega_1^c + 9m^2 = m^2$$

$$\underline{\omega_1^c = \frac{4}{3}m.}$$

fra a)

$$\omega_1^L = \frac{\frac43 m}{m}\left(m\sqrt{\frac{16}{9}+1} + \frac43 m\right) = m\,\frac43\left(\frac53+\frac43\right) = 4m.$$

Oppgave 2.

a)

Partikler: Elementære: leptoner $e\,\mu\,\tau,\ \nu_e\,\nu_\mu\,\nu_\tau$ + anti

Ikke sterk v.v. Ikke S, ikke I, $m_\nu \neq 0$? L-bevart

sammensatte partikler: kvarker: u d s c b t + anti

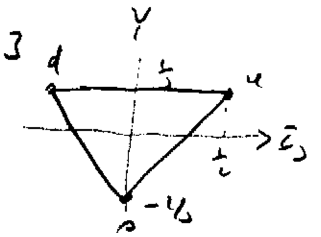
mesoner $q\bar{q}$, baryoner qqq. alle v.v., fengslet. $Q = \frac{1}{3}, \frac{2}{3}$, S, I, C, B, T, bevar. B-tall.

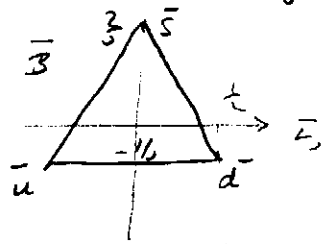
(kjerner, d, α --) gauge partikler: γ $W^\pm$ Z g (graviton)

el.mag, svak, sterk, (gravit) v.v. spinn 1.

$m_\gamma = 0$, $m_g = 0$ fengslet, $m_W, m_Z \sim 90$ GeV.

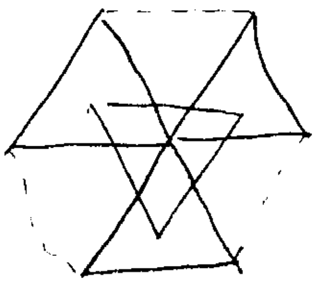
b)

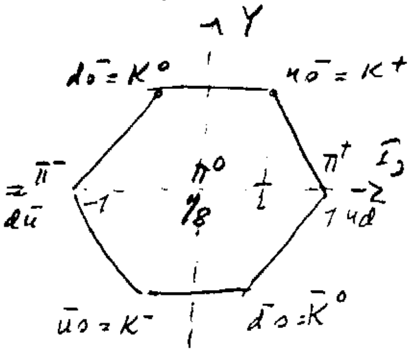




$3 \otimes \bar{3}$

$8 \oplus 1$

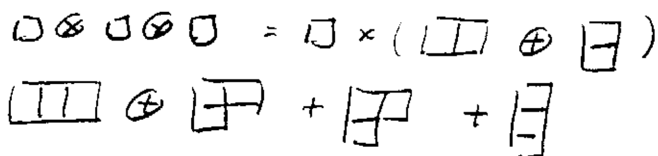




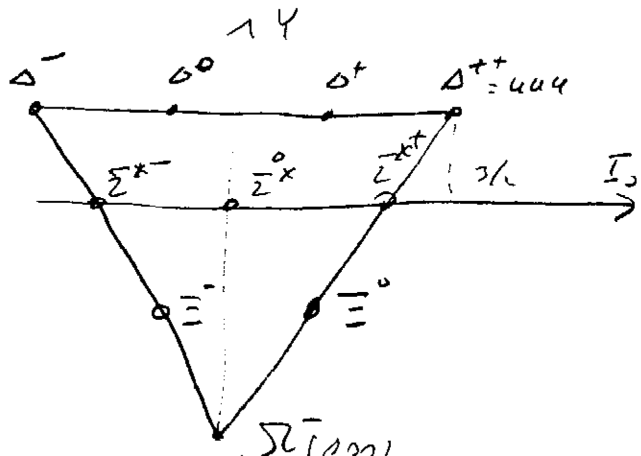
$+ \quad \bullet\ \eta_1$

η, η' er lin.komb. av $\eta_1\ \eta_8$.

c) $3 \otimes 3 \otimes 3$ =



$= 10 \oplus 8 \oplus 8 \oplus 1.$



4 ~ rom. spinn. farge

rom. spinn del er symmetrisk, antisymmetrisk i fargevariable.

Oppgave 3

θ^+ og τ^+ var to partikler med samme masse ladning men forskjellig desintegrasjon

$$\theta^+ \to \pi^+ + \pi^0 \qquad (2\pi)$$

$$\tau^+ \to \pi^+ + \pi^+ + \pi^- \quad (3\pi)$$

Paritetsmessig

$$P_{initial} = P_\theta = P_{final} = (-1)^{\eta_\pi}(-1)^{\eta_\pi}(-1)^{l_{\pi^0\pi^+}} = (-1)^{l_{\pi^0\pi^+}}$$

når $(-1)^{\eta_\pi} = -1$. Dreieimpulskonservering

$$\vec{J}_i = \vec{S}_\theta = \vec{J}_f = \vec{S}_{\pi^0} + \vec{S}_{\pi^+} + \vec{L}_{\pi^0\pi^+} = \vec{L}_{\pi^0\pi^+} \quad \text{for } \vec{S}_\pi = 0, \text{ så}$$

$S_\theta = l_{\pi^+\pi^0}$ og

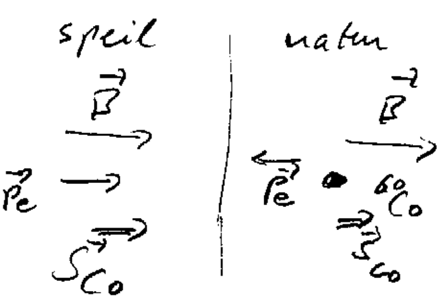
$$P_\theta = (-1)^{S_\theta}.$$

For τ^+ er

$$P_\tau = (-1)^{3\eta_\pi}(-1)^{l_{\pi\pi}} = -(-1)^{S_\tau}$$

så enten $\theta \neq \tau$ og P bevart eller $\theta = \tau$ og P brutt.

Eksperimentet



^{60}Co kjerner innrettes i et magnetfelt $\vec{B}$, β-elektroner ut måles retningen til.

Elektroner ut mest antiparallelt til $\vec{S}_{Co}$, ikke slik i speilbilde som altså ikke er som naturen: P brutt.

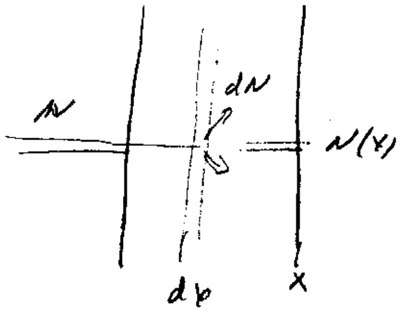
Oppgave 4

$$d\sigma = \frac{r_0^2}{2}(1+\cos\theta)\,d\Omega$$

a)

$$\sigma = \int d\sigma = \frac{r_0^2}{2}\int(1+\cos\theta)\,d\cos\theta\,d\varphi = \frac{r_0^2}{2}\cdot 2\pi\cdot\left(2+\frac{2}{3}\right) = \frac{8\pi}{3}r_0^2.$$

b)



I dx mister vi dN

$$dN = -N\rho_s\sigma\,dx$$

$$\frac{dN}{N} = -\rho_s\sigma\,dx\,, \qquad \kappa = \rho_s\sigma.$$

$$N(x) = N_0\,e^{-\rho_s\sigma x} = N_0\,e^{-\kappa x}.$$

Mister $N_0 - N = N_0(1-e^{-\kappa x})$

c) Intensitet halvert når $e^{-\kappa x} = \frac{1}{2}$ ∴ $x = \frac{\ln 2}{\kappa}$.

For fotoner i karbon:

$$x = \frac{\ln 2}{\sigma\rho_s}\,, \qquad \sigma = \frac{8\pi}{3}r_0^2$$

ρ_s er antall elektroner pr. volumenhet

I Ag er det N_A atomer.

I en cm^3 er det ρ gram eller $\rho\cdot\frac{N_A}{A}$ atomer, eller

$$\rho_s = \rho\frac{N_A}{A}Z \text{ elektroner/vol} = 2\cdot\frac{6.022\cdot10^{23}}{12}\cdot 6\ el/cm^3 = 6.022\cdot10^{23}\ el/cm^3.$$

$$x = \frac{\ln 2}{\frac{8\pi}{3}(2.179\cdot10^{-13})^2\,6.022\cdot10^{23}}\,cm = \frac{0.693\cdot10^3}{8.37\cdot4.75\cdot6.022}\,cm = 2.89\,cm.$$