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Suggested solution for Exam FY3403: Particle Physics

NOTE: The solutions below are meant as guidelines for how the problems may be solved and do not necessarily contain all the required details.

PROBLEM 1

(a) Neutrino: nothing lighter to decay into via weak interaction. Photon: nothing lighter to decay into. Electron: lightest charged particle. Proton: lightest baryon (conservation of baryon number saves it).

(b) First one must be weak since it changes strangeness. Second one must also be weak since it involves an antineutrino. The third one is dominated by the strong interaction.

(c) (i) Pion. (ii) Delta-baryon. (iii) Photon. The K-G equation describes a scalar spin-0 (bosonic) field.

(d) All naturally occuring particles are colorless. Thus, any isolated particle capable of appearing on its own has to be colorless. A single quark has a color, and thus isolated quarks cannot appear - they appear in collections of *e.g.* two (mesons) or three (baryons). Quark confinement refers to precisely this fact. Asymptotic freedom refers to the distance-dependence of the strong coupling constant: for small distances, the strong coupling constant is small whereas it grows larger for larger distances. The interaction strength of quarks thus depends on their separation distance.

(e) See description of experiment in Griffiths book. EM and strong interactions conserve parity.

(f) Renormalization is the procedure of accounting for higher-order Feynman diagrams by obtaining effective coupling constants and/or masses for the theory. The contribution from higher-order diagrams to these renormalized (effective) quantities may contain both infinite and finite momentum-dependent terms. The renormalized coupling constants and/or masses are what is measured experimentally, which means that there evidently are cancelling infinities. The finite contributions make the coupling constants and/or masses "running" in the sense that they depend on the energy (or equivalently separation distance) of the involved particles.

(g) $e+e \to e+e$ scattering can be mediated both via a photon (EM) and a Z^0 particle (weak). The weak coupling constant is stronger than the EM one in magnitude, but the massive Z^0 propagator suppresses the weak contribution for low-energy scattering (far away from the Z^0 pole). However, near the Z^0 pole the weak contribution can completely dominate the EM one due to the (formal) divergence of the Z^0 propagator when $q = M_Z c$ (high-energy scattering).

(h) Let f be a mapping that maps the element G of $\mathcal{G}$ onto G' of $\mathcal{G}'$. If $f(G_iG_j) = f(G_i)f(G_j)$ holds for any two elements, then f is a homomorphic mapping. This means that there is an n-to-one correspondence between the elements of the groups $\mathcal{G}$ and $\mathcal{G}$'. The groups SU(2) and SO(3) are not isomorphic since there is a 2-to-1 correspondence between their elements. SU(2) is not a representation of SO(3), but SO(3) is a representation of SU(2).

(i) Quarkonium consisting of light quarks is intrinsically relativistic and cannot be treated with the Schrödinger equation. The fact that the spatial dependence of the interaction is not well-known is also a complicating matter. More ...

(j) A gauge theory is a field theory in which the Lagrangian has a continuous symmetry. The different choices exploiting this degree of freedom are called gauges. Spontaneous symmetry breaking refers to a situation where the Lagrangian (or equivalently the equations of motion) are invariant under a symmetry transformation whereas the state of the system does not share that symmetry. Physical quantities do not depend on the choice of gauge. The Higgs mechanism is a result of spontaneous symmetry breaking of a local gauge symmetry and renders the gauge boson massive. One way to think of this is that the gauge field ”ate” the Goldstone boson and acquired a mass (longitudinal polarization) whereas the Goldstone bosons disappeared.

PROBLEM 2

(a) The diagrams are shown in the ”Examples” section of the QED chapter in the Griffiths book.

(b) We have that $|\mathcal{M}|^2 = |\mathcal{M}_1|^2 + |\mathcal{M}_2|^2 + \mathcal{M}_1^*\mathcal{M}_2 + \mathcal{M}_1\mathcal{M}_2^*$. Now, $\mathcal{M}_1$ is known. Also, $\mathcal{M}_2$ can be obtained directly from $\mathcal{M}_1$ by letting $p_3 \leftrightarrow p_4$ (since the diagrams only differ by exchanging 3 with 4). It then only remains to evaluate:

$$\mathcal{M}_1^*\mathcal{M}_2 + \mathcal{M}_1\mathcal{M}_2^* = 2\mathrm{Re}\{\mathcal{M}_1\mathcal{M}_2^*\}. \tag{1}$$

Note that we also have to do the spin-averaging, so we really want $\langle\mathcal{M}_1\mathcal{M}_2^*\rangle$. We can use Casimir’s trick to evaluate this term in the same way as we would have done for $|\mathcal{M}_1|^2$ or $|\mathcal{M}_2|^2$. From the diagrams, we have:

$$\begin{aligned}\mathcal{M}_1 &= -\frac{g_e^2}{(p_1-p_3)^2}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)],\\ \mathcal{M}_2 &= +\frac{g_e^2}{(p_1-p_4)^2}[\bar{u}(4)\gamma^\mu u(1)][\bar{u}(3)\gamma_\mu u(2)],\end{aligned} \tag{2}$$

It follows that:

$$\begin{aligned}\langle\mathcal{M}_1\mathcal{M}_2^*\rangle &= \frac{1}{4}\Big[\frac{-g_e^4}{(p_1-p_3)^2(p_1-p_4)^2}\Big]\\ &\times\sum_{\text{spins}}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)][\bar{u}(4)\gamma^\nu u(1)]^*[\bar{u}(3)\gamma_\nu u(2)]^*.\end{aligned} \tag{3}$$

Now rewrite the complex conjugated brackets such that *e.g.* :

$$[\bar{u}(4)\gamma^\nu u(1)]^* = [\bar{u}(1)\gamma^\nu u(4)] \tag{4}$$

we obtain that:

$$\sum_{\text{spins}}\ldots = \sum_{s_3}\bar{u}(3)\gamma^\mu\Big(\sum_{s_1}u(1)\bar{u}(1)\Big)\gamma^\nu\Big(\sum_{s_4}u(4)\bar{u}(4)\Big)\gamma_\mu\Big(\sum_{s_2}u(2)\bar{u}(2)\Big)\gamma_\nu u(3). \tag{5}$$

Use the completeness relations for the terms $\Big(\ldots\Big)$ with $m=0$ to arrive at:

$$\begin{aligned}\sum_{\text{spins}}\ldots &= \sum_{s_3}\bar{u}(3)\gamma^\mu p_1'\gamma^\nu p_4'\gamma_\mu p_2'\gamma_\nu u(3)\\ &= \sum_{i,j,s_3}\bar{u}(3)_i Q_{ij}u(3)_j = \mathrm{Tr}(Qp_3').\end{aligned} \tag{6}$$

Here, we have used the notation ’ meaning ”Feynman slash” and defined the matrix $Q = \gamma^\mu p_1'\gamma^\nu p_4'\gamma_\mu p_2'\gamma_\nu$. Using the trace rules to evaluate $\mathrm{Tr}(Qp_3')$, we find finally that:

$$\langle\mathcal{M}_1\mathcal{M}_2^*\rangle = \frac{8g_e^4}{(p_1-p_3)^2(p_1-p_4)^2}(p_1p_2)(p_3p_4). \tag{7}$$

PROBLEM 3

(a) The procedure is identical to the treatment of the decay of the pion in the Griffiths book (section ”Decay of the pion” in the chapter on Weak Interactions) - just need to replace m_π with m_K. The ratio between the decay rates is then:

$$\frac{\Gamma(K^- \to e^- + \bar{\nu}_e)}{\Gamma(K^- \to \mu^- + \bar{\nu}_\mu)} = \frac{m_e^2(m_K^2 - m_e^2)^2}{m_\mu^2(m_K^2 - m_\mu^2)^2}. \tag{8}$$

Inserting numbers, we find that the ratio is $\simeq 2.5 \times 10^{-5}$, indicating that the muon branch completely dominates.

(b) The explicit expression for the decay rate to the muon derived in (a) is:

$$\Gamma_\mu = \frac{f_K^2}{\pi\hbar m_K^3}\left(\frac{g_W}{4M_W}\right)^4 m_\mu^2(m_K^2 - m_\mu^2)^2. \tag{9}$$

Since $\Gamma_{\text{tot}} = \tau^{-1}$ where τ is the lifetime, and we know from the problem text that $\Gamma_\mu = 0.64\Gamma_{\text{tot}}$, we can identify:

$$f_K^2 = (0.64/\tau)(\pi\hbar m_K^3)\left(\frac{4M_W}{g_W}\right)^4/[m_\mu^2(m_K^2 - m_\mu^2)^2]. \tag{10}$$

One may then simply insert the numerical values provided in the Supplementary Information of the exam.