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Suggested solution for Exam FY3403: Particle Physics

NOTE: The solutions below are meant as guidelines for how the problems may be solved and do not necessarily contain all the detailed steps of the calculations.

PROBLEM 1

(a) See page 80 in 2nd edition of the book. It is not required that the student should remember the detailed analytical form of the expressions associated with the vertices in each case. For full score, it should be clear from the student's answer that there is cross-generational coupling between the quarks via weak interactions (Cabbibo angle) whereas there is not for leptons.

(b) These phenomena are all described in detail in the textbook in the following sections, respectively: b) 10.2 and 10.3, c) 10.8, d) 10.9, e) 8.6, f) 9.7, g) 7.9.

PROBLEM 2

(a) As informed during the exam, it was only necessary to consider the photon-mediated contribution to this process. The figure is given in Fig. 7.2. of the book and the amplitude is given by:

$$\mathcal{M} = -\frac{g_e^2}{(p_1 - p_3)^2}[\bar{u}(3)\gamma^\mu u(1)][\bar{u}(4)\gamma_\mu u(2)]. \tag{1}$$

(b) Casimir's trick is described in detail in section 7.7. and applying this procedure to the above amplitude gives us:

$$\langle|\mathcal{M}|^2\rangle = \frac{g_e^4}{4(p_1-p_3)^4}\text{Tr}[\gamma^\mu(p_1' + mc)\gamma^\nu(p_3' + mc)] \times \text{Tr}[\gamma_\mu(p_2' + Mc)\gamma_\nu(p_4' + Mc)] \tag{2}$$

where $'$ means Feynman slash and m, M is the electron and muon mass, respectively.

(c) Neglecting the masses of the fermions, as can be safely done near the Z^0 pole, the incoming and outgoing particles all have the same energies in the CM frame. By simple insertion of the corresponding 4-momenta into $\langle|\mathcal{M}|^2\rangle$, we then get:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c g_z^2 E}{16\pi[(2E)^2 - (M_z c^2)^2]}\right)^2 \left[[(c_V^f)^2 + (c_A^f)^2][(c_V^e)^2 + (c_A^e)^2](1 + \cos^2\theta) - 8c_V^f c_A^f c_V^e c_A^e \cos\theta\right] \tag{3}$$

Integration over all solid angles then gives:

$$\sigma = F \times G, \tag{4}$$

where

$$F = \frac{1}{3\pi}\left(\frac{\hbar c g_z^2 E}{4[(2E)^2 - (M_Z c^2)^2]}\right)^2,$$
$$G = [(c_V^f)^2 + (c_A^f)^2][(c_V^e)^2 + (c_A^e)^2]. \tag{5}$$