**NTNU** **Institutt for fysikk**

## Midsemester exam in FY3464 QUANTUM FIELD THEORY I

Wednesday october 17, 2007
12:15–14:00

Allowed help: Standard calculator
K. Rottman: *Matematisk formelsamling* or equivalent

Write your *student number* on every sheet of your solution.

This problem set consists of 2 pages.

**Problem 1.**
Consider the model defined by the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial_\mu\varphi\,\partial^\mu\varphi - \frac{1}{2}m^2\varphi^2 + \lambda\varphi\boldsymbol{E}\cdot\boldsymbol{B}, \tag{1}$$

where $\varphi$ is a real scalar field, $\boldsymbol{E} = -\dot{\boldsymbol{A}}$, and $\boldsymbol{B} = \boldsymbol{\nabla}\times A$.

**a)** Find the canonically conjugate field $\Pi_\varphi$ of $\varphi$.

**b)** Find the canonically conjugate field $\Pi_{\boldsymbol{A}}$ of $\boldsymbol{A}$.

**c)** Find the Hamiltonian density $\mathcal{H}$ of this model.

**d)** We use natural units. What is the mass dimension of the coupling parameter $\lambda$: (i) In 4 space-time dimensions? (ii) In $d$ space-time dimensions?

**e)** Find the Euler Lagrange equation for $\varphi$.

**f)** Find the Euler Lagrange equation for $\boldsymbol{A}$.

**g)** The Lagrangian density $\mathcal{L}$ is invariant under the transformation

$$\boldsymbol{A}(\boldsymbol{x},t) \to \boldsymbol{A}'(\boldsymbol{x},t) = \boldsymbol{A}(\boldsymbol{x},t) + \boldsymbol{\nabla}\Lambda(\boldsymbol{x}),$$

for all differentiable functions $\Lambda(\boldsymbol{x})$. Use the Nöther theorem to find the corresponding conserved Nöther current $J_\Lambda$.

**Problem 2.**

The field expansion of the free electromagnetic field in Coulomb gauge is

$$\boldsymbol{A}(x) = \sum_{\boldsymbol{k},r} \frac{1}{\sqrt{2|\boldsymbol{k}|V}} \left( a_{\boldsymbol{k},r}\, \hat{e}_{\boldsymbol{k},r}\, \mathrm{e}^{-ikx} + \text{hermitian conjugate} \right). \tag{2}$$

Then the matrix element $\langle \Omega |\, a_{\boldsymbol{q},s}\, \boldsymbol{A}(x)\, | \Omega \rangle$ equals

A. $0$

B. $\frac{1}{\sqrt{2|\boldsymbol{q}|V}}\, \hat{e}_{\boldsymbol{q},s}\, \mathrm{e}^{-iqx}$

C. $a_{\boldsymbol{q},s}$

D. $\frac{1}{\sqrt{2|\boldsymbol{q}|V}}\, \hat{e}^*_{\boldsymbol{q},s}\, \mathrm{e}^{iqx}$

E. None of the alternatives above.

**Problem 3.**

Let $\mathcal{T}$ be the time ordering operator, and $\varphi(x)$, $\varphi^\dagger(x)$ quantized complex Klein Gordon fields. Then we have (in natural units, i.e. when $\hbar = c = 1$)

A. $\mathcal{T}\left\{\varphi(x)\varphi^\dagger(y)\right\} = \mathcal{T}\left\{\varphi^\dagger(y)\varphi(x)\right\}$

B. $\mathcal{T}\left\{\varphi(x)\varphi^\dagger(y)\right\} = \mathcal{T}\left\{\varphi^\dagger(y)\varphi(x)\right\} + \mathrm{i}G_F(x-y)$

C. $\mathcal{T}\left\{\varphi(x)\varphi^\dagger(y)\right\} = \mathcal{T}\left\{\varphi^\dagger(y)\varphi(x)\right\} - \mathrm{i}G_F(x-y)$

D. $\mathcal{T}\left\{\varphi(x)\varphi^\dagger(y)\right\} = -\mathcal{T}\left\{\varphi^\dagger(y)\varphi(x)\right\} - \mathrm{i}G_F(x-y)$

E. None of the alternatives above.

Here $G_F(x-y)$ is the Feynman propagator for a complex Klein Gordon field.

**Problem 4.**

The Dirac equation

$$\left[\mathrm{i}\left(\gamma^0\partial_0 + \boldsymbol{\gamma}\cdot\boldsymbol{\nabla}\right) - m\right]\psi(x^0,\boldsymbol{x}) = 0$$

is invariant under space inversion (parity transformation), $\boldsymbol{x} \to -\boldsymbol{x}$. I.e, if $\psi(x^0,\boldsymbol{x})$ solves the Dirac equation then so does $\psi_P(x^0,\boldsymbol{x})$, where

A. $\psi_P(x^0,\boldsymbol{x}) = \mathrm{i}\gamma^2\, \psi^*(x^0,-\boldsymbol{x})$

B. $\psi_P(x^0,\boldsymbol{x}) = \gamma^1\,\gamma^3\, \psi^*(x^0,-\boldsymbol{x})$

C. $\psi_P(x^0,\boldsymbol{x}) = \psi(x^0,-\boldsymbol{x})$

D. $\psi_P(x^0,\boldsymbol{x}) = \gamma^0\, \psi(x^0,-\boldsymbol{x})$

E. $\psi_P(x^0,\boldsymbol{x}) = \psi^*(-x^0,-\boldsymbol{x})$