NTNU Trondheim, Institutt for fysikk

Home exam FY3464 Quantum Field Theory 1

Polarised muon decay.
Aim of the exam is to derive the differential decay rate of muon decay, $\mu^-(p_1) \to e^-(p_4)\nu_\mu(p_3)\bar{\nu}_e(p_2)$, via the exchange of a W-boson described by the vertex

$$-\frac{\mathrm{i}g}{\sqrt{2}}\bar{f}\gamma_\mu(1-\gamma^5)f\, W^\mu\,,$$

and to show its spin dependence.
a.) Draw the Feynman diagram and write down the matrix element $\mathcal{M}_{fi}$ of this process at lowest order perturbation theory. Use the Dirac equation to simplify $\mathcal{M}_{fi}$.
b.) Show that the projection operator on states with definite energy and spin are given by

$$u_a(p,s)\bar{u}_b(p,s) = \left[(\not{p}+m)\left(1+\gamma^5\not{s}\right)\right]_{ab} \tag{1}$$

$$v_a(p,s)\bar{v}_b(p,s) = \left[(\not{p}-m)\left(1+\gamma^5\not{s}\right)\right]_{ab}\ . \tag{2}$$

[I use the normalisation $\bar{u}u = 1$ and thus fermion have the same phase space as bosons in the final state. The fermion polarisation vector s was discussed in exercise 4.4.]
c.) Neglect all terms of order $m_e^2/m_W^2, m_\mu^2/m_W^2$. Account for the polarisation of the leptons by including projection operators on their spin, $1+\gamma^5\not{s}$. Show that

$$|\mathcal{M}|^2 = \frac{g^4}{64M_W^4}L_{\nu\mu}N^{\nu\mu} = \frac{g^4}{M_W^4}[p_3\cdot(p_4-m_4s_4)\; p_2\cdot(p_1-m_1s_1)]$$

with

$$L_{\mu\nu} = p_3^\alpha(p_1-m_1s_1)^\beta\,\mathrm{tr}[\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu(1-\gamma^5)]$$

d.) Evaluate the phase space integration over momenta of the unobserved neutrinos. [Note that you can not use Eq. (7.159) from the notes, because it is valid only for spin-averaged decays.] It is useful to write the required integral in a covariant way,

$$I_{\alpha\beta} = \int\frac{\mathrm{d}^3p_2\mathrm{d}^3p_3\; p_{3\alpha}p_{2\beta}\delta^{(4)}(p-p_2-p_3)}{E_2E_3} \tag{3}$$

and to express it using the "tensor method" as

$$I = \int\frac{\mathrm{d}^3p_2\mathrm{d}^3p_3\delta^{(4)}(p-p_2-p_3)}{E_2E_3} \tag{4}$$

and two scalar functions. Next evaluate I in an easy frame, e.g. the cm frame of the two neutrinos.
e.) Neglect finally the electron mass, simplify the formula for a decay of a muon at rest, and introduce the angle $\cos\vartheta = \hat{\boldsymbol{p}}_4\boldsymbol{s}_1$ and $x = E_4/E_4^{\max}$ in the differential decay width $\mathrm{d}\Gamma/(\mathrm{d}x\,\mathrm{d}\cos\vartheta\,\mathrm{d}\phi)$.