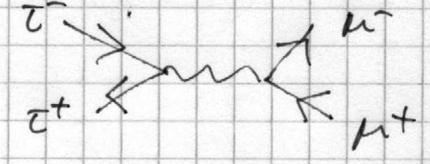
FY3464 QUANTUM FIELD THEORY I

Soln exam Nov. 30 2007

Problem 1

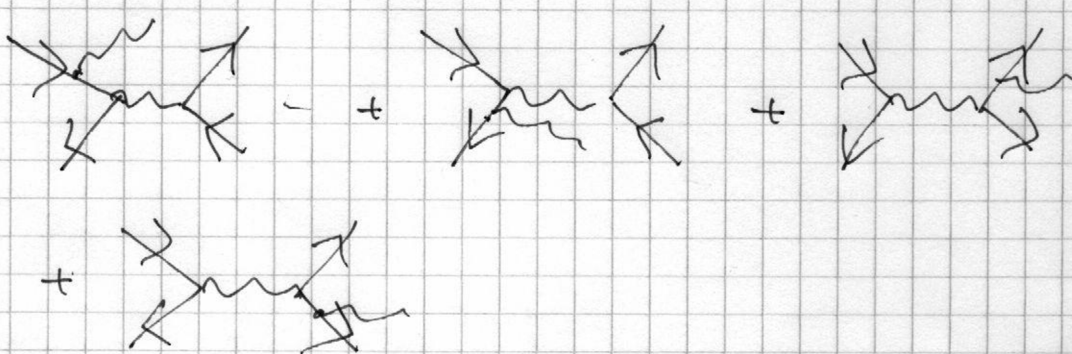
a) $\tau^+ \to \mu^+ e^+ e^-$ Impossible. Violates conservation of μ- and τ-numbers.

b) $\tau^+\tau^- \to \mu^+\mu^-$

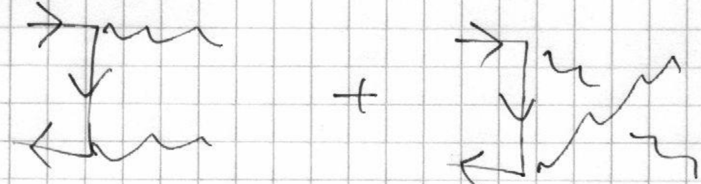


c) $\tau^+\mu^- \to \mu^+ e^-$ Impossible. Violates conservation of e-, μ- and τ-numbers.

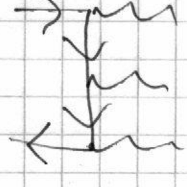
d) $\tau^+\tau^- \to \mu^+\mu^-\gamma$



e) $\tau^+\tau^- \to \gamma\gamma$

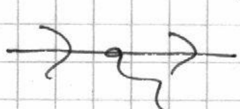


f) $\tau^+\tau^- \to \gamma\gamma\gamma$



+ 5 permutation of γ-lines

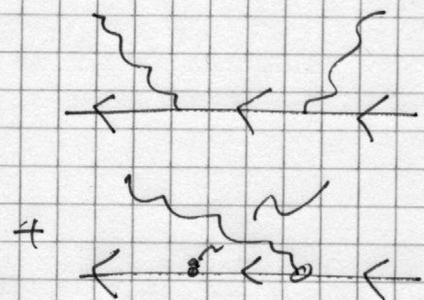
g) ~~[illegible]~~ $\tau^+ \to \tau^+\gamma$



Impossible. Violates conservation of 4-momentum.

Problem 1

f) $\tau^+ \gamma \to \tau^+ \gamma$

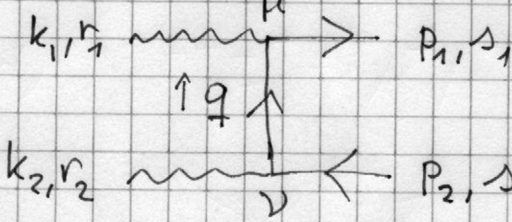


(Compton process)

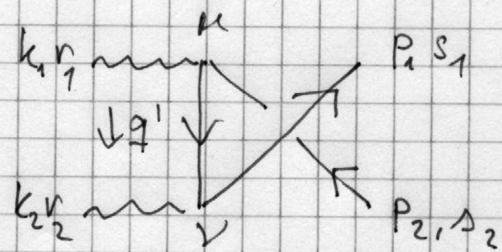
Problem 2.

$\gamma\gamma \to \tau^+ \tau^-$

a)



(a)



(b)

b)

$$\mathcal{M}_{fi} = \mathcal{M}_{fi}^{(a)} + \mathcal{M}_{fi}^{(b)}$$

$$-i\mathcal{M}_{fi}^{(a)} = (ie)^2 i * \left[\bar{u}(p_1,s_1)\gamma^\mu(\not{q}+m_\tau)\gamma^\nu v(p_2,s_2)\right] * e_\mu(k_1,r_1)\, e_\nu(k_2,r_2) * \frac{1}{q^2-m_\tau^2}$$

$$-i\mathcal{M}_{fi}^{(b)} = (ie)^2 i * \left[\bar{u}(p_1,s_1)\gamma^\nu(\not{q}'+m_\tau)\gamma^\mu v(p_2,s_2)\right] * e_\mu(k_1,r_1)\, e_\nu(k_2,r_2) * \frac{1}{q'^2-m_\tau^2}$$

c) $\sigma \propto e^4 \propto \alpha^2$. Scale set by m_τ.

I.e.

$$\sigma(E=2m_\tau) \simeq \alpha^2\left(\frac{\hbar}{m_\tau c}\right)^2 = \cdots$$

Problem 2

d) If $p_{2\mu} e_1^\mu = p_{2\mu} e_2^\mu = 0$ we would have $\not{p}_2 \not{e}_1 = -\not{e}_1 \not{p}_2$ and $\not{p}_2 \not{e}_2 = -\not{e}_2 \not{p}_2$. I.e.

$$(-\not{p}_2 + m_\tau)\not{e}_1 v(p_2, s_2) = \not{e}_1 (\not{p}_2 + m_\tau) v(p_2, s_2) = 0$$

and

$$(-\not{p}_2 + m_\tau)\not{e}_2 v(p_2, s_2) = \not{e}_2 (\not{p}_2 + m_\tau) v(p_2, s_2) = 0$$

since $(\not{p} + m) v(p, s) = 0$.

Now. Assume we have polarization vectors such that

$$(p_2 e_1) \neq 0 \quad , (p_2 e_2) \neq 0$$

Then we may choose new vectors

$$e_1'^\mu = e_1^\mu - c_1 k^\mu, \quad e_2'^\mu = e_2^\mu - c_2 k^\mu$$

with

$$c_1 = \frac{(p_2 k)}{(p_2 e_1)} \quad , \quad c_2 = \frac{(p_2 k)}{(p_2 e_2)} .$$

e) ~~With $\not{q} = \not{p}_1 - \not{k}_1$ we m~~

With $\not{q} = \not{k}_2 - \not{p}_2$ we may simplify

$$\bar{u}(p_1, s_1) \not{e}_1 (\not{q} + m_\tau) \not{e}_2 v(p_2, s_2) =$$

$$\bar{u}(p_1 s_1) \not{e}_1 (\not{k}_2 - \not{p}_2 + m_\tau) \not{e}_2 v(p_2, s_2)$$

$$= \bar{u}(p_1 s_1) \not{e}_1 \not{k}_2 \not{e}_2 v(p_2 s_2) + \bar{u}(p_1, s_1) \not{e}_1 \not{e}_2 \underbrace{(\not{p}_2 + m_\tau) v(p_2, s_2)}_{=0}$$

$$= \bar{u}(p_1 s_1) \not{e}_1 \not{k}_2 \not{e}_2 v(p_2, s_2).$$

Likewise

$$q^2 - m_\tau^2 = (k_2 - p_2)^2 - m_\tau^2 = -2k_2 p_2 + \underbrace{k_2^2}_{=0} + \underbrace{p_2^2 - m_\tau^2}_{=0}$$

$$= -2k_2 p_2$$

Hence: $\underline{-i\mathcal{M}_{fi}^{(a)} = \frac{-ie^2}{(-2k_2 p_2)} \bar{u}(p_1 s_1) \not{e}_1 \not{k}_2 \not{e}_2 v(p_2 s_2)}$

Problem 3

a) $[\varphi] = \text{mass} \qquad [\psi] = \text{mass}^{3/2}$

$[\lambda] = 1 \quad$ (dimensionless)

b)
$$\partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi^*} = \frac{\partial \mathcal{L}}{\partial \varphi^*}$$
$$\Updownarrow$$
$$\underline{\underline{\Box \varphi + \frac{\lambda}{2} |\varphi|^2 \varphi = 0}}$$

$$\partial_\mu \frac{\partial \mathcal{L}}{\partial \partial_\mu \overline{\psi}} = \frac{\partial \mathcal{L}}{\partial \overline{\psi}}$$
$$\Updownarrow$$
$$\underline{\underline{i \not\partial \psi = 0}}$$

c) Only combinations $\varphi^* \varphi$ occur

$\Rightarrow$ $\varphi \to e^{i\alpha} \varphi$ is ~~[illegible]~~ a symmetry ~~for~~ $\mathcal{L}$. $\delta\varphi = i\cancel{\alpha}\varphi$, $\delta\varphi^* = -i\cancel{\alpha}\varphi^*$

$\Rightarrow$
$$\underline{\underline{j^\mu_\varphi}} = \frac{\partial \mathcal{L}}{\partial_\mu \varphi} \delta\varphi + \frac{\partial \mathcal{L}}{\partial \partial_\mu \varphi^*} \delta\varphi^*$$
$$= \underline{\underline{i\left[(\partial^\mu \varphi^*)\varphi - \varphi^*(\partial^\mu \varphi)\right]}}$$

d) Likewise
$$\underline{\underline{j^\mu_\psi = \overline{\psi} \gamma^\mu \psi}}$$

e)
$$\underline{\underline{j^\mu_5 = \overline{\psi} \gamma^\mu \gamma^5 \psi}}$$

f) See soln. ~~problemset~~ Problemset 4.