Besvarelseark del A: Navn FASIT

LF
Eksamen FY6013
Mekanikk H2015

1: A

2: D

3: A

4: D

5: B

6: C

7: A

8: C

9: C

10: A

TFY6013

MEKANIKK FASIT DEL A

EKSAMEN

16/12 - 2015

KOMPIS

1. (A) $\frac{dx}{dt} = 0$

2. $v = 80 \text{ km/h} \Rightarrow \frac{80}{3.6} \frac{\text{m}}{\text{s}}$, $r = \frac{100 \text{ m}}{2\pi}$

$$a = \frac{v^2}{r} = \frac{80^2 \cdot 2\pi}{3.6^2 \cdot 100} \simeq 10 \frac{\text{m}}{\text{s}^2}$$

(D)

3. $$\alpha(t) = \frac{d\omega(t)}{dt} = -\frac{\omega_0^2}{50} e^{(-\omega_0 t/50)}$$ (A)

N.b. 'hvor stor' $\omega_0 = 0.5 \frac{1}{\text{s}} \Rightarrow -\frac{0.5^2}{50} = -0.005 \text{ s}^{-2} \simeq 0.005$

4. FOR Å FINNE TOTALT OMLØP $\omega = \frac{d\phi}{dt} \Rightarrow d\phi = \omega \cdot dt$

$$\phi_{tot} = \int_0^\infty \omega(t)\, dt = \int_0^\infty \omega_0 e^{-\omega_0 t/50}$$

$$= \omega_0 \left[\frac{-50}{\omega_0} \cdot e^{(-\omega_0 t/50)} \right]_0^\infty = 0 - (-50) \cdot e^0 = 50 \text{ rad}$$

ANTALL OMLØP $= \frac{\phi_{tot}}{2\pi} = \frac{50}{2\pi} \simeq 8$ (D)

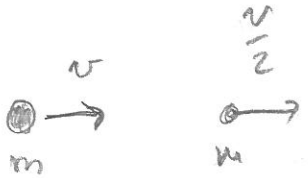
5. Newton: $m \cdot g = 4m \cdot a \Rightarrow a = \frac{g}{4}$

$$v = a \cdot t \Rightarrow h = \frac{at^2}{2} = \frac{g t^2}{8} \Rightarrow t = \sqrt{\frac{8h}{g}}$$ (B)

$$v = a \cdot t = \frac{g}{4} \cdot \sqrt{\frac{8h}{g}} = \sqrt{\frac{g \cdot g}{4 \cdot 4}} \cdot \sqrt{\frac{8h}{g}} = \sqrt{\frac{gh}{2}}$$

6. (C)

7.



$$mv + \frac{mv}{2} = 2mv_f$$

$$\Rightarrow \frac{3mv}{2} = 2mv_f \Rightarrow v_f = \frac{3v}{4}$$

K: FØR: $\frac{9mv^2}{4\cdot 2} + \frac{mv^2}{8} = \frac{5mv^2}{8}$

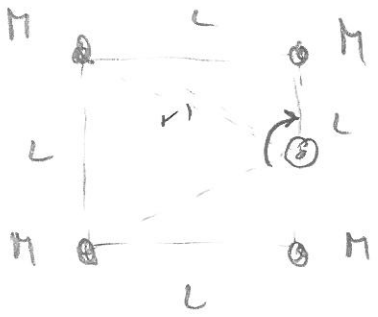
K: ETTER: $\frac{2m\,9v^2}{8\cdot 16} = \frac{9mv^2}{16}$

$$K_{før} - K_{etter} = \frac{5mv^2}{8} - \frac{9mv^2}{16} = \frac{mv^2}{16}$$

$$\Rightarrow \text{tap}: \frac{\frac{mv^2}{16}}{2\cdot\frac{5mv^2}{8\cdot 2}} = \frac{1}{10} \quad 10\%$$

(A)

8.



HVERT HJØRNE: MR^2 – R AVSTAND FRA ROT-AKSJE

$$I = 2M\left(\frac{L}{2}\right)^2 + 2M(r')^2$$

$$(r')^2 = L^2 + \left(\frac{L}{2}\right)^2 = \frac{5L^2}{4}$$

$$\Rightarrow I = \frac{ML^2}{2} + 2M\cdot\frac{5L^2}{4} = \underline{3ML^2}$$

(C)

9. $K_{trans} = \frac{1}{2} M v^2$

$K_{rot} = 4 \cdot \frac{1}{2} I \cdot \omega^2 = 4 \cdot \frac{1}{2} \cdot \frac{1}{2} m r^2 \cdot \left(\frac{v^2}{r^2}\right)$ ($\omega^2 \downarrow \frac{v^2}{r^2}$)

$= m v^2$

$\frac{K_{rot}}{K_{trans}} = \frac{m v^2}{\frac{1}{2} M v^2} = \frac{2m}{M} = \frac{10}{1000} \sim 1\%$ (C)

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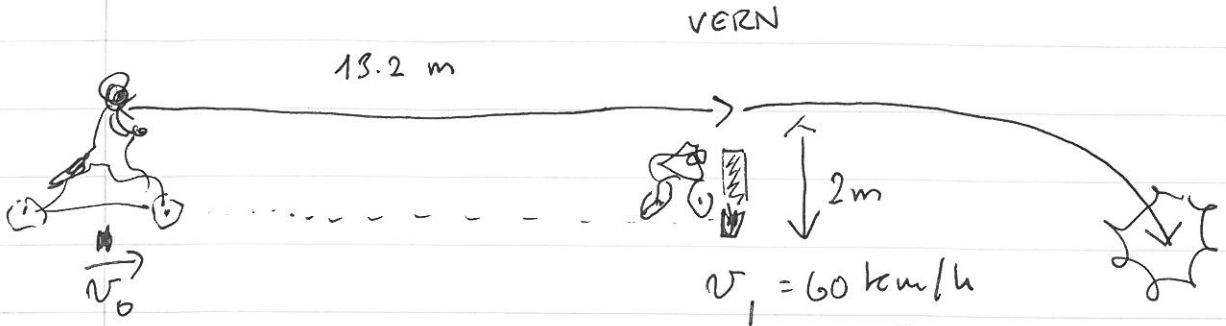
10. $m_1 \cdot g \cdot R_1 = S_2 \cdot R_2$

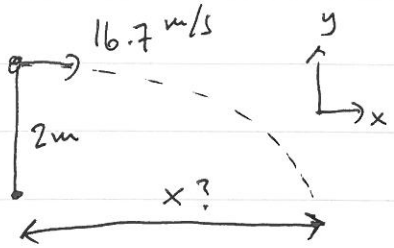
$\Rightarrow S_2 = \frac{m_1 g R_1}{R_2}$ (A)

Del B

Oppgave 11.



a)



I : BEREGN TIDEN

$a_y = -g \qquad y = v_0 \cdot t + \frac{1}{2} a_y t^2$

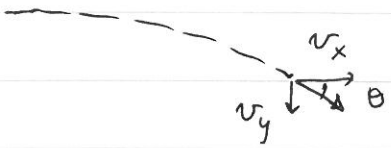
$y = 0$

$-2 = \frac{1}{2}(-g) \cdot t^2 \Rightarrow t = \sqrt{\frac{4}{g}} \simeq 0.639\,s$

II : HVOR LANGT LENGS x ? $\qquad x = v_0 \cdot t = 16.7 \cdot 0.639 \simeq 10.7\,m$

Svar: 11 m

b)



$\tan\theta = \frac{v_y}{v_x}$; $v_y = -gt \simeq 6.27 \frac{m}{s}$

$v_x = 16.7 \frac{m}{s}$

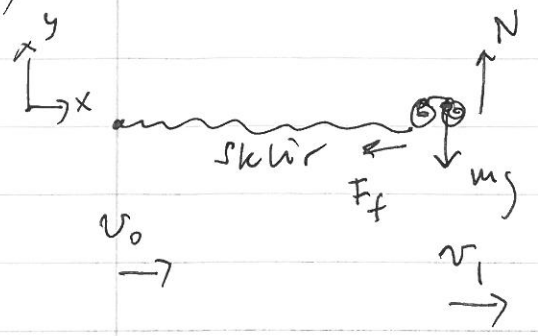
$\Rightarrow \tan\theta = \frac{-6.27}{16.7} \Rightarrow \theta = -20.6^\circ$ vi kan se bort fra tegn.

Svar: 21°

Oppgave 11, kont.

d)

$F_f = \mu \cdot N$



Newton: y-ledd $N - mg = 0$ (1)

x-ledd $-F_f = m \cdot a_x$ (2)

(1) gir $N = mg \Rightarrow F_f = \mu \cdot mg$

→ (2) gir $-\mu mg = m \cdot a_x$ $\therefore a = -\mu \cdot g$ QED

d) ENERGI BETRAKTNING

ENERGI FØR: $\frac{m v_0^2}{2}$

ETTER: $\frac{m v_1^2}{2}$

FRIKSJONS ARBEID: $W_f = F_f \cdot x = -\mu mg \cdot x$

$\frac{1}{2} m v_0^2 - \mu mg \cdot x = \frac{1}{2} m v_1^2$ (1)

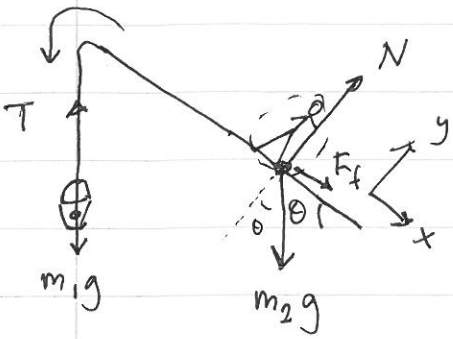
$\frac{1}{2} m (v_0^2 - v_1^2) = \mu mg \cdot x \Rightarrow x = \frac{(v_0^2 - v_1^2)}{2\mu g}$ QED

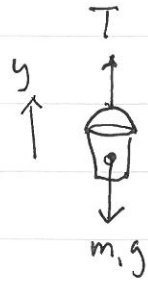
(1) $\Rightarrow v_0^2 = v_1^2 + 2\mu g \cdot x = 434 \frac{m^2}{s^2} \Rightarrow v_0 = 21 \frac{m}{s}$

$16.7 \frac{m}{s}$ ↑ 0.6 ↑ ↑ $13.2\,m$

Oppgave 12

a) HVA ER SNORKRAFTEN?





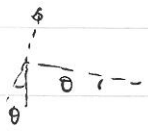
Newton: $T - m_1 \cdot g = m_1 \cdot a_y$

! Vi trenger å beregne a_y

$m_1 = 75\,kg$ (gutt)

$m_2 = 80\,kg$ (mann)

$\mu = 0.1$ friksjon



→ Vi ser på hele systemet i xy-fasene

x-ledd $\quad -m_1 \cdot g + F_f + m_2 \cdot g \cdot \sin\theta = (m_1 + m_2)\, a_x$

Friksjon $\quad F_f = \mu \cdot N = \mu \cdot m_2 \cdot g \cos\theta$

y-ledd $\quad -m_2 \cdot g \cos\theta + N = 0 \Rightarrow N = m_2 g \cos\theta$

$$\Rightarrow -m_1 \cdot g + \mu m_2 \cdot g \cos\theta + m_2 \cdot g \sin\theta = (m_1 + m_2)\, a_x$$

$$\Rightarrow a_x = \frac{1}{(m_1 + m_2)} \cdot g\left(\mu m_2 \cos\theta + m_2 \sin\theta - m_1\right)$$

$$= \frac{1}{(75+80)} \cdot 9.81\left(0.1 \cdot 80 \cdot \cos 45^\circ + 80 \sin 45^\circ - 75\right) = 0.808\ \frac{m}{s}$$

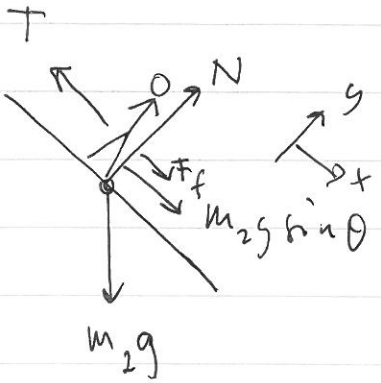
Vi ser i a) oppgaven tilsvarer $a_y = a_x$

$$T - m_1 g = m_1 \cdot a_y \Rightarrow T = m_1(g + a_x) = 75 \cdot (9.81 - 0.808) = 675\,N$$

SVAR 670N

Oppgave 12, forts

Vi sjekker T/snoret på andre siden, må være lik då ikke noen friksjon i hjul...



NEWTON $\quad -T + F_f + m_2 g \sin\theta = m_2 \cdot a_x$

$$\Rightarrow T = F_f + m_2 g \sin\theta - m_2 a_x$$

$$= \mu \cdot m_2 g \cos\theta + m_2 \cdot g \sin\theta - m_2 a_x$$

$$= 0.1 \cdot 80 \cdot 9.81 \cdot \frac{1}{\sqrt{2}} + 80 \cdot \frac{1}{\sqrt{2}} \cdot 9.81 - (-0.808) \cdot 80 = 674\,N$$

Innen feilmargin samme

b) $v = v_0 + a \cdot t$ ($v_0 = 0$)

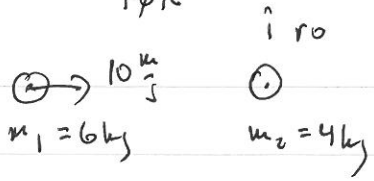
$$\frac{dx}{dt} = a \cdot t \Rightarrow x - x_0 = \frac{1}{2} a t^2 \Rightarrow t^2 = \frac{2x}{a} = \frac{10}{0.808}$$

$$\Rightarrow t = 3.52\ s$$

$$v = a \cdot t = 0.808 \cdot 3.52 = 2.84\ \frac{m}{s} \qquad \text{Fasit: } 2.8\ \frac{m}{s}$$

FØR ETTER?

Oppgave 13.



BEVARINGSLOV:

$\vec{p}$ $\quad m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$ $\quad (1)$

energi $\quad \frac{m_1 v_{1i}^2}{2} = \frac{m_1 v_{1f}^2}{2} + \frac{m_2 v_{2f}^2}{2} \Rightarrow m_1 (v_{1i}^2 - v_{1f}^2) = m_2 v_{2f}^2 \quad (2)$

$(2) \Rightarrow m_1 (v_{1i} + v_{1f})(v_{1i} - v_{1f}) = m_2 v_{2f}^2$

$(1) \Rightarrow m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f}) \Rightarrow (v_{1i} - v_{1f}) = \frac{m_2}{m_1} v_{2f}$

$\Rightarrow m_1 (v_{1i} + v_{1f}) \cdot \frac{m_2}{m_1} \cdot v_{2f} = m_2 v_{2f}$

$\therefore \underline{v_{1i} + v_{1f} = v_{2f}} \rightarrow (1)$

$m_1 v_{1i} = m_1 v_{1f} + m_2 (v_{1i} + v_{1f}) \Rightarrow v_{1i} (m_1 - m_2) = v_{1f} (m_1 + m_2)$

$\Rightarrow \underline{v_{1f} = v_{1i} \left(\frac{m_1 - m_2}{m_1 + m_2} \right)}$

MED TALL $\quad v_{1f} = 10 \cdot \left(\frac{6-4}{6+4} \right) = 2$

$v_{2f} = v_{1i} + v_{1f} = 10 + 2 = 12$

Svar: $\begin{cases} v_{1f} = 2 \frac{m}{s} \\ v_{2f} = 12 \frac{m}{s} \end{cases}$

KONTROLL

$(1) \Rightarrow 6 \cdot 10 = 6 \cdot 2 + 4 \cdot 12$ OK $\quad (2) \Rightarrow \frac{6 \cdot 100}{2} = \frac{6 \cdot 4}{2} + \frac{4 \cdot 144}{2}$ OK