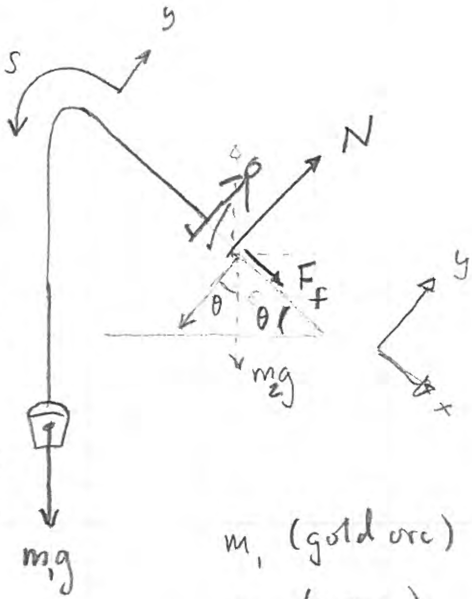
TFY4104 ; 17 des, 2011 LØSNINGSFORSLAG

P1:

TO SOLVE EQ. OF MOTIONS FOR BUCKET IN yS-FRAME AND CALC TENSION WE NEED TO FIND ACCELERATION FROM NEWTON'S 2ND LAW



IN xy-FRAME:

$$-m_2 g\cdot\cos\theta + N = 0$$

FRICTION FORCE:

$$F_f = \mu\cdot N = \mu m_2 g\cos\theta$$

m_1 (gold ore) $= 75$ kg ; $\mu = 0.1$

m_2 (man) $= 80$ kg

IN yS-FRAME:

Newton 2nd:

$$m_1\cdot g - F_f - m_2 g\cdot\sin\theta = (m_1+m_2)\cdot a_s$$ ACCELERATION ALONG $\hat{S}$

F_f: put in

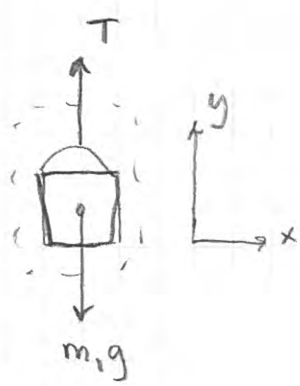
$$m_1 g - \mu m_2 g\cos\theta - m_2 g\sin\theta = (m_1+m_2)\,a_s$$

$$\Rightarrow\quad a_s = \frac{1}{(m_1+m_2)}\cdot g\cdot\left(m_1 - \mu m_2\cos\theta - m_2\sin\theta\right)$$

WITH NUMBERS: $$\underline{a_s} = \frac{1}{(75+80)}\cdot 9.81\,(75 - \overbrace{0.1\cdot 80\cdot\cos 45^\circ}^{5.657} - \overbrace{80\cdot\sin 45^\circ}^{56.57})$$

$$\underline{= 0.808\ \mathrm{m/s^2}}$$

a) TENSION IN THE ROPE ?

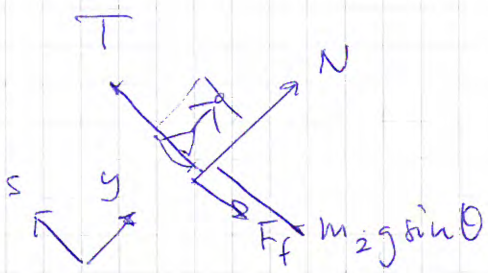


$$T - m_1 g = -m_1\cdot a_s \qquad \text{for BUCKET}$$

$$\Rightarrow T = m_1\,(g - a_s) = 75\,(9.81 - 0.808) = \underline{675\ \mathrm{N}}$$

P1, cont

a) TENSION IN THE ROPE, OTHER SIDE



$$T - m_2 g \sin\theta - F_f = m_2 \cdot a_s$$

$$T = m_2 a_s + m_2 g \sin\theta + F_f$$

$$= \underbrace{80 \cdot 0.808}_{64.6\,N} + \underbrace{80 \cdot 9.81 \cdot \frac{1}{\sqrt{2}}}_{554\,N} + \underbrace{0.1 \cdot 80 \cdot 9.81 \cdot \frac{1}{\sqrt{2}}}_{55.5\,N} = 674\,N$$

OK; SAME AS FOR m_1

Ans. 670 N

b) ACCELERATION CONSTANT, USE $v - v_0 = a \cdot t$ $\quad v_0 = 0$

TIME? USE $\underbrace{x - x_0}_{5m} = \underbrace{v_0 t}_{0m} + \frac{a t^2}{2} \Rightarrow t^2 = \frac{2 \cdot x}{a} = \frac{10}{0.808} \frac{m}{m/s^2}$

$= 12.4\ s^2$

$$t = \sqrt{12.4} = 3.52\ s$$

$$v = a \cdot t = 0.808 \cdot 3.52 \left[\frac{m}{s^2}\right] \cdot [s] = 2.84\ \frac{m}{s}$$

Ans. $2.8\ \frac{m}{s}$

P2

Løsning: eksamen i TFY 4104

a) Tilført varme:

$$Q = C_p \cdot \Delta T = (C_v + nR) \cdot \Delta T = \frac{7}{2} nR\Delta T$$

$$Q = \frac{7}{2} \cdot 1\,\text{mol} \cdot 8{,}314\ \text{J/mol}\cdot\text{K} \cdot 80\text{K}$$

$$\underline{Q = 2{,}3\ \text{kJ}}$$

Volum før oppvarming:

Ideell gasslov: $PV = nRT$

$$V = \frac{nRT}{P} = \frac{1\,\text{mol} \cdot 0{,}08206\ \text{L}\cdot\text{atm/mol}\cdot\text{K} \cdot 293\text{K}}{1\ \text{atm}}$$

$$\underline{V = 24\ \text{L}}$$

Arbeid gjort på omgivelsene:

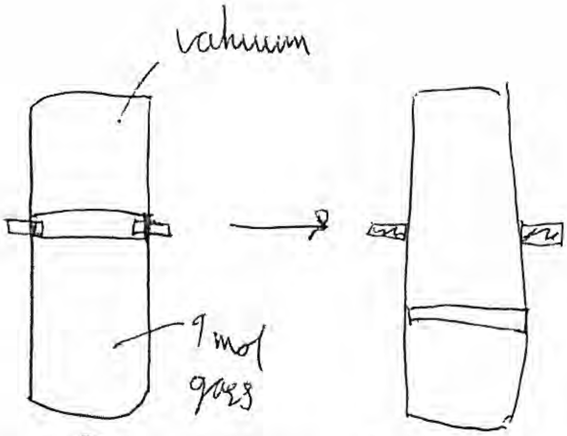
$$W_{\text{by gas}} = \int P\,dV = P\Delta V = P(V_2 - V_1)$$

$$= PV_2 - PV_1$$

$$= nRT_2 - nRT_1$$

$$W_{by} = nR(T_2 - T_1) = nR\Delta T$$

$$W_{by} = 1\,\text{mol} \cdot 8{,}314\ \text{J/mol}\cdot\text{K} \cdot 80\text{K} = \underline{0{,}67\ \text{kJ}}$$



b) En prosess uten varmeoverføring kalles adiabatisk.
For en adiabatisk kvasistatisk prosess gjelder (for ideell gass):

er denne oppgitt?

$$T_1 V^{\gamma-1} = \text{konstant} \qquad \gamma = \frac{C_p}{C_V} = \frac{\frac{7}{2}nR}{\frac{5}{2}nR} = \frac{7}{5}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300\,K \cdot 2^{2/5} = \underline{396\,K}$$

Arbeid gjort på gassen:

$$\Delta E_{int} = 0 + W_{on}$$

$$W_{on} = C_V \Delta T = \frac{5}{2} nR (T_2 - T_1)$$

$$W_{on} = \frac{5}{2} \cdot 9\,mol \cdot 8{,}314\,J/mol\cdot K \cdot 96\,K$$

$$\underline{W_{on} = 2\,kJ}$$

c) Utført arbeid:

$$W = Q_h - Q_c$$

Fra termodynamikkens første lov: $\Delta E_{int} = 0$ for en hel syklus =>

$$0 = Q_{in} + W_{on} = Q_{in} - W$$

$$W = Q_h - Q_c$$

$$W = 200J - 150J$$

$$\underline{W = 50\ J}$$

Effektivitet:

$$\varepsilon = \frac{W}{Q_h} = \frac{50J}{200J} = 0,25 = \underline{25\ \%}$$

En Carnotmaskin har maksimal effektivitet:

$$\varepsilon_c = \left(1 - \frac{T_c}{T_h}\right)$$

Tapt arbeid:

$$W_{lost} = W_{max} - W = \varepsilon_c Q_h - W = \left(1 - \frac{273K}{573K}\right) \cdot 200J - 50J$$

$$W_{lost} = 0,524 \cdot 200J - 50J = 104,8J - 50J = \underline{54,8J}$$

Endring i entropi

$\Delta S_{system} = 0$ fordi S en en tilstandsvariabel, og må være det samme etter en hel syklus

$\Delta S_{omgivelser} = \Delta S_h + \Delta S_c$ Entropiendring i kaldt og varmt reservoar.

$= \frac{-Q_h}{T_h} + \frac{Q_c}{T_c}$

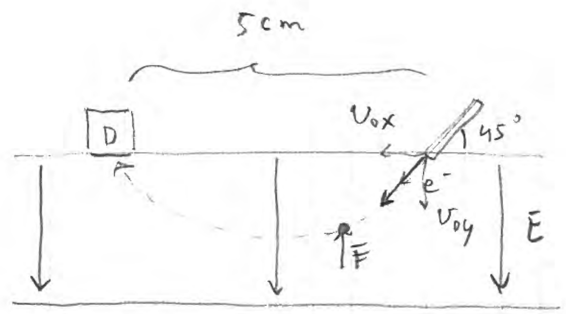
$\Delta S_{omgivelser} = \frac{-200J}{573K} + \frac{150J}{273K} = 0{,}20\ J/K$

$\Delta S_u = \Delta S_{system} + \Delta S_{omgivelser} = 0 + 0{,}20\ J/K = 0{,}20\ J/K$

En maksimalt effektiv varmemaskin må være reversibel (Carnotmaskin).
For en reversibel prosess er

$\Delta S_u = 0$

P3:



$|E| = 4\,kN/C$

$m_e = 9.1 \cdot 10^{-31}$ kg

a) DIRECTION OF FIELD; AS e^- NEGATIVE; FIELD SHOULD GO FROM TOP → DOWN; SEE FIG. (POSITIVE CHARGE IS 'FOLLOWING' FIELD LINES, NEG OTHER WAY)

b) TO FIND W_{kin}; WE NEED VELOCITY FROM $\frac{mv^2}{2} = W_{kin}$

NEWTON $\bar{F} = q \cdot \bar{E}$ IS ACCELERATING ELECTRONS UPWARDS

$$\Rightarrow m_e \cdot a_y = q \cdot E \quad \Rightarrow a_y = \frac{q \cdot E}{m_e}$$

WITH NUMBERS $a_y = \dfrac{1.602 \cdot 10^{-19}\,C \cdot 4000\,N\frac{1}{C}}{9.1 \cdot 10^{-31}\,kg} = 7.04 \cdot 10^{14}\,m/s^2$

ALSO,

$$v_{ox} = v_{oy} = \frac{1}{\sqrt{2}} \cdot v \qquad (\theta = 45°)$$

WE HAVE TWO EXPRESSIONS TO CALCULATE TIME TO REACH DETECTOR:

(1) DISTANCE $\Delta x = 5\,cm = v_{ox} \cdot \Delta t$

(2) TIME TO SLOW TO ZERO (v_y VELOCITY), AND ACCELERATE BACK

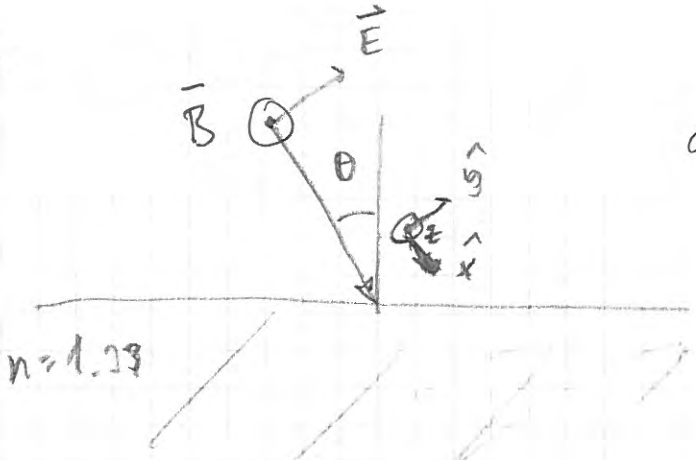
$$v_y - v_{oy} = -a_y \Delta t' \quad \Rightarrow \Delta t' = \frac{v_{oy}}{a_y} \quad ; \quad \text{N.B. } \Delta t = 2\Delta t'$$

$$\Rightarrow \Delta x = v_{ox} \cdot \underbrace{2 \cdot \frac{v_{oy}}{a_y}}_{\Delta t = 2\Delta t'} \Rightarrow \Delta x \cdot a_y = v^2 \quad \text{AS } v_{ox} = v_{oy} = \frac{1}{\sqrt{2}} v$$

$$\Rightarrow W_{kin} = \frac{m_e \cdot v^2}{2} = \frac{9.1 \cdot 10^{-31}\,kg\, 0.05\,m \cdot 7.04 \cdot 10^{14}\,m/s^2}{2} = \underline{1.60 \cdot 10^{-17}\,J}$$

$$= 100\,eV$$

P4:



a) FIND DIRECTION OF B-FIELD GIVEN $\bar{E}$ AS IN FIG.

INTRODUCE xyz AS SHOWN

FARADAY LAW

$$\boxed{\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

Let prop direction be $\hat{x}$ AND $\bar{E} = E_0 \cdot \hat{y}$ AS IN FIG

$$\Rightarrow \quad \bar{E} = E_0 \cos(kx - \omega t)\, \hat{y} \quad ;$$

$\bar{\nabla} \times \bar{E}$ gives

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 \cos(kx - \omega t) & 0 \end{vmatrix} = \hat{z} E_0 k(-\sin(kx - \omega t)) - \underbrace{\hat{x} \cdot \frac{\partial}{\partial z}(\quad)}_{=0}$$

NO Z-DEPENDENCE

$$\therefore -\hat{z} \cdot E_0 k \sin(kx - \omega t) = \bar{\nabla} \times \bar{E}$$

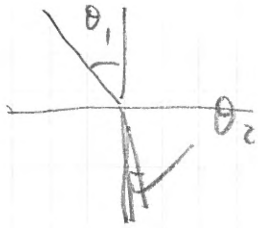
$$-\frac{\partial \bar{B}}{\partial t} = -\left(\frac{\partial B_x}{\partial t} \hat{x} + \frac{\partial B_y}{\partial t} \hat{y} + \frac{\partial B_z}{\partial t} \cdot \hat{z}\right)$$

INTEGRATION

IDENTIFY: $-\frac{\partial B_z}{\partial t} = -k E_0 \sin(kx - \omega t) \Rightarrow -B_z = -k\left(\frac{E_0}{-\omega}\right)(-)\cos(kx - \omega t)$

$\therefore B_z = \frac{k E_0}{\omega} \cos(kx - \omega t)$ Directed along $\hat{z}$; out from the plane of the paper, see fig.

P4) b



Snell's law

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$n_1 = 1$; $\sin 30^\circ = \frac{1}{2}$; $n_2 = 1.33$

$\Rightarrow \sin \theta_2 = \frac{0.5}{1.33}$ $\Rightarrow \theta_2 = 22.1^\circ$

Ans. IT PROPAGATES AT AN ANGLE 22.1° FROM THE NORMAL DOWNWARDS