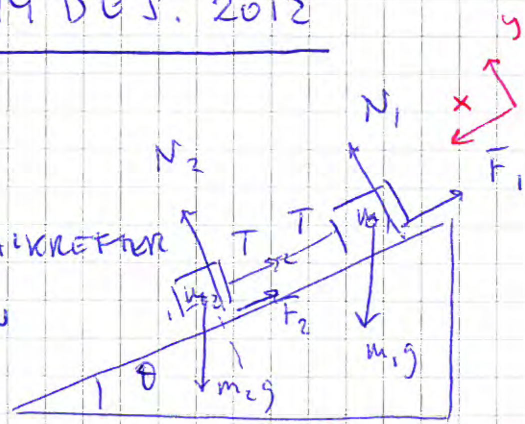
TFY4104 – 14 DES. 2012

OPPGAVE 1

1)



$m_1 = 0.9$ kg

$m_2 = 0.3$ kg

$\mu_1 = 0.2$

$\mu_2 = 0.1$

N_1, N_2 NORMALKREFTER

F_1, F_2 FRIKSJON

2x FRITT LEGEMEDIAGRAM

a) NÅR BEVEGELSE STARTER $\bar{a} \simeq 0$

$$N_1 = m_1 g \cos\theta_c \; ; \; N_2 = m_2 g \cos\theta_c \; ; \; F_1 = \mu_1 N_1 \; ; \; F_2 = \mu_2 N_2$$

$$\hat{x}1: \quad m_1 g \sin\theta_c + T - F_1 = 0 \quad \Rightarrow \quad m_1 g \sin\theta_c + T - \mu_1 m_1 g \cos\theta_c = 0$$

$$\hat{x}2: \quad m_2 g \sin\theta_c - T - F_2 = 0 \quad \Rightarrow \quad m_2 g \sin\theta_c - T - \mu_2 m_2 g \cos\theta_c = 0$$

$$\Sigma \quad (m_1+m_2)\cancel{g}\sin\theta_c - (\mu_1 m_1 + \mu_2 m_2)\cancel{g}\cos\theta_c = 0$$

$$\Rightarrow \tan\theta_c = \frac{\mu_1 m_1 + \mu_2 m_2}{(m_1+m_2)} = \frac{0.9 \cdot 0.2 + 0.3 \cdot 0.1}{0.9 + 0.3} = 0.175 \Rightarrow \underline{\underline{\theta_c = 9.9^\circ}}$$

b) I BEVEGELSE $\mu_1 = 0.18$; $\mu_2 = 0.09$; $\theta = 20^\circ$

LEGG TIL m_1-a OG m_2-a TIL H.L. I $\hat{x}1$ OG $\hat{x}2$

$$\Rightarrow (m_1+m_2) g \sin\theta - (\mu_1 m_1 + \mu_2 m_2) g \cos\theta = (m_1+m_2) a$$

$$\Rightarrow a = g \cdot \left[\sin\theta - \frac{(\mu_1 m_1 + \mu_2 m_2)}{(m_1 + m_2)} \cdot \cos\theta \right]$$

$$= 9.81 \cdot \left[\underbrace{\sin 20^\circ}_{0.3420} - \underbrace{\frac{(0.18 \cdot 0.9 + 0.09 \cdot 0.3)}{(0.9 + 0.3)}}_{0.1575} \cdot \underbrace{\cos 20^\circ}_{0.9396} \right] = 1.903$$

$$\Rightarrow \underline{\underline{a = 1.9 \ m/s^2}}$$

$$T = -m_1 g \sin 20^\circ + \mu_{1k} \cdot m_1 g \cos 20^\circ + m_1 \cdot a = 0.186 \, N$$

$\hat{x}1 = m_1 a$; $\mu_{1k} = 0.18$; $m_1 = 0.9$; $a = 1.9 \ m/s^2$

$\underline{\underline{T = 0.19 \ N}}$

OPPGAVE 1, FORTS.

c) $a = 1.903\ m/s^2$ $v = v_0 + at$; $\Delta x = v_0 t + \frac{1}{2}at^2$ FÅS

LETT VED INTEGRERING AV $v = \frac{dx}{dt}$ OG $a = \frac{dv}{dt}$

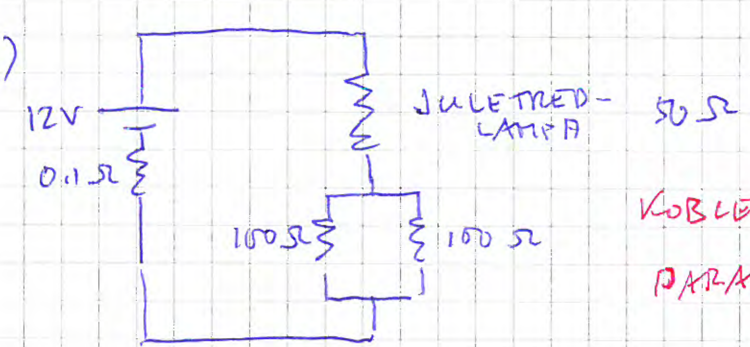
$$\Delta x = 0.25 = \frac{1}{2}at^2 \quad (v_0 = 0) \implies t = \sqrt{\frac{2 \cdot 0.25}{a}} = 0.5126\ s$$

$$v = v_0 + at \implies v = 1.903 \cdot 0.5126 = 0.9754\ m/s$$

$$\underline{\underline{v = 0.98\ m/s}}$$

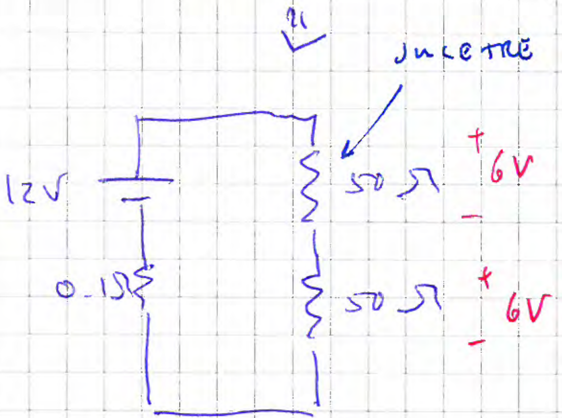
OPPGAVE 2.

a)



KOBLE 2 100 Ω RES I PARALELL $\frac{1}{R_{tot}} = \frac{1}{100} + \frac{1}{100} = \frac{2}{100} = \frac{1}{50}$

⇓



VI NEGLISJERER 0.1 Ω OG SER APPROX 6V OVER JULETREET.

$$R_{tot} = 50 + 50 + 0.1 = 100.1\ \Omega$$

b) $12V = R_{tot} \cdot I \implies I = \frac{12}{100.1} = 0.120\ A$

JA DEN KLARER DET...

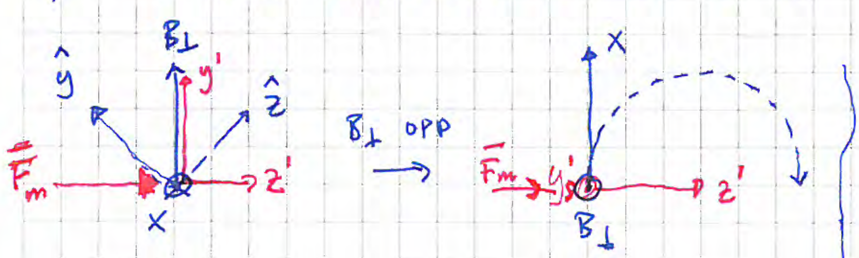
3) $v_x = 8.0\cdot 10^5 \frac{m}{s}$; $\bar{B} = (1,1,1)\,0.1\ T$; $q = 2e = 2\cdot 1.609\cdot 10^{-19}\ C$

a) $$\bar{F}_m = q(\bar{v}\times\bar{B}) = \underbrace{\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{vmatrix}}\cdot \underbrace{8\cdot 10^5\cdot 0.1\cdot 2\cdot 1.609\cdot 10^{-19}}_{2.57\cdot 10^{-14}\ N}$$

$$(-\hat{y}+\hat{z}) = \sqrt{2}\cdot\underbrace{\left(-\frac{1}{\sqrt{2}}\hat{y}+\frac{1}{\sqrt{2}}\hat{z}\right)}_{\text{unit vector}} \longrightarrow F_m = \left(-\frac{1}{\sqrt{2}}\hat{y}+\frac{1}{\sqrt{2}}\hat{z}\right)\cdot 3.64\cdot 10^{-14}\ N$$

ans.. $3.6\cdot 10^{-14}\ N$ rettet $\left(-\frac{1}{\sqrt{2}}\hat{y}+\frac{1}{\sqrt{2}}\hat{z}\right)$

b) $\bar{B}$ HAR KOMPONENT $B_\perp$ $\bar{v}$ (v_x) i yz-PLANET



MAGNETFELT $B_\perp$ VIL DREIE JON I SIRKULAER BANE, MEN, SOM DE

FINNES $\bar{B}$ KOMPONENT LANGS $\hat{x}$ VIL SIRKULAER-BANEN BLI HELIX, HVOR ER HELIX RETTET? (OPP ELLER NED?)

$B_\perp$ HAR RETNING $\frac{2}{\sqrt{6}}$ LANGS $\hat{y}'$ OG $\frac{1}{\sqrt{3}}$ LANGS $\hat{x}$ (0 LANGS $\hat{z}'$)

$\bar{F}_m$ GIR LITEN HASTIGHET LANGS $\hat{z}'$ ⇒ MINKNING LANGS $\hat{x}$

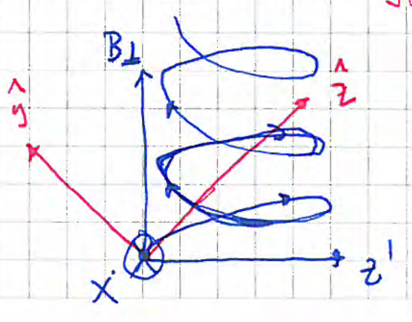
⇒ ETTER LITEN TID $\bar{v} = (\underset{\text{MINKER LANGS } \hat{x}}{v_x - \Delta}, 0, \underset{\uparrow\text{ØKER LANGS } z'}{+\Delta'})$; HVA BLIR KRAFT FRA B_x?

$$\bar{F}'_{m,ny} = \begin{vmatrix} \hat{x} & \hat{y}' & \hat{z}' \\ v_x-\Delta & 0 & \Delta' \\ \frac{1}{\sqrt{3}} & \frac{2}{\sqrt{6}} & 0 \end{vmatrix} |B|q = \left[\hat{x}\left(-\Delta'\cdot\frac{2}{\sqrt{6}}\right) + \hat{y}'\left(\Delta'\frac{1}{\sqrt{3}}\right) + \hat{z}'\frac{(v_x-\Delta)\cdot 2}{\sqrt{6}}\right]|B|\cdot q$$

DREIER SIRKULAER BANE

~~DY~~ ~~FL~~TER JONER OPP-OVER $\left[\hat{y}'\ (B_\perp)\right]$

⇒ JONENS BANE



VENSTRE SKRU

[SE KAP 26.2 I TB]

3 FORTS.

c)

MED ELEKTRISK FELT KAN $\bar{F}_m$ KANSELLERES

$\bar{F}_e = q \cdot \bar{E}$; RETTET MOT $\bar{F}_m$

$\bar{F}_m$ HAR RETNING $\left(-\frac{1}{\sqrt{2}} \hat{y} + \frac{1}{\sqrt{2}} \hat{z} \right)$

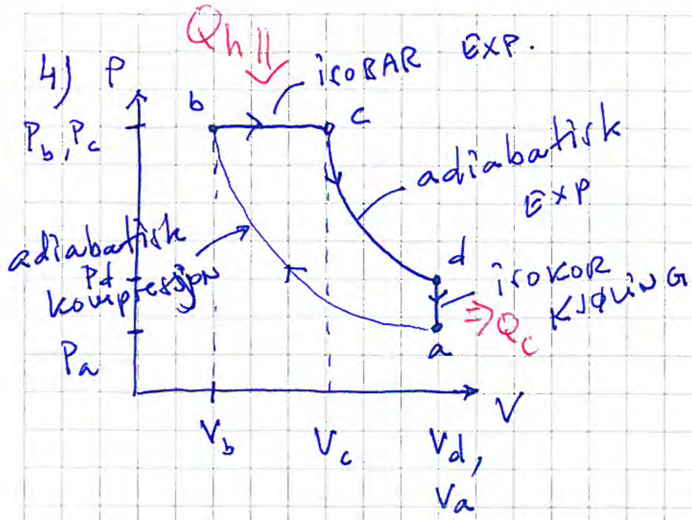
$\Rightarrow \bar{F}_e$ MÅ HA RETNING $\left(\frac{1}{\sqrt{2}} \hat{y} - \frac{1}{\sqrt{2}} \hat{z} \right)$

d) FELTETS STYRKE GIS AV

$$|\bar{F}_m| = q \cdot |\bar{E}| \Rightarrow |\bar{E}| = \frac{|\bar{F}_m|}{2e} = \frac{3.62 \cdot 10^{-14}\,N}{2 \cdot 1.602 \cdot 10^{-19}\,C}$$

$\Rightarrow$ FELTETS STYRKE $|E| \simeq 110 \ \frac{kV}{m}$

4)



a) SE BILDE

b) Effektivitet er gitt av

$$\epsilon = 1 - \frac{Q_c}{Q_h}$$

N.B. DEFINERT VAERME AVGITT

IKKE NOEN VAERMEOVERFØRING FOR DI ADIABATISKE PROSESSENE $c \to d$ og $a \to b$

$$b \to c \qquad dQ = C_p\, dT \Rightarrow Q_h = C_p (T_c - T_b)$$

MÅ HA − FEGN DÅ Q_c' ER VAERME GJORT PÅ GAS

$$d \to a \qquad dQ' = C_v\, dT \Rightarrow -Q_c = C_v (T_a - T_d)$$

Q_{in} TILL GAS

$$\Rightarrow \epsilon = 1 - \frac{-C_v (T_a - T_d)}{C_p (T_c - T_b)} = 1 + \frac{1}{\gamma} \frac{(T_a - T_d)}{(T_c - T_b)} \text{ : ans.}$$

SETT INN

c) $$\frac{P_b V_b}{T_b} = \frac{P_c V_c}{T_c} \overset{P_b = P_c}{\Longrightarrow} \frac{T_c}{T_b} = \frac{V_c}{V_b} \Rightarrow T_b = T_c \cdot \frac{V_b}{V_c}$$

$$\Rightarrow \epsilon = 1 + \frac{1}{\gamma} \frac{1}{T_c} \frac{(T_a - T_d)}{\left(1 - \frac{V_b}{V_c}\right)} \quad ; \quad \frac{P_c V_c}{T_c} = \frac{P_a V_a}{T_a} \quad ; \quad \frac{P_c V_c}{T_c} = \frac{P_d V_d}{T_d}$$

SETT INN

$$T_a = T_c \cdot \frac{P_a V_a}{P_c V_c} \qquad T_d = T_c \frac{P_d V_d}{P_c V_c}$$

$$\Rightarrow \epsilon = 1 + \frac{1}{\gamma} \frac{1}{\cancel{T_c}} \frac{\left[\cancel{T_c}\left(\frac{P_a V_a}{P_c V_c}\right) - \cancel{T_c}\left(\frac{P_d V_d}{P_c V_c}\right)\right] \times V_c}{\left(1 - \frac{V_b}{V_c}\right) \times V_c}$$

$$\Rightarrow \epsilon = 1 + \frac{1}{\gamma} \frac{\left(\frac{P_a V_a}{P_c} - \frac{P_d V_d}{P_c}\right)}{(V_c - V_b)} \quad P_c = P_b \Rightarrow$$

$$\epsilon = 1 + \frac{1}{\gamma} \frac{\frac{P_a V_a}{P_b} - \frac{P_d V_d}{P_c}}{(V_c - V_b)}$$

FORTS.

4) c) FORTS.

NÅ BRUK ADIABATISKE PROSESSENE $a \to b$; $c \to d$

$$P_a V_a^{\gamma} = P_b V_b^{\gamma} \Rightarrow \frac{P_a}{P_b} = \frac{V_b^{\gamma}}{V_a^{\gamma}}$$

$$P_c V_c^{\gamma} = P_d V_d^{\gamma} \Rightarrow \frac{P_d}{P_c} = \frac{V_c^{\gamma}}{V_d^{\gamma}}$$

SETT INN I ϵ

$$\Rightarrow \epsilon = 1 + \frac{1}{\gamma} \frac{\left(V_b^{\gamma} V_a^{1-\gamma} - V_c^{\gamma} V_d^{1-\gamma}\right)}{(V_c - V_b)}$$

SETT IN

VI HADDE $V_a = V_d$

$$\epsilon = 1 + \frac{1}{\gamma} \cdot \frac{V_a^{1-\gamma}\left(V_b^{\gamma} - V_c^{\gamma}\right)}{(V_c - V_b)} \overset{\text{tegn}}{=} 1 - \frac{1}{\gamma} \frac{V_a^{1-\gamma} V_b^{\gamma}\left(V_c^{\gamma} - V_b^{\gamma}\right) \frac{1}{V_b^{\gamma}}}{V_b (V_c - V_b) \frac{1}{V_b}} =$$

$$= 1 - \frac{1}{\gamma} \frac{V_a^{1-\gamma}}{V_b^{1-\gamma}} \frac{\left[\left(\frac{V_c}{V_b}\right)^{\gamma} - 1\right]}{\left[\left(\frac{V_c}{V_b}\right) - 1\right]} = 1 - \frac{1}{\left(\frac{V_a^{\gamma-1}}{V_b^{\gamma-1}}\right)\gamma} \frac{\left[\left(\frac{V_c}{V_b}\right)^{\gamma} - 1\right]}{\left[\left(\frac{V_c}{V_b}\right) - 1\right]} =$$

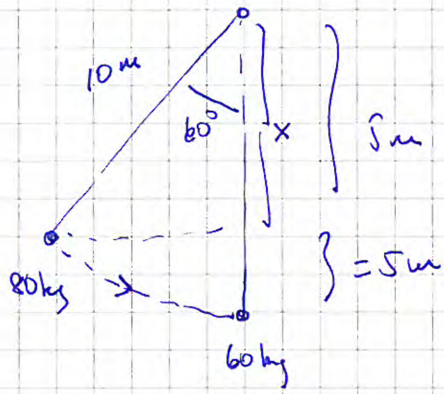
$$= 1 - \frac{1}{\gamma \left(\frac{V_a}{V_b}\right)^{\gamma-1}} \frac{\left[\left(\frac{V_c}{V_b}\right)^{\gamma} - 1\right]}{\left[\left(\frac{V_c}{V_b}\right) - 1\right]} = 1 - \frac{1}{\gamma r^{\gamma-1}} \left[\frac{\alpha^{\gamma} - 1}{\alpha - 1}\right] =$$

MED $r = \frac{V_a}{V_b}$ OG

$\alpha = \frac{V_c}{V_b}$

QED

5)



$\} = 5\,m = h$

$\frac{x}{10} = \cos 60^\circ \Rightarrow x = 10 \cos 60^\circ = 5\,m$

a) BRUK ENERGIPRINSIPPET: $\cancel{m} g h = \frac{\cancel{m} v^2}{2}$ $\quad v = \sqrt{2gh}$

$g = 9.81\ m/s^2$; $h = 5\,m \Rightarrow \underline{v = 9.9\ m/s}$

b) TARZAN PRESIS FØR TREFF

$T - mg = \frac{m v^2}{r}$ (F) SIRKELBEVEGELSE $\Rightarrow$ CENTRIPETAL AKSELERASJON

POSITIV INN MOT SIRKEL-SENTRUM

$\Rightarrow T = mg + \frac{mv^2}{r} = m\left(g + \frac{2gh}{r}\right) = mg\left(1 + \underbrace{\frac{2 \cdot h}{r}}_{=1}\right) = 2mg$

SIFFER: $T = 2 \cdot 80\,kg \cdot 9.81\,m/s^2 = 1570\ N \simeq \underline{1600\ N}$

c) $m_1 v_1 + m_2 v_2 = (m_1 + m_2) v_3$

FØR $\quad$ ETTER

$v_1 = 9.9\ m/s$ (TARZAN), $v_2 = 0$ (JANE)

$v_3 = \frac{m_1}{(m_1 + m_2)} \cdot v_1$

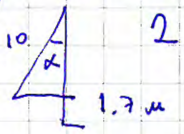
$= \frac{80\ kg}{140\ kg} \cdot 9.9\ \frac{m}{s} \simeq \underline{5.7\ \frac{m}{s}}$

d) SOM (b) NÅ $m = m_1 + m_2$

$T - (m_1 + m_2) g = \frac{(m_1 + m_2) v_3^2}{r} \Rightarrow T = 140\,kg \left(9.81\ \frac{m}{s^2} + \frac{5.7^2 \left(\frac{m}{s}\right)^2}{10\ m}\right)$

$\simeq \underline{1800\ N}$

e) $\frac{\cancel{(m_1 + m_2)} v_3^2}{2} = \cancel{(m_1 + m_2)} g h \Rightarrow h = \frac{v_3^2}{2g} = \frac{5.7^2\ \frac{m^2}{s^2}}{2 \cdot 9.81\ \frac{m}{s^2}} = 1.7\ m$



$\cos \alpha = \left(\frac{10 - 1.7}{10}\right) \Rightarrow \underline{\alpha = 33^\circ}$