Løsningsforslag Eksamen 19. mai 2009. (1)

Oppgave 1.

a) Horisontal bevegelsen: $v_x = v \cdot \cos\theta \qquad x(t) = v_x \cdot t$

Vertikal bevegelsen: $v_y = v \cdot \sin\theta - g \cdot t$

$$y(t) = v_y t - \tfrac{1}{2} g t^2$$

Varighet av flyturen: t_1

Fra horisontal bevegelsen: $\underline{t_1} = \dfrac{x(t_1)}{v_x} = \underline{\dfrac{s_0}{v \cdot \cos\theta}}$

Fra vertikal bevegelsen: $y(t_1) = h_0$

$$\Rightarrow \quad h_0 = v_y \cdot t_1 - \tfrac{1}{2} g t_1^2$$

$$h_0 = \cancel{v} \cdot \sin\theta \cdot \frac{s_0}{\cancel{v} \cdot \cos\theta} - \frac{1}{2} g \frac{s_0^2}{v^2 \cos^2\theta} \qquad \Big| \; v^2 \cdot \cos^2\theta \cdot 2$$

$$2v^2 \cdot h_0 \cos^2\theta = 2v^2 \cdot s_0 \sin\theta \cdot \cos\theta - g s_0^2$$

$$\Rightarrow \quad v^2 = \frac{s_0^2 \cdot g}{2\cos\theta\,(s_0 \sin\theta - h_0 \cos\theta)}$$

$$\Rightarrow \quad \underline{\underline{v = s_0 \sqrt{\frac{g}{2\cos\theta\,(s_0 \sin\theta - h_0 \cos\theta)}}}} \qquad \text{q.e.d.}$$

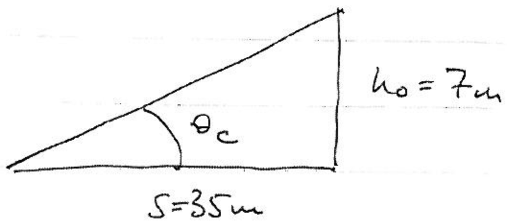
b) $v(\theta) \to \infty$ for $\theta \to \theta_c$

θ_c gitt ved at $s_0 \sin\theta_c - h_0 \cos\theta_c = 0$

$$\Rightarrow \underline{\underline{\tan\theta_c = \frac{h_0}{s_0}}} \Rightarrow \underline{\underline{\theta_c = \arctan\left(\frac{h_0}{s_0}\right)}}$$

Tallverdi: $\underline{\underline{\theta_c}} = \arctan\left(\frac{7.0}{35.0}\right) = \underline{\underline{11,3°}}$

Tolkning:



For $\theta = \theta_c$ må bilen "fly" rettlinjet fra ó nå treffpunktet, dvs. det kan ikke tillates noen avbøyning pga. tyngdens akselerasjon, dvs. "flytiden" må gå mot null, og tilsvarende $v \to \infty$.

Hvis $\theta < \theta_c$ vil ~~bilen~~ bilen treffe $y < h_0$ selv om $v \to \infty$. Matematisk blir v da gitt som $\sqrt{-\text{tall}^2}$ dvs. et imaginært tall. Begge deler angir at problemet ikke har løsning.

c) Fra grafen estimeres $v(\theta)$ å ha minimalverdi for $\theta \simeq 50°$

$$v_{min} = 35\,\text{m}\sqrt{\frac{9{,}81\,\text{m/s}^2}{2\cdot\cos 50°[35\,\text{m}\cdot\sin 50° - 7\,\text{m}\cdot\cos 50°]}} = 35\sqrt{\frac{9{,}81}{28{,}68}}\ \text{m/s}$$

$$\underline{\underline{v_{min}}} = 35.0 \cdot 0{,}585 \text{ m/s} = \underline{\underline{20{,}5 \text{ m/s} = 74 \text{ km/t}}}$$

Dette synes ikke å være en "svært høy fart", men er allikevel langt over fartsgrensen for tettbygd strøk. Fartsgrensen på stedet er ukjent.

d) Kollisjonskraften F stanser bilen over strekningen $s = 2$ m.
Hastighet like før kollisjonen settes til v_1.

$$\Rightarrow \quad F \cdot s = \frac{1}{2} m v_1^2$$ fra energibetraktning.

Finner v_1 fra energibetraktning:

$$\frac{1}{2} m v_{min}^2 = \frac{1}{2} m v_1^2 + m g h_0$$

$$\Rightarrow \frac{1}{2} m v_1^2 = \frac{1}{2} m v_{min}^2 - m g h_0$$

$$\Rightarrow F = \frac{1}{2} m v_1^2 / s = \frac{m}{s}\left[\frac{1}{2} v_{min}^2 - g h_0\right] = \frac{1400 \text{ kg}}{2 \text{ m}}\left[\frac{1}{2} \cdot 20{,}5^2 - 9{,}81 \cdot 7\right] \frac{\text{m}^2}{\text{s}^2}$$

$$\underline{\underline{F = 99 \text{ kN}}}$$

Alternativt: $\underline{t_1} = \frac{s_0}{v_{min} \cos\theta} = \frac{35 \text{ m}}{20{,}5 \text{ m/s} \cdot \cos 50°} = \underline{\cancel{2{,}8 \text{ m/s}} = 2{,}72 \text{ s}}$

$v_x = v_{min} \cos\theta = 20.5 \cdot \cos 50 = \underline{12{,}8 \text{ m/s}}$

$v_y = v \cdot \sin\theta - g \cdot t_1 = (20.5 \cdot \sin 50° - 9{,}81 \cdot 2{,}72) \text{ m/s} = \underline{\underline{-11.0 \text{ m/s}}}$

$v_1 = \sqrt{v_x^2 + v_y^2} = \sqrt{164 + 121} \text{ m/s} = \sqrt{285} \text{ m/s} = \underline{\underline{16.9 \text{ m/s}}}$

Energibetraktning som før:

$$F\cdot s = \frac{1}{2} m v_1^2$$

$$\Rightarrow \underline{F} = \frac{m}{2s}\cdot v_1^2 = \frac{1400\,\text{kg}}{2\cdot 2\,\text{m}}\cdot 285\,\frac{\text{m}^2}{\text{s}^2} = \underline{99750\,\text{N}}$$

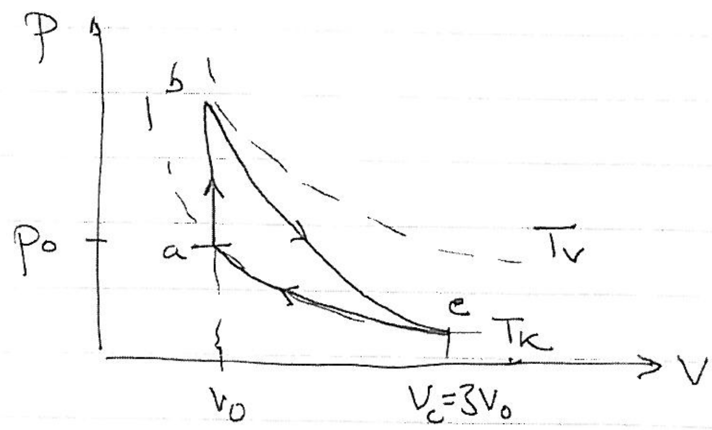
$$\underline{\underline{F \simeq 100\,\text{kN}}} \qquad (\text{mot fartsretningen})$$

Kollisjonstiden t_2 bestemmes fra overført impuls

$$F\cdot t_2 = |\Delta p| = m v_1$$

$$\underline{\underline{t_2}} = \frac{m v_1}{F} = \frac{1400\,\text{kg}\cdot 16{,}9\,\text{m/s}}{99750} = \underline{\underline{0{,}24\,\text{s}}}$$

Oppgave 2.



Diatomær gass, $\gamma = \frac{C_p}{C_v} = \frac{7}{5} = 1{,}4.$

a) Gitt $p_0, V_0, V_c = 3V_0, T_k = T_0$
Bestem T_v, p_b, p_c

Isoterm c–a : $\frac{p_a V_a}{T_a} = \frac{p_c V_c}{T_c}$

Innsatte, kjente verdier: $\frac{p_0 V_0}{T_0} = \frac{p_c \cdot 3V_0}{T_0}$

$$\Rightarrow \quad \underline{\underline{p_c = \frac{1}{3} p_0}}$$

Adiabat b–c : $T_b V_b^{(\gamma-1)} = T_c V_c^{(\gamma-1)}$

Innsatt kjente verdier $T_b \cdot V_0^{(\gamma-1)} = T_0 \cdot V_0^{(\gamma-1)} \cdot 3^{(\gamma-1)}$

$$\Rightarrow \quad \underline{\underline{T_b \equiv T_v = T_0 \cdot 3^{(\gamma-1)}}}$$

Tilstandsligningen: $\frac{p_b \cdot V_b}{T_b} = \frac{p_c V_c}{T_c}$

Innsatt kjente verdier: $\frac{p_b \cdot V_0}{T_0 \cdot 3^{(\gamma-1)}} = \frac{\frac{1}{3} p_0 \cdot 3 \cdot V_0}{T_0}$

$$\Rightarrow \quad \underline{\underline{p_b = p_0 \cdot 3^{(\gamma-1)}}}$$

b) Arbeid utført i hver av delprosessene:

__Isokor a→b:__ $\underline{\underline{W_{ab}}} = \int_a^b p \cdot dV = \underline{\underline{0}}$ fordi $V =$ konstant

__Adiabat b→c:__ $Q_{bc} = \Delta U + W_{bc} = 0$

$$\Rightarrow W_{bc} = -\Delta U = -\left[n C_V \cdot \Delta T \right] = n C_V \left[T_b - T_c \right]$$

Fra hint: $C_V = \frac{R}{(\gamma-1)}$

$$\Rightarrow \underline{\underline{W_{bc}}} = \frac{nRT_b - nRT_c}{(\gamma-1)} = \underline{\underline{\frac{1}{(\gamma-1)} \left[p_b V_b - p_c V_c \right]}}$$

$$W_{bc} = \frac{1}{(\gamma-1)} \left[p_0 \cdot 3^{(\gamma-1)} \cdot V_0 - \frac{1}{3} p_0 \cdot 3 V_0 \right]$$

$$\underline{\underline{W_{bc} = \frac{p_0 V_0}{(\gamma-1)} \left[3^{(\gamma-1)} - 1 \right]}}$$

Isoterm c→a: $W_{ca} = \int_c^a p \cdot dV$

$p \cdot V = nRT =$ konstant $\Rightarrow p = \frac{nRT_0}{V}$

$$\underline{\underline{W_{ca}}} = \int_c^a nRT_0 \cdot \frac{dV}{V} = nRT_0 \cdot \ln \frac{V_a}{V_c} = \underline{\underline{-nRT_0 \cdot \ln 3}}$$

$$p_0 \cdot V_0 = nRT_0$$

$$\Rightarrow \underline{\underline{W_{ca} = -p_0 V_0 \cdot \ln 3}}$$

c) Totalarbeid for et helt omløp:

$$W_{net} = W_{ab} + W_{bc} + W_{ca} = 0 + \frac{p_0 V_0}{(\gamma-1)}\left[3^{(\gamma-1)} - 1\right] - p_0 V_0 \ln 3$$

$$\underline{\underline{W_{net} = p_0 V_0 \left[\frac{(3^{(\gamma-1)} - 1)}{(\gamma-1)} - \ln 3\right]}} \qquad \text{q.e.d.}$$

Tallverdier:

$$\left\{\frac{3^{(\gamma-1)} - 1}{\gamma - 1} - \ln 3\right\} = \left[\frac{3^{0,4} - 1}{0,4} - 1,0986\right] = 0,28$$

$$\Rightarrow \underline{\underline{W_{net}}} = 2,0 \cdot 10^5 \frac{N}{m^2} \cdot 2,0 \cdot 10^{-3} m^3 \cdot 0,28 = 1,12 \cdot 10^2 Nm = \underline{\underline{112 J}}$$

$$P = \frac{W_{net}}{T} = \frac{W_{net}}{1/f} = f \cdot W_{net} = \frac{1000 \frac{sykl}{min}}{60 \frac{s}{min}} \cdot 112 J = \underline{\underline{1870 W}}$$

d) Tilførte varmemengder.

Bruker termodynamikkens 1. lov. $\underline{Q = \Delta U + W}$

$$Q_{ab} = \Delta U + W_{ab} \qquad W_{ab} = 0$$

$$\Rightarrow Q_{ab} = \Delta U = U_b - U_a = nC_v(T_b - T_a) = nC_v(T_v - T_0)$$

$$Q_{ab} = n\frac{R}{(\gamma-1)}\left(T_v - T_0\right) = \frac{1}{(\gamma-1)}\left(nRT_v - nRT_0\right)$$

$$\underline{\underline{Q_{ab}}} = \frac{1}{(\gamma-1)}\left[p_b \cdot V_0 - p_0 V_0\right] = \underline{\underline{\frac{p_0 V_0}{(\gamma-1)}\left[3^{(\gamma-1)} - 1\right]}}$$

$$\underline{\underline{Q_{ab}}} = \frac{400\,J}{0.4}\left[0.5518\right] = \underline{\underline{552\,J}} \quad \text{(absorbert)}$$

$$\underline{\underline{Q_{bc} = 0}} \quad \text{pr. def. av adiabat.}$$

$$Q_{ca} = \Delta U + W_{ca} \qquad \Delta U = 0 \text{ pga. isoterm.}$$

$$\Rightarrow \underline{\underline{Q_{ca}}} = W_{ca} = -\underline{\underline{p_0 V_0 \cdot \ln 3}} \qquad (< 0)$$

$$\underline{\underline{Q_{ca}}} = -400\,J \cdot \ln 3 = \underline{\underline{-439\,J}} \qquad \text{(: }\underline{\text{avgitt varme}}\text{)}$$

e) Verkningsgrad: $\varepsilon = \frac{W_{net}}{Q_H} = \frac{W_{net}}{Q_{ab}}$

$$\underline{\underline{\varepsilon}} = \frac{p_0 V_0 \left[\frac{3^{(\gamma-1)} - 1}{(\gamma-1)} - \ln 3\right]}{p_0 V_0 \left[\frac{3^{(\gamma-1)} - 1}{(\gamma-1)}\right]} = \underline{\underline{1 - \frac{\ln 3(\gamma - 1)}{3^{(\gamma-1)} - 1}}}$$

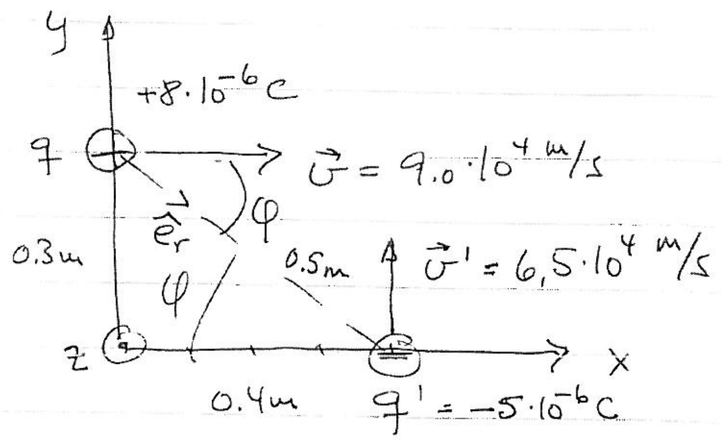
$$\underline{\underline{\varepsilon}} = 1 - \frac{\ln 3 \cdot 0{,}4}{0{,}5518} = \underline{\underline{0{,}20}}$$

For Carnot prosess:

$$\underline{\underline{\varepsilon_c}} = 1 - \frac{T_c}{T_H} = 1 - \frac{T_0}{T_0 \cdot 3^{(\gamma-1)}} = 1 - \frac{1}{3^{(\gamma-1)}} = \underline{\underline{1 - 3^{(1-\gamma)}}}$$

$$\underline{\underline{\varepsilon_c}} = 1 - 0{,}644 \approx \underline{\underline{0{,}36}}$$

Oppgave 3.



r = Avst. qq'

$$\underline{\underline{r}} = \sqrt{0{,}3^2 + 0{,}4^2}\ \text{m} = \underline{\underline{0{,}5\ \text{m}}}$$

a) Magnetfelt $\vec{B}$ generert av q i pos. for q':

$$\vec{B} = \frac{\mu_0}{4\pi}\cdot q\frac{(\vec{v}\times\hat{e}_r)}{r^2}$$

$\hat{e}_r$ enhetsvektor fra kilde (q) i retning P (dvs. q')

$$\underline{|\vec{v}\times\hat{e}_r|} = v\cdot 1\cdot\sin\varphi = \underline{\frac{3}{5}v}$$

Tallverdi: $|B| = \frac{\mu_0}{4\pi}\cdot\frac{q v\cdot\sin\varphi}{r^2}$

$$\underline{|B|} = \frac{\cancel{4\pi}\cdot10^{-7}\ \text{N/A}^2}{\cancel{4\pi}}\cdot\frac{8\cdot10^{-6}\ \text{As}\cdot 9\cdot10^{4}\ \text{m/s}\cdot\frac{3}{5}}{(0.5\ \text{m})^2} \stackrel{\sim}{=} \underline{173\cdot10^{-9}\ \text{T}}$$

Retning: Ved høyre håndsregel blir $\vec{v}\times\hat{e}_r = -v\cdot k$
dvs. retning <u>inn i papirplanet</u>

$\Rightarrow$ I posisjon for q': $\underline{\underline{\vec{B} = -173\cdot10^{-9}\,\text{T}\ k}}$

Kraftvirkning på q' pga. magnetfeltet B:

$$\vec{F} = q'(\vec{v}\,'\times\vec{B}) \qquad \vec{v}\,' \perp \vec{B}$$

$$\vec{F} = q'v'B[\mathbb{j}\times(-\mathbb{k})] = -5\cdot10^{-6}\,C\cdot 6{,}5\cdot10^{4}\,m/s\cdot(-173\cdot10^{-9}\,T)(\mathbb{i})$$

$$\underline{\underline{\vec{F} = 5{,}6\cdot10^{3}\cdot10^{-11}\,\mathbb{i}\,N = 56\,nN\,\mathbb{i}}}$$

$$\vec{v}\,'\times\vec{B} = v'B[\mathbb{j}\times(-\mathbb{k})] = -v'B\,\mathbb{i}$$

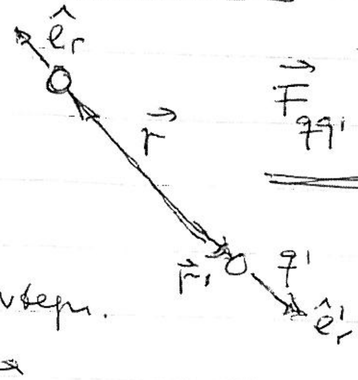
Kraften virker i positiv x-retn. siden $q'<0 \Rightarrow q'(\vec{v}\,'\times\vec{B})$ gir $F_x > 0$.

b) Elektrostatisk kraft har retning langs $\vec{r}$.

Kraft fra q' på q: $\vec{F}_q = \frac{q'q}{4\pi\varepsilon_0}\cdot\frac{1}{r^2}\hat{e}_r$

$$\vec{F}_q = \frac{-5\cdot10^{-6}\cdot8\cdot10^{-6}\,C^2}{4\pi\cdot8{,}85\cdot10^{-12}\,C^2/Nm^2}\cdot\frac{1}{(0{,}5\,m)^2}\,\hat{e}_r$$

$$\vec{F}_q = -1{,}44\cdot10^{-12+12}\,N\,\hat{e}_r = \underline{\underline{-1{,}44\,N\,\hat{e}_r}}$$



$\vec{r}$ har retning fra q' til q

$$\underline{\underline{\vec{F}_{qq'} = -\vec{F}_{q'q}}}$$

Kraften virker tiltrekkende fordi q og q' har motsatt ladningsfortegn.

Tilsvarende kraft på q': $\underline{\underline{\vec{F}_{q'} = -1{,}44\,N\,\hat{e}_r'}}$

tiltrekkende

c) Verdien av elektrostatisk potensial generert av q i posisjon fra q' (d.v.s. i avstand r)

$$V(r) = \frac{q}{4\pi\varepsilon_0} \cdot \frac{1}{r}$$

$$= \frac{8 \cdot 10^{-6} C}{4\pi \cdot 8{,}85 \cdot 10^{-12} \frac{C^2}{Nm^2}} \cdot \frac{1}{0{,}5m}$$

$$\underline{\underline{V(r)}} = 0{,}144 \cdot 10^{6} V = \underline{\underline{0{,}144 \ MV}}$$

Elektrostatisk potensiell energi av systemet

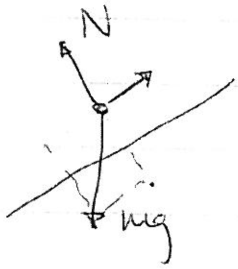
$$U = \sum_{i<j} \frac{q_i q_j}{4\pi\varepsilon_0 r_{ij}} = \frac{q q'}{4\pi\varepsilon_0 \cdot r} = q' \cdot V$$

$$\Rightarrow \underline{\underline{U}} = 0{,}144 \cdot 10^{6} V \cdot (-5{,}0 \cdot 10^{-6} C) = \underline{\underline{-0{,}72 \ J}} < 0$$

Potensiell energi er negativ fordi ladningene har motsatt fortegn og ikke er i ∞ avstand fra hverandre.

Oppgave 4. Flervalgsspørsmål

1)



svar: $\underline{\underline{C}}$ (3)

2) $W = \vec{F} \cdot \vec{s}$

$$W = (-3\mathbf{i} + 6\mathbf{j} - 9\mathbf{k}) \cdot (6\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})\ J$$

$$\underline{\underline{W}} = (-18 - 24 - 18)\ J = \underline{\underline{-60 J}}$$ svar: $\underline{\underline{E}}$

3) $E = \frac{1}{2} k A^2$

$\Rightarrow 2E = \frac{1}{2} k (A\sqrt{2})^2$ svar: $\underline{\underline{C}}$ faktor $1.4 = \sqrt{2}$

4) $F = ma = -kx$

maks $|a|$ for maks $|x|$ svar: $\underline{\underline{D}}$ (4)

5) $\frac{1}{2} m \vec{v}^2 = \frac{3}{2} kT$

$\langle K \rangle = \frac{3}{2} kT$ svar: $\underline{\underline{B}}$

$m_{O_2} > m_{N_2}$

$\Rightarrow v_{O_2} < v_{N_2}$

6) ~~$\langle K \rangle = \frac{3}{2}kT$~~

~~$\frac{\langle K_1 \rangle}{\langle K_2 \rangle} = \frac{T_1}{T_2} = \frac{293}{333} = \underline{0,88}$ Svar: D~~

(Se siste side)

7) U, S, T uavh. av prosessen
Q, W avh. — " —

$\Rightarrow$ Svar: A

8) $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\varepsilon_0} = \frac{2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$

$\Rightarrow$ Svar: A

9) $V_{ab} = 12\,V \quad t = 2,5\,s \quad I = 60\,A$

$W = P \cdot t = V_{ab} \cdot I \cdot t = 12 \cdot 60 \cdot 2,5\,J = \underline{1800\,J}$

Svar: A

10) $\Phi_m = 6t^2 + 7t + 1 \qquad \mathcal{E} = -\frac{d\Phi_m}{dt}$

$\mathcal{E} = -\frac{d\Phi_m}{dt} = -\left[12t + 7\right]$

$t = 2s \Rightarrow$

$\Rightarrow |\mathcal{E}| = +\left[12 \cdot 2 + 7\right]V = 31\,V$

Svar: D

Flervalgsoppgave 6

$\tau = J\alpha = Tr$ — ligninger for dreiemoment på hjulet.

$T - mg = -ma$ — N2 for loddet.

$$\Rightarrow a = -\frac{T}{m} + g = -\frac{J\alpha}{mr} + g$$

Snor festet til hjul $\Rightarrow$ $a = \alpha r$ ("ikke-skli")

Dermed

$$a\left(1 + \frac{J}{mr^2}\right) = g$$

$$a = g\left(1 + \frac{J}{mr^2}\right)^{-1} = 9.81\,\frac{m}{s^2}\left(1 + \frac{0.50\,kgm^2}{15\,kg\,(0.5\,m)^2}\right)^{-1}$$

$$= \underline{\underline{8.7\ m/s^2}}$$

Svar: B.