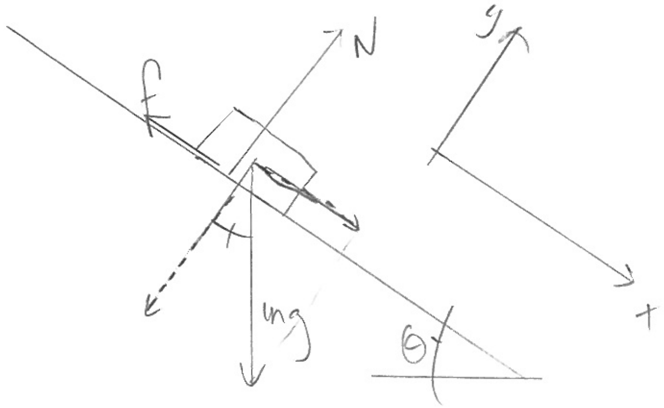
Løsningsforslag,
TFY 4125 Kont. eksamen aug. 2010.

1a)



$$\sum \vec{F} = m\vec{a}.$$

y-retning: $N - mg\cos\theta = 0 \Rightarrow N = mg\cos\theta$

x-retning: $mg\sin\theta - f = ma, \quad f = \mu_k N$

Vi eliminerer f og finner

$$a = g(\sin\theta - \mu_k\cos\theta)$$

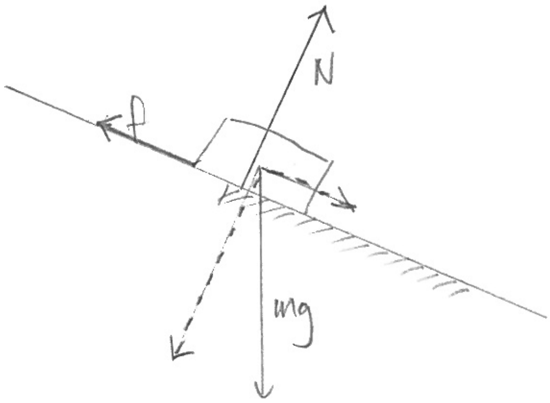
Det er rimelig å anta at friksjonen skalerer med kontaktarealet mellom klossen og skråplanets ru del. For $x \leq 0$ er dette arealet null, mens for $x > s$ er kontaktarealet lik klossens areal, som er dens bredde b · lengde s.

Dermed fås: $x \leq 0$: $\mu_{eff} = 0$

$x \in (0, s)$: $\mu_{eff} = Cx, \quad C = \frac{\mu_k}{s}$

$x > s$: $\mu_{eff} = \mu_k$

b) $a = 0$, dvs $\sum F = 0$.



$x = x_0$. $\mu_{eff} = \frac{x_0}{s}\mu_k$.

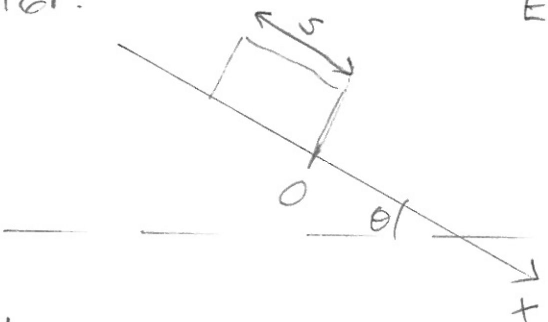
$$g(\sin\theta - \mu_{eff}\cos\theta) = 0$$

$$\mu_{eff} = \frac{x_0}{s}\mu_k = \frac{\sin\theta}{\cos\theta}$$

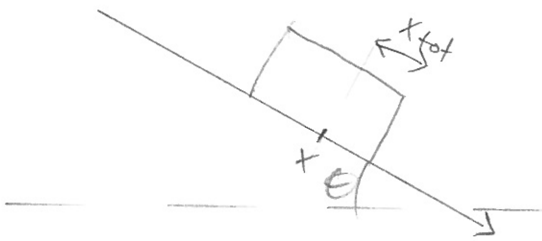
$$\underline{\underline{x_0 = \frac{1}{\mu_k}\tan\theta \; s.}}$$

c) Energibetraktninger

Før:



Etter:



Endring i potensiell energi: $\Delta U = -mgh = -mg x_{tot}\sin\theta$

Endring i kinetisk energi: $\Delta K = 0$.

Endring i termisk energi: $W_{lost} = \int_0^{x_{tot}} f_{eff}\,dx = \int_0^{x_{tot}} \mu_{eff}\, mg\cos\theta\, dx$

$$= \int_0^{x_{tot}} \frac{x}{s}\mu_k mg\cos\theta\, dx = \underline{\frac{1}{2s}\mu_k mg\cos\theta\, x_{tot}^2}$$

1c forts.

Tapet av potensiell energi er lik økningen i termisk energi

$$\cancel{mg}\cancel{x_{tot}} \sin\theta = \frac{1}{2s}\mu_k \cancel{mg}\cos\theta\, x_{tot}^{\cancel{2}}$$

$$2\frac{1}{\mu_k}\tan\theta\, s = x_{tot} \qquad \text{evt. } x_{tot} = 2x_0$$

$x_{tot} \leq s$:

$$2\frac{1}{\mu_k}\tan\theta\, s \leq s$$

$$\mu_k \geq 2\tan\theta$$

Oppg 2

a) $V_D = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{x^2+R^2} - x\right), \quad x > 0$

Ladning Q på skive: $Q = \pi R^2 \sigma$

$$\underline{\underline{V_D = \frac{Q}{2\pi\varepsilon_0 R^2}\left(\sqrt{x^2+R^2} - x\right)}}$$

$V_R = \frac{\lambda}{2\varepsilon_0} \frac{R}{\sqrt{x^2+R^2}}, \quad x \in (-\infty, \infty)$

Ladning Q på ring: $Q = 2\pi R \lambda$

$$\underline{\underline{V_R = \frac{Q}{4\pi\varepsilon_0} \frac{1}{\sqrt{x^2+R^2}}}}$$

b)

$$V_R^\infty = \frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{x^2+R^2}} = \frac{Q}{4\pi\varepsilon_0}\left(x^2+R^2\right)^{-\frac{1}{2}}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left(1+\frac{R^2}{x^2}\right)^{-\frac{1}{2}}\left(x^2\right)^{-\frac{1}{2}}$$

$$\underset{(\text{små } x)}{\simeq} \frac{Q}{4\pi\varepsilon_0}\frac{1}{x}\left(1-\frac{R^2}{2x^2}\right) \qquad // \text{2. orden}$$

$$\underline{\underline{\simeq \frac{Q}{4\pi\varepsilon_0}\frac{1}{x}}} \qquad // \text{1. orden.}$$

$$V_D^\infty = \frac{Q}{2\pi\varepsilon_0 R^2}\left(\sqrt{x^2+R^2} - x\right)$$

$$= \frac{Q}{2\pi\varepsilon_0 R^2}\left(x\left(1+\frac{R^2}{x^2}\right)^{\frac{1}{2}} - x\right)$$

$$\simeq \frac{Q}{2\pi\varepsilon_0 R^2}\, x\left(\cancel{1} + \frac{R^2}{2x^2} - \cancel{1}\right)$$

$$\underline{= \frac{Q}{4\pi\varepsilon_0}\frac{1}{x}} = V_R^\infty,$$ <u>som forventet fra Gauss' lov.</u>

2c) $V_R = \frac{Q}{4\pi\varepsilon_0} \frac{1}{\sqrt{x^2+R^2}}$

Kraft $\vec{F} = q\vec{E}$, $\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\hat{x}$

$$\vec{E} = -\frac{\partial}{\partial x}\left(\frac{Q}{4\pi\varepsilon_0}\frac{1}{\sqrt{x^2+R^2}}\right)\hat{x} = \frac{(-Q)(-1)}{4\pi\varepsilon_0 \, 2}(x^2+R^2)^{-\frac{3}{2}} 2x\hat{x}$$

$$= \frac{Qx}{4\pi\varepsilon_0}\frac{1}{(x^2+R^2)^{3/2}}\hat{x}$$

$$\vec{F} = q\vec{E} = \frac{eQx}{4\pi\varepsilon_0}\frac{1}{(x^2+R^2)^{3/2}}\hat{x}$$

$$= \frac{(-1{,}6\cdot 10^{-19})(1{,}0\cdot 10^{-9})\,0{,}2}{4\pi\, 8{,}85\cdot 10^{-12}\;(0{,}2^2+0{,}3^2)^{3/2}}\hat{x} = \underline{\underline{-6.1\cdot 10^{-18}\,\mathrm{N}\hat{x}}}$$

(rettet mot ringen)

2d) $W = \Delta U = e\Delta V = e(V_f - V_i)$

$$= \frac{eQ}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x_f^2+R^2}} - \frac{1}{\sqrt{x_i^2+R^2}}\right)$$

$$= \frac{(-1{,}60\cdot 10^{-19})(1{,}0\cdot 10^{-9})}{4\pi\, 8{,}85\cdot 10^{-12}}\left(\frac{1}{\sqrt{1{,}0^2+0{,}3^2}} - \frac{1}{\sqrt{0{,}2^2+0{,}3^2}}\right)\mathrm{J}$$

$$= \underline{\underline{2.6\cdot 10^{-18}\,\mathrm{J}}}$$

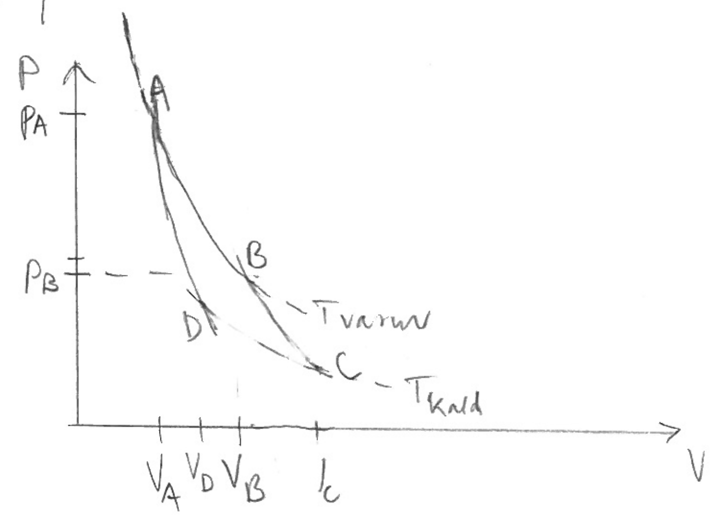
Arbeidet tilsvarer endring i kinetisk energi:

$$\frac{1}{2}mv^2 = W$$

$$v = \sqrt{\frac{2W}{m}} = \underline{\underline{2{,}4\cdot 10^6\,\mathrm{m/s}}} \qquad (\ll c, \text{ ok!})$$

Oppg 3. Varmekraftmaskin.

b)



a) p_B.

$$pV = nRT \Rightarrow p_A V_A = p_B V_B$$

$$p_B = p_A \frac{V_A}{V_B} = \underline{\frac{1}{2} p_A} = \underline{50.5\ \text{kPa}}$$

p_C

$$p_B V_B^{\gamma} = p_C V_C^{\gamma}$$

$$p_C = p_B \left(\frac{V_B}{V_C}\right)^{\gamma} = p_A \frac{1}{2}\left(\frac{2\cancel{V_A}}{3\cancel{V_A}}\right)^{\gamma} = \underline{\underline{p_A \frac{2^{\gamma-1}}{3^{\gamma}}}}$$

$$\underline{\underline{p_C = 25.7\ \text{kPa}}} \qquad \underline{\underline{= p_A \frac{2^{2/3}}{3^{5/3}} \simeq 0{,}25 p_A}}$$

V_A

$$p_A V_A = nRT_A,\ n = 1,\ T_A = 400\text{K},\ p_A = 1{,}01 \cdot 10^5\ \text{Pa}$$

$$V_A = \frac{RT_A}{p_A} = \frac{8{,}314 \cdot 400}{1.01 \cdot 10^5}\ \text{m}^3 = \underline{\underline{32.9\ \text{dm}^3}}$$

T_C

$$p_C V_C = nRT_C$$

$$T_C = \frac{\cancel{p_C} V_C}{\cancel{n}R} = \cancel{p_A} \frac{2^{\gamma-1}}{3^{\gamma}} \frac{3\left(\frac{\cancel{R} T_A}{\cancel{p_A}}\right)}{\cancel{R}}$$

$$\underline{\underline{= T_A \left(\frac{2}{3}\right)^{\frac{2}{3}}}} \qquad \underline{\underline{T_C = 305\ \text{K}}}$$

3c) Ved D må begge ligningene være oppfylt:

$$p_C \frac{V_C}{V_D} = p_A \left(\frac{V_A}{V_D}\right)^{\gamma}$$

$$\cancel{p_A} \frac{2^{\gamma-1}}{3^{\gamma}} \frac{3V_A}{V_D} = \cancel{p_A} \left(\frac{V_A}{V_D}\right)^{\gamma}$$

$$V_D^{\gamma-1} = V_A^{\gamma-1} \frac{3^{\gamma-1}}{2^{\gamma-1}}$$

$$\underline{\underline{V_D = \frac{3}{2} V_A = 49.4\,L}}$$

$$p_D V_D = nRT_D, \quad T_D = T_C = T_A \left(\frac{2}{3}\right)^{\gamma-1}$$

$$p_D = \frac{RT_D}{V_D} = RT_A \left(\frac{2}{3}\right)^{\gamma-1} \frac{1}{\frac{3}{2} V_A}$$

$$= RT_A \left(\frac{2}{3}\right)^{\gamma} \cdot \frac{1}{V_A}$$

$$\underline{\underline{= p_A \left(\frac{2}{3}\right)^{\frac{5}{3}}}}$$

3d) $Q_{varm} = Q_{AB}$

$Q = \Delta U + W$, $\Delta U = 0$ fordi $U = U(T)$ for ideell gass.

$$Q = W = \int_A^B p(V)\,dV = \int_A^B \frac{nRT_A}{V}\,dV = nRT_A \ln\frac{V_B}{V_A}$$

$$= nRT_A \ln 2 = \underline{\underline{p_A V_A \ln 2}}$$

$$= 1{,}01 \cdot 10^5 \cdot 32{,}9 \cdot 10^{-3} \ln 2 \text{ J}$$

$$\underline{\underline{= 2{,}3 \text{ kJ}}}$$

3e) Virkningsgrad for Carnotprosess,

$$\varepsilon_C = 1 - \frac{T_{kald}}{T_{varm}} = 1 - \frac{T_C}{T_A} = 1 - \left(\frac{2}{3}\right)^{\frac{2}{3}} \quad \underline{\underline{\simeq 0.24}}$$

$$\varepsilon = \frac{W_{net}}{Q_{varm}} \qquad W_{net} = \varepsilon Q_{varm} = \varepsilon_C Q_{varm}$$

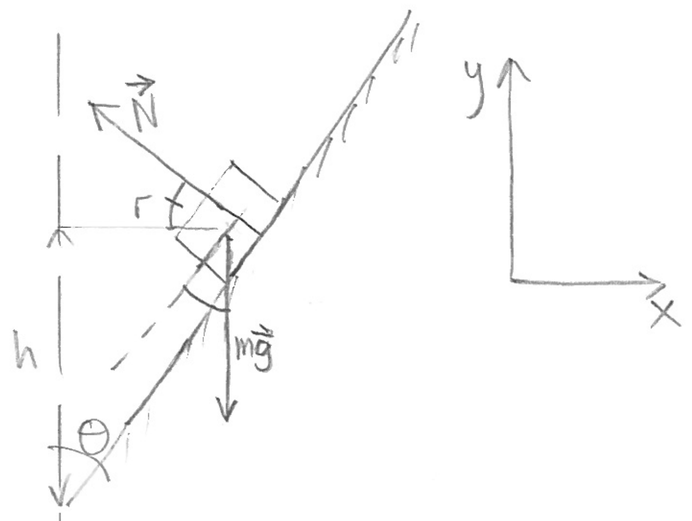
$$\underline{\underline{= \left(1 - \left(\frac{2}{3}\right)^{\frac{2}{3}}\right) p_A V_A \ln 2}}$$

$$\underline{\underline{\simeq 0{,}16\, p_A V_A}}$$

$$\underline{\underline{= 546 \text{ J}}}$$

Oppg 4

To krefter virker på klossen: tyngdekraften og normalkraften.



Radielt er klossen akselerert, $a_x = -\frac{v^2}{r}$.

Kraftbalanse:

Ingen akselerasjon i y-retning, $a_y = 0$.

$$\Rightarrow \sum F_y = 0: \quad -mg + N\sin\theta = 0.$$

$$\therefore N = \frac{mg}{\sin\theta}$$

$$\sum F_x = ma_x = -N\cos\theta = -m\frac{v^2}{r}$$

Eliminerer N: $\frac{\cancel{m}g}{\sin\theta}\cos\theta = \cancel{m}\frac{v^2}{r}$ (*)

Potensiell energi $U = mgh$

Kinetisk energi $K = \frac{1}{2}mv^2$

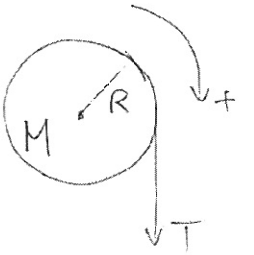
$\frac{U}{K} = \frac{\cancel{m}gh}{\frac{1}{2}\cancel{m}v^2}$. Vi ser at $\frac{r}{h} = \tan\theta$ (fra figur)

og $v^2 = \frac{gr}{\tan\theta}$ (fra *).

Dermed:

$$\frac{U}{K} = \frac{g\frac{r}{\tan\theta}}{\frac{1}{2}\frac{gr}{\tan\theta}} = 2$$

Oppg 5

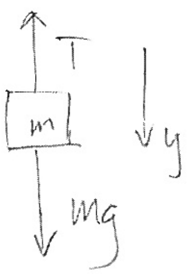


$$\sum \tau = J\alpha, \quad J = \frac{1}{2} M R^2$$

$$\tau = RT$$

$$M = \pi R^2 H \rho$$

$$\alpha = \frac{a}{R} \quad (\text{"non-slip"})$$



$$mg - T = ma$$

eliminerer T: $T = m(g-a)$

Dermed, $m(g-a)\cancel{R} = \frac{1}{2}(\pi R^2 H \rho) R^{\cancel{2}} \frac{a}{\cancel{R}}$

Løser mhp ρ:

$$\rho = \frac{2m(g-a)}{a \pi R^2 H} = 2 \frac{m}{\pi R^2 H} \frac{g-a}{a}$$

Med $a = \frac{g}{2}$, $H = 0{,}30\,m$, $R = 0{,}10\,m$ og $m = 18{,}0\,kg$ fås

$$\rho = \frac{\cancel{2} \cdot 18 \cdot \frac{\cancel{9,81}}{\cancel{2}}}{3{,}14(0{,}1)^2\, 0{,}3 \cdot \frac{\cancel{9,81}}{2}} \frac{kg}{m^3}$$

$$= 3{,}82 \cdot 10^3\, kg/m^3$$

$$= 3{,}82\, g/cm^3$$