TFY 4165 - Termisk fysikk

19. desember 2018

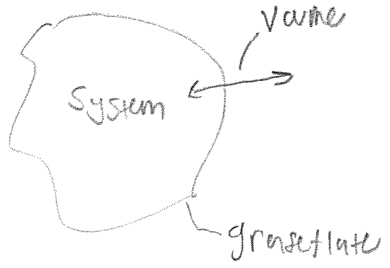
1) $$\mu_{JT} = -\frac{1}{C_p}\left\{V + p\left(\frac{\partial V}{\partial p}\right)_T\right\}$$

Ideell gass: $V = \frac{nRT}{p} \Rightarrow \left(\frac{\partial V}{\partial p}\right)_T = -\frac{nRT}{p^2} = -\frac{V}{p}$

$$\mu_{JT} = -\frac{1}{C_p}\underbrace{\left\{V + p \cdot -\frac{V}{p}\right\}}_{=0} = \underline{\underline{0}}$$

2) Ekstensive: U, S, V, N

Intensive: P, μ, T

3) 

Omgivelser

energi som krysser grenseflaten pga en temperaturforskjell

4) En prosess kan bare være reversibel hvis prosessen er kvasistatisk og dissipative effekter er fraværende

5)

$$C_p - C_V = T \left(\frac{\partial p}{\partial T}\right)_V \left(\frac{\partial V}{\partial T}\right)_p$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{Nk}{V-Nb}$$

$$\left(\frac{\partial V}{\partial T}\right)_p = \left(\frac{\partial T}{\partial V}\right)_p^{-1} = \left[\frac{\partial}{\partial V}\left(\frac{p(V-Nb)}{Nk}\right)\right]^{-1} = \left[\frac{p}{Nk}\right]^{-1}$$

$$\Rightarrow C_p - C_V = T \underbrace{\left(\frac{Nk}{V-Nb}\right)}_{\frac{p}{T}} \left(\frac{Nk}{p}\right) = T \; \frac{p}{T} \cdot \frac{Nk}{p} = Nk$$

$$\Rightarrow C_p = Nk + C_V = Nk + \frac{3}{2} Nk = \underline{\underline{\frac{5}{2} Nk}}$$

6) Adiabat: $Q = 0$ & $p(V-Nb)^\gamma = konst.$

1. HS: $Q = \Delta U + W \Rightarrow \Delta U = -W$

$$W = \int_{V_1}^{V_2} p \, dV = \int_{V_1}^{V_2} \frac{konst}{(V-Nb)^\gamma} dV = konst \left[\frac{(V-Nb)^{-\gamma+1}}{1-\gamma}\right]_{V_1}^{V_2}$$

$$= \frac{konst}{1-\gamma}\left[\frac{(V_2-Nb)}{(V_2-Nb)^\gamma} - \frac{(V_1-Nb)}{(V_1-Nb)^\gamma}\right] = \frac{1}{1-\gamma}\left[p_2(V_2-Nb) - p_1(V_1-Nb)\right]$$

$$\Rightarrow \Delta U = -W = \underline{\underline{\frac{1}{1-\gamma}\left[p_1(V_1-Nb) - p_2(V_2-Nb)\right]}}$$

7) Legendre: ny funksjon = gammel funk. − (ny variabel)(gammel variabel)

konjugat

$$\Rightarrow \underline{\underline{\beta(x,v) = \alpha(x,y) - vy}}$$

8) 1D oscillator: $E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} K x^2$

2 kvadratiske frihetsgrader $\Rightarrow U = 2 \cdot \frac{1}{2} kT = kT$

$$\Rightarrow \underline{\underline{C_v = k}}$$

9) System = {gass + pendel} = Isolert fra omgivelsene

1. lov $\Rightarrow \underbrace{Q}_{0} = \Delta U + W$

Her blir arbeid utført av tyngdekraften på pendelen ergo må $W < 0$.

energien absorbert av {gass + pendel}.
Siden pendelen er isolert er det kun gassens indre energi som endrer seg, derfor har vi at $\Delta U = n C_v \Delta T$

$$W = E_p(\theta = \theta_0) - E_p(\theta = 0) = mgL\cos\theta_0 - mgL = -mgL(1 - \cos\theta_0)$$

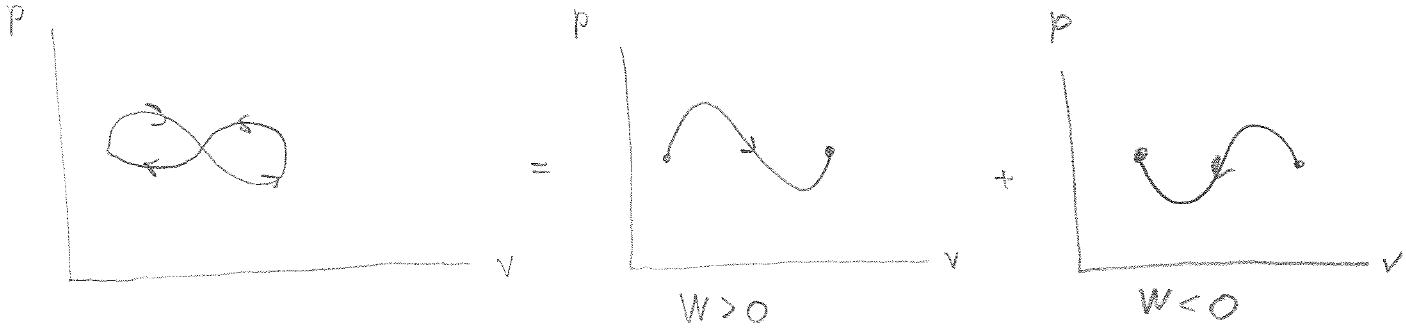
$$\cos\theta_0 \simeq 1 - \frac{\theta_0^2}{2}$$

$$\Rightarrow W = -mgL\left(1 - 1 + \frac{\theta_0^2}{2}\right) = -mgL\frac{\theta_0^2}{2}$$

$$\Rightarrow 0 = n C_v \Delta T - mgL\frac{\theta_0^2}{2} \Rightarrow \boxed{\Delta T = \frac{mgL\theta_0^2}{2 n C_v}}$$

10) $T = \frac{2}{3k}\langle E_k \rangle$

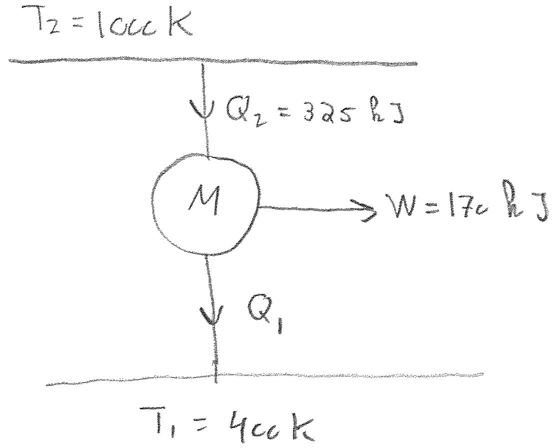
11)



$W = \oint_c p \, dV = \underline{\underline{0}}$

12) Carnotmaskinens virkningsgrad er uavhengig av arbeidssubstans $\Rightarrow \eta = \eta_c = \underline{\underline{1 - T_1/T_2}}$

13) $Q_1 = 325 - 170 = 155 \text{ kJ}$



$\eta_M = \frac{W}{Q_2} = \frac{170}{325} \simeq \underline{\underline{0.52}}$

$\eta_c = 1 - \frac{T_1}{T_2} = 1 - \frac{400}{1000} = \underline{\underline{0.6}}$

$\eta_M < \eta_c \Leftrightarrow$ M er mulig, men ikke reversibel

14) Entropien i en irreversibel varmeisolert prosess må øke*

15) $G = H - TS = U + pV - TS$

$$dG = \underbrace{dU + p\,dV}_{\cancel{T\,dS}} + V\,dp \cancel{- T\,dS} - S\,dT \quad \& \quad T\,dS = dU + p\,dV$$

$$dG = V\,dp - S\,dT \Rightarrow V = \left(\frac{\partial G}{\partial p}\right)_T$$

$$\Rightarrow V = \frac{NkT}{p} + B + Cp + Dp^2$$

16) Entropi er en tilstandsfunksjon og er derfor veiuavhengig.

$\Delta S_A = \Delta S_B = \Delta S_C$

17) $\Delta S_{f \to g}$ må være størst fordi at volumendringen er størst når faststoff omdannes til gass. Større volum betyr flere tilgjengelige mikrotilstander.

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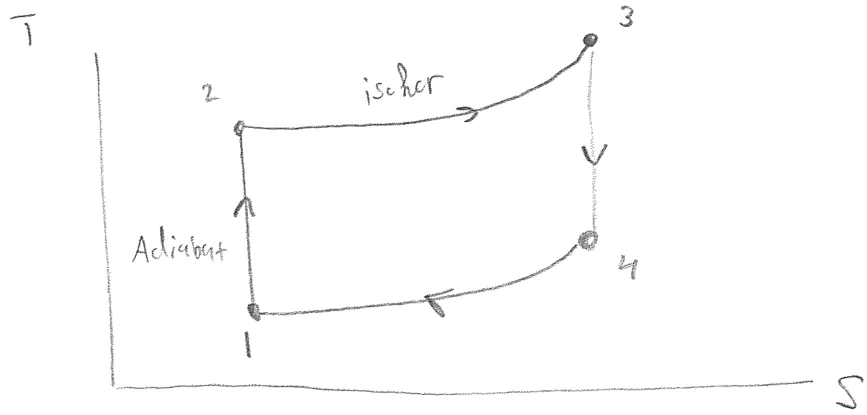
* Alternativ B og C er feil fordi det ikke nevnes at dette må være "det eneste resultat".

Eks: Isoterm med ideell gass $\Rightarrow U = U(T,V) \equiv U(T) \Rightarrow \Delta U = 0$

1. lov $\Rightarrow$ $Q = W$ mengden tilført varme er her lik mengden utført arbeid.

18)



$$\eta = \frac{\text{arbeid}}{\text{Tilført varme}} = \frac{\text{Tilført varme + avgitt varme}}{\text{Tilført varme}}$$

$$Q = \underbrace{\Delta U}_{\to\, 0 \text{ for syklisk prosess}} + W = W$$

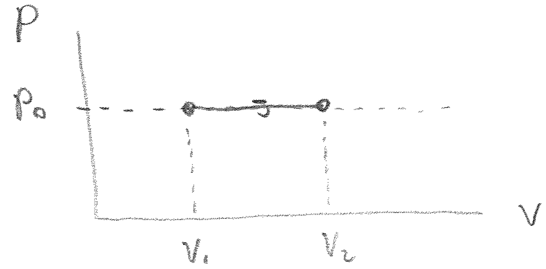
Tilført varme: $C_V (T_3 - T_2) > 0$

Avgitt varme: $C_V (T_1 - T_4) < 0$

$$\Rightarrow \eta = \frac{C_V [T_3 - T_2 + T_1 - T_4]}{C_V [T_3 - T_2]} = \underline{\underline{1 + \frac{T_1 - T_4}{T_3 - T_2}}}$$

Merk: dette kan forkortes videre, men det er ikke nødvendig.

1a)



Reversibel prosess:

$$dS = C_V \frac{dT}{T} + \underbrace{\left(\frac{\partial p}{\partial T}\right)_V}_{\frac{nR}{V}} dV$$

$$\Delta S = C_V \ln \frac{T_2}{T_1} + nR \ln \frac{V_2}{V_1}$$

Isobar: $pV = nRT \Rightarrow \frac{T}{V} = \text{konstant} \Rightarrow \frac{T_1}{V_1} = \frac{T_2}{V_2} \Rightarrow \frac{T_2}{T_1} = \frac{V_2}{V_1}$

$$\Delta S = \ln \frac{V_2}{V_1} \underbrace{\left[C_V + nR\right]}_{C_p} = \underline{\underline{C_p \ln \frac{V_2}{V_1}}}$$

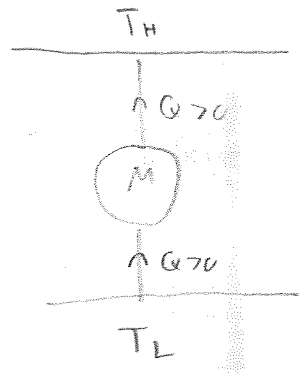
2c) Merk at temperaturen er den samme i både start- og slutt-tilstanden!

For å beregne ΔS for den irreversible prosessen kan vi bruke en reversibel isoterm prosess:

$$dS = C_V \underbrace{dT}_{0} + \left(\frac{\partial P}{\partial T}\right)_V dV = \frac{nR}{V}\, dV$$

$$\Rightarrow \underline{\underline{\Delta S = nR \ln \frac{V_2}{V_1}}}$$

21)



- Clausis' formulering garanterer at denne prosessen er <u>umulig</u>.
- I dette tilfellet er <u>energien bevart</u>

$$\text{System} = \{T_H, M, T_L\} = \text{isolert}$$

$$\Delta S_{tot} = \underbrace{\Delta S_{T_L}}_{-\frac{Q}{T_L}} + \underbrace{\Delta S_M}_{\substack{0 \\ \text{(syklisk} \\ \text{prosess)}}} + \underbrace{\Delta S_{T_H}}_{\frac{Q}{T_H}} = Q\left(\frac{1}{T_H} - \frac{1}{T_L}\right) < 0$$

2. lov: ΔS for isolert system er ≥ 0.

I dette tilfellet er $\Delta S_{tot} < 0$.

$\Rightarrow$ <u>Prosessen er umulig fordi den totale entropiendringen er negativ.</u>

22) Alle prosessene krever uendelig mange reservoarer for å være reversibel. <u>Ingen av kretsprosessene kan benyttes.</u>

23) $dH = T\,dS + V\,dp$

$T = \left(\frac{\partial H}{\partial S}\right)_p$ & $V = \left(\frac{\partial H}{\partial p}\right)_S$

$$\left(\frac{\partial^2 H}{\partial p \partial S}\right)_S = \left(\frac{\partial^2 H}{\partial S \partial p}\right)_p$$

$$\Rightarrow \left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p$$

24) $\uparrow\uparrow\uparrow$, $\uparrow\uparrow\downarrow$, $\uparrow\downarrow\uparrow$, $\uparrow\downarrow\downarrow$, $\downarrow\uparrow\uparrow$, $\downarrow\uparrow\downarrow$, $\downarrow\downarrow\uparrow$, $\downarrow\downarrow\downarrow$

E: $J/2$, 0, $-J/2$, 0, 0, $-J/2$, 0, $J/2$

$S(E=0) = k \ln 4$ og $S(E=J/2) = k \ln 2$

$$\Delta S = S(J/2) - S(0) = k\ln 2 - k\ln 4 = k \ln \frac{2}{4} = \underline{\underline{-k\ln 2}}$$

25)

Mest sannsynlige fart: $\frac{df}{dv} = 0$

$$\ln f = \ln A v^{\frac{1}{v}} = \ln A + \frac{1}{v}\ln v$$

$$\underbrace{\frac{1}{f}\frac{df}{dv}}_{0} = -\frac{1}{v^2}\ln v + \frac{1}{v^2} = \frac{1}{v^2}(1-\ln v)$$

$$\Rightarrow \frac{1}{v^2}(1-\ln v) = 0$$

$\Rightarrow \underline{\underline{v = e}}$ siden $0 \le v \le 10$

26) $e^x \simeq \underbrace{1}_{\text{laveste orden}} + x$

Små v: $f(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 \cdot \underbrace{1}_{\text{rekkeutviklet } e^{-\frac{mv^2}{2kT}}}$

$$P = \int_0^{v_0} f(v)\,dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\underbrace{\int_0^{v_0} v^2\,dv}_{\frac{v_0^3}{3}} = \underline{\underline{\frac{4\pi}{3}\left(\frac{m}{2\pi kT}\right)^{3/2} v_0^3}}$$

27) $E_2 = +\Delta/2$

$E_1 = -\Delta/2$

$$Z = \sum_{n=1}^{2} e^{-\beta E_n} = e^{-\beta E_1} + e^{-\beta E_2} = e^{\frac{\beta\Delta}{2}} + e^{-\frac{\beta\Delta}{2}} = 2\cosh\left(\frac{\beta\Delta}{2}\right)$$

$$\underline{\underline{U}} = -\frac{d}{d\beta}\ln Z = -\frac{d}{d\beta}\left\{\ln 2 + \ln\cosh\left(\frac{\beta\Delta}{2}\right)\right\}$$

$$= -\left\{\frac{\sinh\left(\frac{\beta\Delta}{2}\right)}{\cosh\left(\frac{\beta\Delta}{2}\right)}\cdot\frac{\Delta}{2}\right\} = \underline{\underline{-\frac{\Delta}{2}\tanh\left(\frac{\beta\Delta}{2}\right)}}$$

28) LV kriteriet : $dG = 0$ eller $\frac{\partial G}{\partial m} = 0$ (begge gir samme svar)

$$0 = \frac{\partial G}{\partial m} = -J_0 m - B_0 + \frac{kT_0}{2}\left[\ln(1+m) + 1 - \ln(1-m) - 1\right]$$

$$= -J_0 m - B_0 + kT_0 \underbrace{\frac{1}{2}\ln\frac{1+m}{1-m}}_{\tanh^{-1}(m)}$$

$$= -J_0 m - B_0 + kT_0 \tanh^{-1}(m)$$

$$\Rightarrow \quad \tanh^{-1}(m) = \frac{J_0 m + B_0}{kT_0}$$

$$\Rightarrow \quad \underline{\underline{m = \tanh\left(\frac{J_0 m + B_0}{kT_0}\right)}}$$

29) $M = N\mu\left(1 - e^{-\mu\beta H}\right)$ $\qquad \beta = \frac{1}{kT}$

$$\left(\frac{\partial M}{\partial T}\right)_H = \underbrace{\frac{\partial \beta}{\partial T}}_{-k\beta^2} \underbrace{\frac{\partial M}{\partial \beta}}_{-N\mu \cdot e^{-\mu\beta H} \cdot -\mu H} = -k\beta^2 \cdot N\mu^2 H e^{-\mu\beta H}$$

$$dT = \frac{\mu_0 T}{C_H}\left|\left(\frac{\partial M}{\partial T}\right)_H\right| dH = \mu_0 T \cdot \frac{kT^2}{\mu_0 N H^2 \mu^2 e^{-\mu\beta H}} \cdot \frac{1}{kT^2} N\mu^2 H e^{-\mu\beta H} \, dH$$

$$= \frac{T}{H} \, dH$$

$$\Rightarrow \frac{dT}{T} = \frac{dH}{H} \Rightarrow \ln T = \ln H + \text{konst}$$

$$\Rightarrow \underline{\underline{T = \text{konst.} \, H}}$$

30) $T = \frac{\hbar c^3}{8\pi G M k} \cdot \frac{c^2}{c^2} = \frac{\hbar c^5}{8\pi G k U}$

$$T\,ds = dU + \overset{\to 0}{dW} = dU$$

$$\Rightarrow \frac{\hbar c^5}{8\pi G k U} ds = dU$$

$$\Rightarrow \frac{\hbar c^5}{8\pi G k} \underbrace{\int ds}_{S} = \int U \, dU = \frac{1}{2} U^2 + \text{konst}$$

$$\Rightarrow \underline{\underline{S = \frac{4\pi G k}{\hbar c^5} U^2 + \text{konstant}}}$$

31) $dG = Vdp - SdT$

$$\left.\begin{aligned} dW &= p\,dV \\ dW &= y\,dx \end{aligned}\right\} \quad y \longleftrightarrow p \quad \text{og} \quad V \longleftrightarrow x \quad \Rightarrow \quad dG = x\,dy - S\,dT$$

$\Rightarrow$ G sine naturlige variable er $\underline{\underline{(y, T)}}$

32) Et punkt i faserommet = alle posisjon- og impuls-koordinatene til partiklene

$$\text{dimensjon} = d = N \left\{ \overbrace{\begin{matrix} x^1 & y^1 & z^1 & p_x^1 & p_y^1 & p_z^1 \\ x^2 & y^2 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ x^N & y^N & \cdot & \cdot & \cdot & \cdot \end{matrix}}^{6} \right. = \underline{\underline{6N}}$$

33) Likevekt : $\mu_{2D} = \mu_g$

$$-\cancel{kT_0} \ln\left(\frac{1}{\Theta} - 1\right) = -\cancel{kT_0} \ln\left[\frac{kT_0}{p}\left(\frac{2\pi m kT_0}{h^2}\right)^{3/2}\right]$$

$$\Rightarrow \frac{1-\Theta}{\Theta} = \frac{1}{p}\left(\frac{2\pi m}{h^2}\right)^{3/2} (kT_0)^{5/2}$$

$$\Rightarrow \underline{\underline{\frac{1}{(kT_0)^{5/2}} \frac{1-\Theta}{\Theta} = \frac{1}{p}\left(\frac{2\pi m}{h^2}\right)^{3/2}}}$$

34) $G = H - TS = U + pV - TS$

$$dG = \underbrace{dU + p\,dV}_{T\cancel{dS} + \mu\,dN} + V\,dp - \cancel{T\,dS} - S\,dT = \mu\,dN + V\,dp - S\,dT$$

$$\Rightarrow \underline{\underline{\mu = \left(\frac{\partial G}{\partial N}\right)_{P,T}}}$$

35) Et differensial på formen $df = M\,dx + N\,dy$ er eksakt hvis (og bare hvis) $M = \frac{\partial f}{\partial x}$ og $N = \frac{\partial f}{\partial y}$

Siden $\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y}$ er et ekvivalent krav at $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Her:

$$\left.\begin{aligned} M &= \frac{R}{P} + \frac{3b}{T^4} \qquad \frac{\partial M}{\partial y} = \frac{\partial M}{\partial P} = -\frac{R}{P^2} \\ N &= -\frac{RT}{P^2} \qquad \frac{\partial N}{\partial x} = \frac{\partial N}{\partial T} = -\frac{R}{P^2} \end{aligned}\right\} \underline{\underline{\frac{\partial M}{\partial P} = \frac{\partial N}{\partial T}}}$$

$\Rightarrow$ <u>dV er eksakt uavhengig av b & R</u>

36) $G = \mu N \Rightarrow dG = \mu\,dN + N\,d\mu = \mu\,dN + V\,dp - S\,dT$

$$\Rightarrow d\mu = \frac{V}{N}\,dp - \frac{S}{N}\,dT$$

Økt $P \Rightarrow$ Større μ Økt $T \Rightarrow$ Mindre μ

37) Gibbs' faseregel: $q \leq c + 2$

$$q_{max} = \overset{\rightarrow 3}{c} + 2 = \underline{\underline{5}}$$

38)

ukjent: $v_u = \frac{1}{pT}$ $\xrightarrow[\ell]{\text{faseovergang}}$ gass: $v_g = \frac{T}{p}$

Faselikevektslinjen oppfyller

$$\frac{dp}{dT} = \frac{\ell}{T \Delta v}$$

$$\Delta v = v_g - v_u = \frac{T}{p} - \frac{1}{pT} = \frac{T^2 - 1}{pT}$$

$$\Rightarrow \frac{dp}{dT} = \frac{\ell}{T} \cdot \frac{pT}{T^2 - 1} = \frac{p\ell}{T^2 - 1}$$

$$\frac{1}{\ell} \int \frac{dp}{p} = \int \frac{dT}{T^2 - 1}$$

$$\frac{1}{\ell} \ln p = \frac{1}{2} \ln \frac{1 - T}{1 + T} + C$$

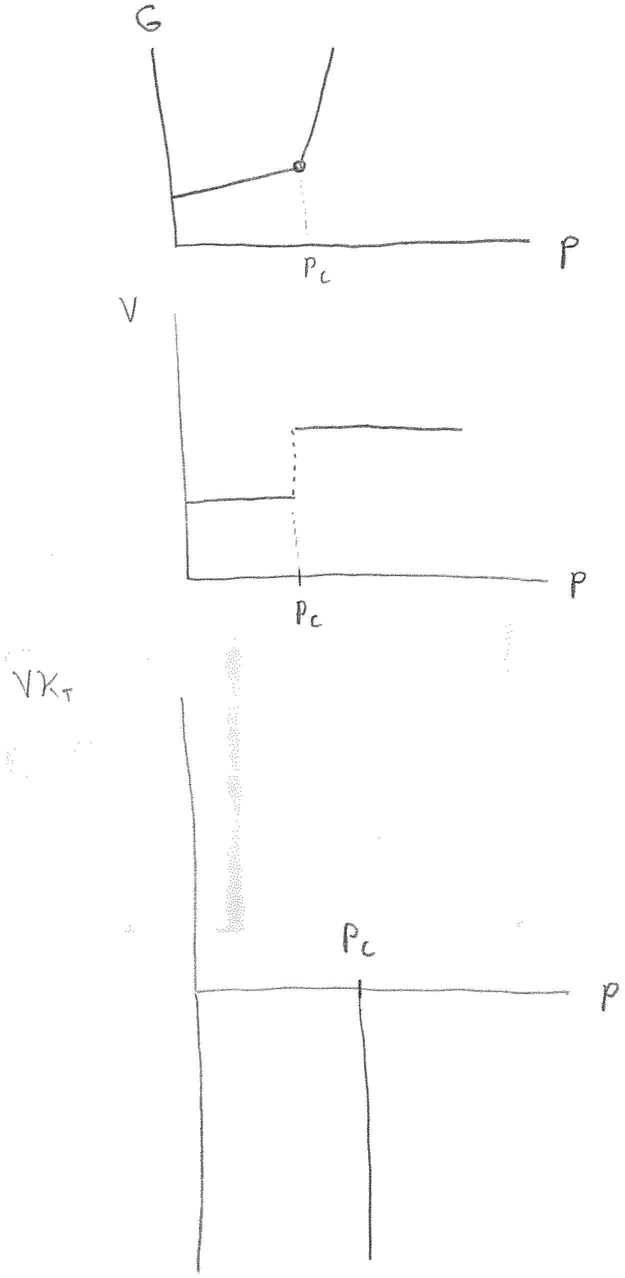
$$p(T = 0) = 1 \Rightarrow C = 0$$

$$\ln p = \ln \left(\frac{1 - T}{1 + T} \right)^{\ell/2}$$

$$\Rightarrow \underline{\underline{p = \left(\frac{1 - T}{1 + T} \right)^{\ell/2}}}$$

3a) $V = \left(\frac{\partial G}{\partial p} \right)$ Se tidligere

$V K_T = -\left(\frac{\partial V}{\partial p} \right)_T$ Se formelark



Figur A

40)
$$P = V^3 - TV^2 + V + 100$$

Kritisk temp: $\frac{\partial P}{\partial V} = \frac{\partial^2 P}{\partial V^2} = 0$

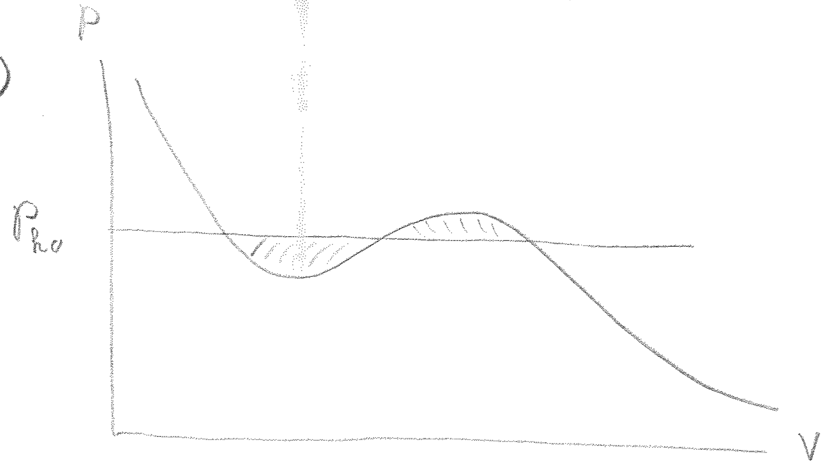
$$\frac{\partial P}{\partial V} = 3V^2 - 2TV + 1 \Rightarrow 3V_c^2 - 2T_cV_c + 1 = 0$$

$$\frac{\partial^2 P}{\partial V^2} = 6V - 2T \Rightarrow 6V_c - 2T_c = 0 \Rightarrow V_c = \frac{T_c}{3}$$

$$\Rightarrow 3\left(\frac{T_c}{3}\right)^2 - 2T_c \cdot \frac{T_c}{3} + 1 = 0$$

$$\frac{T_c^2}{3} - \frac{2T_c^2}{3} + 1 = -\frac{T_c^2}{3} + 1 = 0 \Rightarrow \underline{\underline{T_c = \sqrt{3}}}$$

41)



Kan brukes til å bestemme <u>koeksistenstrykket</u> mellom væske og gass.

42) $\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_H \longrightarrow \frac{\Delta T}{\Delta P}$

$$\Rightarrow \Delta T = \mu_{JT} \cdot \Delta P$$

Hvis $\Delta T < 0$ og $\Delta P < 0$ så må $\underline{\underline{\mu_{JT} > 0}}$

43)

Stasjonær Varmeligning:

$$0 = \nabla^2 T = \left(\underbrace{\frac{\partial^2}{\partial r^2}}_{0} + \underbrace{\frac{1}{r}\frac{\partial}{\partial r}}_{0} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \right) T = \frac{1}{r^2}\frac{d^2 T}{d\theta^2}$$

Symmetri:

$T = \underline{\underline{T(\theta)}}$

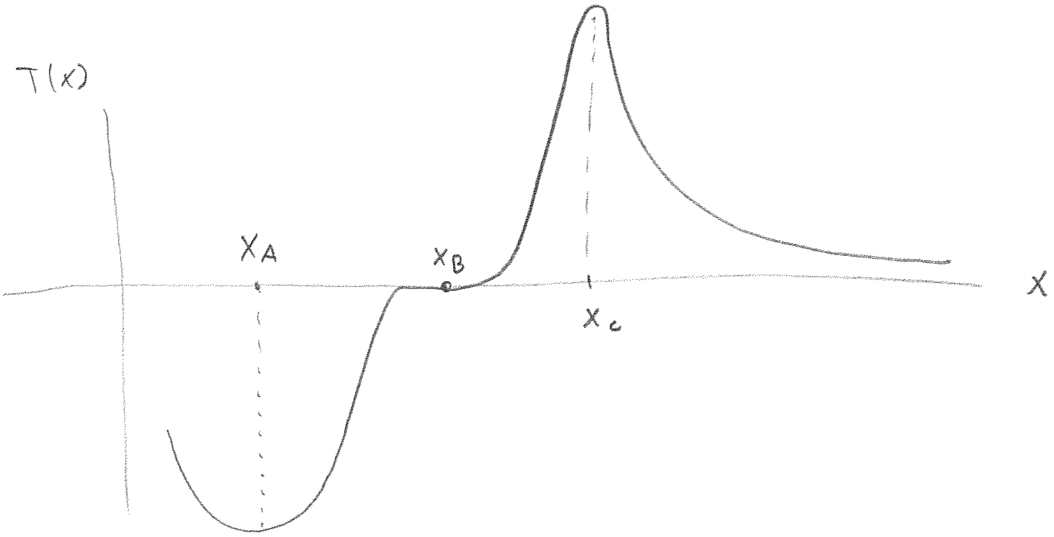
$\Rightarrow \quad T(\theta) = C\theta + D$

$T(0) = T_B = D$

$T(\pi) = T_A = C\pi + T_B \Rightarrow C = \frac{T_A - T_B}{\pi}$

$$\Rightarrow \quad \underline{\underline{T(\theta) = (T_A - T_B)\frac{\theta}{\pi} + T_B}}$$

44) $\frac{\partial T}{\partial t} = D_T \nabla^2 T$



A: $\frac{\partial T}{\partial t} > 0$ B: $\frac{\partial T}{\partial t} = 0$ C: $\frac{\partial T}{\partial t} < 0$

45) $\underline{\underline{\langle x \rangle = 0}}$ $\tilde{p}$ er symmetrisk om origo

$\sqrt{\langle x^2 \rangle} \propto \sqrt{t}$ for diffusion $\Rightarrow$ $\underline{\underline{\langle x^2 \rangle \propto t}}$